Gateway Regional School District

Atlas

Unit Plan

6.7 Rational Numbers

Gateway Regional Middle School / Grade 6 / Mathematics

Week 25 - Week 28 | 6 Curriculum Developers | Last Updated: Apr 8, 2024 by LeBlanc, Deanna

Style Guide

What is the purpose of the unit? What are the major take-aways?

Standards

MA: Mathematics (2017)

MA: Grade 6

Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

- [x] 7. Look for and make use of structure. Show Details
- [x] 8. Look for and express regularity in repeated reasoning. Show Details
- [x] 4. Model with mathematics. Show Details
- [x] 6. Attend to precision. Show Details
- [x] 1. Make sense of problems and persevere in solving them. Show Details
- [x] 2. Reason abstractly and quantitatively. Show Details
- [x] 3. Construct viable arguments and critique the reasoning of others. Show Details

The Number System

6.NS Compute fluently with multi-digit numbers and find common factors and multiples.

- [x] Use prime factorization to find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two relatively prime numbers. Show Details

6.NS Apply and extend previous understandings of numbers to the system of rational numbers.

- [x] 6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
- [x] 7. Understand ordering and absolute value of rational numbers.
- [x] 7c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. Show Details
- [x] 7d. Distinguish comparisons of absolute value from statements about order. Show Details
- [x] 8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
- [x] 6c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
- [x] 6b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
- [x] 6a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.
- [x] 5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge). Use positive and negative numbers (whole numbers, fractions, decimals) to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
- [x] 7b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. Show Details
- [x] 7a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. Show Details

Expressions & Equations

6.EE Apply and extend previous understandings of arithmetic to algebraic expressions.

- [x] 2b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. Show Details

6.EE Reason about and solve one-variable equations and inequalities.

- [x] 8. Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
- [x] 5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- [x] 6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Geometry

6.G Solve real-world and mathematical problems involving area, surface area, and volume.

- [x] 3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Enduring Understandings

- Rational numbers can be represented in various forms (fractions, decimals, and percents), and understanding these representations is crucial for modeling real-world scenarios and solving problems.
- Mathematical precision in expressing and operating with rational numbers is essential for clear communication and accurate calculations.

Essential Questions

- How can we compare and order rational numbers in various forms?
- What strategies can we use to add, subtract, multiply, and divide rational numbers effectively?
- How do rational numbers relate to real-world situations?

- Problems involving rational numbers can be complex and multifaceted; perseverance and strategic approaches are key to finding solutions.
- Rational numbers require abstract and quantitative reasoning, enabling students to make connections between conceptual understanding and numerical expression.
- Constructing arguments and critiquing others' reasoning with rational numbers fosters deeper understanding and mathematical discourse, emphasizing the importance of logic and evidence in mathematics.

- Why is it important to understand the properties of operations when working with rational numbers?
- How do we use rational numbers to represent quantities in different contexts?

Content

In this unit, students interpret signed numbers in contexts (e.g., temperature above or below zero, elevation above or below sea level). They understand and use the terms “positive number,” “negative number,” “rational number,” “opposite,” “sign,” “absolute value,” “a solution to an inequality,” “less than,” “greater than,” and the corresponding symbols. They plot points with signed rational number coordinates on the number line, and recognize and use the connection between relative position of two points on the number line and inequalities involving the coordinates of the points. (These are limited to strict inequalities rather than inequalities such as $2 \leq x$ which occur in grade 7.) They understand and use absolute value notation, understanding that the absolute value of a number as its distance from zero on the number line. Students graph inequalities in one variable on number line diagrams, using a circle or disk to indicate when a given point is, respectively, excluded or included. They solve simple inequalities, understanding that there may be infinitely many solutions, and show solutions symbolically and on the number line. They interpret solutions of inequalities in contexts, understanding that some solutions do not make sense in some contexts. Students plot pairs of signed number coordinates in the plane, understanding the relationship between the signs of a pair of coordinates and the quadrant of the corresponding point, and use coordinates to calculate horizontal and vertical distances between two points. Students understand and use the terms “common factor,” “greatest common factor,” “common multiple,” and “least common multiple,” and solve problems set in real-world contexts in which common factors or multiples occur.

In this unit, students are introduced to signed numbers and plot points in all four quadrants of the coordinate plane for the first time. They work with simple inequalities in one variable and learn to understand and use “common factor,” “greatest common factor,” “common multiple,” and “least common multiple.”

The first section of the unit introduces signed numbers. Students begin by considering examples of positive and negative temperatures, plotting each temperature on a vertical number line on which 0 is the only label. Next, they consider examples of positive and negative numbers used to denote height relative to sea level. In the second lesson, they plot positive and negative numbers on horizontal number lines, including “opposites”—pairs of numbers that are the same distance from zero. They use “less than,” “greater than,” and the corresponding symbols to describe the relationship of two signed numbers, noticing correspondences between the relative positions of two numbers on the number line and statements that use these symbols, e.g., $0.8 > -1.30$ means that 0.8 is to the right of -1.3 on the number line. Students learn that the sign

Skills

Student-facing learning targets

of a number indicates whether the number is positive or negative, and that zero has no sign. They learn that the absolute value of a number is its distance from zero, how to use absolute value notation, and that opposites have the same absolute value because they have the same distance from zero.

Previously, when students worked only with non-negative numbers, magnitude and order were indistinguishable: if one number was greater than another, then on the number line it was always to the right of the other number *and* always farther from zero. In comparing two signed numbers, students distinguish between magnitude (the absolute value of a number) and order (relative position on the number line), distinguishing between "greater than" and "greater absolute value," and "less than" and "smaller absolute value."

Students examine opposites of numbers, noticing that the opposite of a negative number is positive.
The second section of the unit concerns inequalities. Students graph simple inequalities in one variable on the number line, using a circle or disk to indicate when a given point is, respectively, excluded or included. In these materials, inequality symbols in grade 6 are limited to < and > rather than ≤≤ and ≥.≥. However, in this unit students encounter situations when they need to represent statements such as 2<x or 2=x.

Students represent situations that involve inequalities, symbolically and with the number line, understanding that there may be infinitely many solutions for an inequality. They interpret and graph solutions in contexts (MP2), understanding that some results do not make sense in some contexts, and thus the graph of a solution might be different from the graph of the related symbolic inequality. For example, the graph describing the situation "A fishing boat can hold fewer than 9 people" omits values other than the whole numbers from 0 to 8, but the graph of x<8 includes all numbers less than 8. Students encounter situations that require more than one inequality statement to describe, e.g., "It rained for more than 10 minutes but less than 30 minutes" (t>10 and t<30, where t is the amount of time that it rained in minutes) but which can be described by one number line graph.

The third section of the unit focuses on the coordinate plane. In grade 5, students learned to plot points in the coordinate plane, but they worked only with non-negative numbers, thus plotted points only in the first quadrant. In a previous unit, students again worked in the first quadrant of the coordinate plane, plotting points to represent ratio and other relationships between two quantities with positive values. In this unit, students work in all four quadrants of the coordinate plane, plotting pairs of signed number coordinates in the plane. They understand that for a given data set, there are more and less strategic choices for the scale and extent of a set of axes. They understand the correspondence between the signs of a pair of coordinates and the quadrant of the corresponding point. They interpret the meanings of plotted points in given contexts (MP2), and use coordinates to calculate horizontal and vertical distances between two points.

The last section of the unit returns to consideration of whole numbers. In the first lesson, students are introduced to "common factor" and "greatest common factor," and solve problems that illustrate how the greatest common factor of two numbers can be used in real-world situations, e.g., determining the largest

- I can explain what 0, positive numbers, and negative numbers mean in the context of temperature and elevation. (Lesson 1)
- I can use positive and negative numbers to describe temperature and elevation. (Lesson 1)
- I know what positive and negative numbers are. (Lesson 1)
- I can determine or approximate the value of any point on a number line. (Lesson 2)
- I can represent negative numbers on a number line. (Lesson 2)
- I understand what it means for numbers to be opposites. (Lesson 2)
- I can explain how to use the positions of numbers on a number line to compare them. (Lesson 3)
- I can explain what a rational number is. (Lesson 3)
- I can use inequalities to compare positive and negative numbers. (Lesson 3)
- I can compare and order rational numbers. (Lesson 4)
- I can use phrases like "greater than," "less than," and "opposite" to compare rational numbers. (Lesson 4)
- I can explain and use negative numbers in situations involving money. (Lesson 5)
- I can interpret and use negative numbers in different contexts. (Lesson 5)
- I can explain what the absolute value of a number is. (Lesson 6)
- I can find the absolute values of rational numbers. (Lesson 6)
- I can recognize and use the notation for absolute value. (Lesson 6)
- I can explain what absolute value means in situations involving elevation. (Lesson 7)
- I can use absolute values to describe elevations. (Lesson 7)
- I can use inequalities to compare rational numbers and the absolute values of rational numbers. (Lesson 7)
- I can graph inequalities on a number line. (Lesson 8)
- I can write an inequality to represent a situation. (Lesson 8)
- I can determine if a particular number is a solution to an inequality. (Lesson 9)
- I can explain what it means for a number to be a solution to an inequality. (Lesson 9)
- I can graph the solutions to an inequality on a number line. (Lesson 9)
- I can explain what the solution to an inequality means in a situation. (Lesson 10)
- I can write inequalities that involves more than one variable. (Lesson 10)
- I can describe a coordinate plane that has four quadrants. (Lesson 11)
- I can plot points with negative coordinates in the coordinate plane. (Lesson 11)
- I know what negative numbers in coordinates tell us. (Lesson 11)
- When given points to plot, I can construct a coordinate plane with an appropriate scale and pair of axes. (Lesson 12)
- I can explain how rational numbers represent balances in a money context. (Lesson 13)
- I can explain what points in a four-quadrant coordinate plane represent in a situation. (Lesson 13)
- I can plot points in a four-quadrant coordinate plane to represent situations and solve problems. (Lesson 13)
- I can find horizontal and vertical distances between points on the coordinate plane. (Lesson 14)

rectangular tile with whole-number dimensions that can tile a given rectangle with whole-number dimensions. The second lesson introduces “common multiple” and “least common multiple,” and students solve problems that involve listing common multiples or identifying common multiples of two or more numbers. In the third and last lesson, students solve problems that revisit situations similar to those in the first two lessons and identify which of the new concepts is involved in each problem. This lesson includes two optional classroom activities.

- I can find the lengths of horizontal and vertical segments in the coordinate plane. (Lesson 15)
- I can plot polygons on the coordinate plane when I have the coordinates for the vertices. (Lesson 15)
- I can explain what a common factor is. (Lesson 16)
- I can explain what the greatest common factor is. (Lesson 16)
- I can find the greatest common factor of two whole numbers. (Lesson 16)
- I can explain what a common multiple is. (Lesson 17)
- I can explain what the least common multiple is. (Lesson 17)
- I can find the least common multiple of two whole numbers. (Lesson 17)
- I can solve problems using common factors and multiples. (Lesson 18)
- I can use ordered pairs to draw a picture. (Lesson 19)

How will you gauge student learning?

Assessments

6.7 End-of-Unit Assessment | Summative | Written Test

Grade6-7-End-of-Unit-Assessment-assessment.pdf

8 State Standards Assessed

How will students learn?

Learning Activities

Inequalities	Lesson 8 • Coordinate verbal, algebraic, and number line representations of inequalities. • Critique (orally and in writing) possible values given for a situation with a constraint, including determining whether the boundary value is included and making sense of situations with discrete quantities. • Interpret phrases that describe a quantity constrained by a maximum or minimum acceptable value, e.g. “at least,” “at most,” “up to,” “more than,” “less than”, etc., and write an inequality statement to represent the constraint. Lesson 9 • Draw and label a number line diagram to represent the solutions to an inequality. • Recognize and explain (orally and in writing) that an inequality may have infinitely many solutions. • Use substitution to justify (orally) whether a given value is a “solution” to a given inequality. Lesson 10 • Critique (orally and in writing) possible values given for a situation with more than one constraint, including whether fractional or negative values are reasonable. • Interpret unbalanced hanger diagrams (orally and in writing) and write inequality statements to represent relationships between the weights on an unbalanced hanger diagram. • Write and interpret inequality statements that include more than one variable.
The Coordinate Plane	Lesson 11 • Generalize about the signs of coordinates that represent locations in each “quadrant” of the coordinate plane. • Plot a point given its coordinates or identify the coordinates of a given point on the coordinate plane. • Recognize that the axes of the coordinate plane can be extended to represent negative numbers. Lesson 12 • Choose and label appropriate scales for the axes of the coordinate plane, based on the coordinates to be plotted, and explain (orally and in writing) the choice. • Compare and contrast different scales for the axes of the coordinate plane. Lesson 13 • Compare points on a graph, including statements about relative position and the vertical distance between points. • Describe (using words and inequality symbols) and interpret the range of coordinates on a graph, including the meaning of y-values that are negative. • Identify and interpret points on a graph to answer questions about situations involving temperature or money. Lesson 14 • Compare and contrast (orally and in writing) the coordinates for points in different locations on the coordinate plane. • Determine the vertical or horizontal distance between two points on the coordinate plane that share the same x- or y-coordinate. • Generalize (orally) about the coordinates of points that are reflected across the x- or y-axis. Lesson 15 • Determine the total length of multiple horizontal and vertical segments in the coordinate plane that are connected end-to-end. • Draw a polygon in the coordinate plane given the coordinates for its vertices. • Explain (orally) that coordinates can be a useful way of describing geometric figures or modeling real-world locations.

Common Factors and Common Multiples	Lesson 16 • Comprehend (orally and in writing) the terms “factor,” “common factor,” and “greatest common factor.” • Explain (orally and in writing) how to determine the greatest common factor of two whole numbers less than 100. • List the factors of a number and identify common factors for two numbers in a real-world situation. Lesson 17 • Comprehend (orally and in writing) the terms “multiple,” “common multiple,” and “least common multiple.” • Explain (orally and in writing) how to calculate the least common multiple of 2 whole numbers. • List the multiples of a number and identify common multiples for two numbers in a real-world situation. Lesson 18 • Choose to calculate the greatest common factor or least common multiple to solve a problem about a real-world situation, and justify (orally) the choice. • Present (orally, in writing, and using other representations) the solution method for a problem involving greatest common factor or least common multiple. Lesson 19 • Generate a list of ordered pairs to create an image in the coordinate plane, and explain (orally) the reasoning.
Sections	Teacher-facing learning goals
Negative Numbers and Absolute Value	Lesson 1 • Comprehend the words “positive” and “negative” (in spoken and written language) and the symbol “-” (in written language). Say “negative” when reading numbers written with the “-” symbol. • Interpret positive and negative numbers that represent temperature or elevation, and understand the convention of what “below zero” typically means in each of these contexts. • Recognize that the number line can be extended to represent negative numbers. Lesson 2 • Comprehend that two numbers are called “opposites” when they are the same distance from zero, but on different sides of the number line. • Interpret a point on the number line that represents a positive or negative rational number. • Plot a point on a number line to represent a positive or negative rational number. Lesson 3 • Compare rational numbers in the context of temperature or elevation, and express the comparisons (in writing) using the symbols > and <. • Comprehend the word “sign” (in spoken language) to refer to whether a number is positive or negative. • Critique (orally and in writing) statements comparing rational numbers, including claims about relative position and claims about distance from zero. Lesson 4 • Compare rational numbers without a context and express the comparisons using the terms “greater than,” “less than,” and “opposite” (orally and in writing). • Comprehend that all negative numbers are less than all positive numbers. • Order rational numbers from least to greatest, and explain (orally and through other representations) the reasoning. Lesson 5 • Interpret a table of signed numbers that represent how a quantity changed. • Recognize that signed numbers can be useful to represent changes in a quantity in opposite directions, e.g., money received and money paid, inventory bought and inventory sold, etc. Lesson 6 • Compare rational numbers and their absolute values, and explain (orally and in writing) the reasoning. • Comprehend the phrase “absolute value” and the symbol |||| to refer to a number’s distance from zero on the number line. • Interpret rational numbers and their absolute values in the context of elevation or temperature. Lesson 7 • Critique comparisons (expressed using words or symbols) of rational numbers and their absolute values. • Generate values that meet given conditions for their relative position and absolute value, and justify the comparisons (using words and symbols). • Recognize that the value of -a can be positive or negative, depending on the value of a.

Differentiated Instruction

Technology Integration

21st Century Skills

Positive Behavior

CASEL

Collaborative for Academic, Social, and Emotional Learning

Resources

Teacher Notes and Reflections