Gateway Regional School District

Atlas

Unit Plan

6.6 Expressions and Equations

Gateway Regional Middle School / Grade 6 / Mathematics

Week 21 - Week 24 | 6 Curriculum Developers | Last Updated: Apr 8, 2024 by LeBlanc, Deanna

Style Guide

What is the purpose of the unit? What are the major take-aways?

Standards

MA: Mathematics (2017)
MA: Grade 6
Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

- [x] 7. Look for and make use of structure. Show Details
- [x] 8. Look for and express regularity in repeated reasoning. Show Details
- [x] 3. Construct viable arguments and critique the reasoning of others. Show Details

Ratios & Proportional Relationships

6.RP Understand ratio and rate concepts and use ratio and rate reasoning to solve problems.

- [x] 3c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

The Number System

6.NS Compute fluently with multi-digit numbers and find common factors and multiples.

- [x] 3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Expressions & Equations

6.EE Apply and extend previous understandings of arithmetic to algebraic expressions.

- [x] 4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). Show Details
- [x] 2c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). Show Details
- [x] 3. Apply the properties of operations to generate equivalent expressions. Show Details
- [x] 2. Write, read, and evaluate expressions in which letters stand for numbers.
- [x] 1. Write and evaluate numerical expressions involving whole-number exponents.
- [x] 2a. Write expressions that record operations with numbers and with letters standing for numbers. Show Details

6.EE Reason about and solve one-variable equations and inequalities.

- [x] 5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- [x] 6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- [x] 7. Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.

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Enduring Understandings

Enduring Understandings for the unit "6.6 Expressions and Equations":

- Mathematical expressions and equations are powerful tools that can represent real-world situations and help us solve problems involving quantities, including finding parts, wholes, and percentages.
- Understanding the structure of mathematical expressions and equations is essential for manipulating and solving them effectively, recognizing patterns, and predicting outcomes.
- Fluency with operations on multi-digit decimals (addition, subtraction, multiplication, and division) is crucial for accurately solving problems in mathematical and real-world contexts.
- The concept of percent as a rate per 100 provides a universal method for comparing quantities and solving problems related to discounts, tax, and percentages of amounts.
- Mathematical reasoning involves constructing coherent arguments, critiquing the reasoning of others, and justifying one's own reasoning through the use of expressions and equations, fostering a deeper understanding and ability to apply mathematics logically and creatively.
- Identifying regularity in repeated reasoning allows students to predict outcomes, simplify complex problems, and develop strategies for solving a wide range of problems.

Essential Questions

- How can we use variables to represent numbers and write expressions when solving a real-world or mathematical problem?
- What are the different ways to generate equivalent expressions and how can they be used to solve problems?
- How can understanding the properties of operations help in generating equivalent expressions?
- Why is it important to understand and use the correct order of operations in mathematical expressions?
- How can we use equations to represent and solve problems involving addition, subtraction, multiplication, and division?
- What does it mean for two expressions to be equivalent, and how can we determine equivalence?
- How can identifying and writing expressions and equations help us solve real-world problems involving quantitative relationships?

Content

Students begin the unit by working with linear equations that have single occurrences of one variable, . They represent relationships with tape diagrams and with linear equations, explaining correspondences between these representations. They examine values that make a given linear equation true or false, and what it means for a number to be a solution to an equation. Solving equations of the form px=q where p and q are rational numbers can produce complex fractions (i.e., quotients of fractions), so students extend their understanding of fractions to include those with numerators and denominators that are not whole numbers.

The second section introduces balanced and unbalanced “hanger diagrams” as a way to reason about solving the linear equations of the first section. Students write linear equations to represent situations, including situations with percentages, solve the equations, and interpret the solutions in the original contexts (MP2), specifying units of measurement when appropriate (MP6). They represent linear expressions with tape diagrams and use the diagrams to identify values of variables for which two linear expressions are equal. Students write linear expressions and represent them with area diagrams, noting the connection with the distributive property (MP7). They use the distributive property to write equivalent expressions.

In the third section of the unit, students write expressions with whole-number exponents and whole-number, fraction, or variable bases. They evaluate such expressions, using properties of exponents strategically (MP5). They understand that a solution to an equation in one variable is a number that makes the equation true when the number is substituted for all instances of the variable. They represent algebraic expressions and equations in order to solve problems. They determine whether pairs of numerical exponential expressions are equivalent and explain their reasoning (MP3). By examining a list of values, they find solutions for simple exponential equations, and simple quadratic and cubic equations.

In the last section of the unit, students represent collections of equivalent ratios as equations. They use and make connections between tables, graphs, and linear equations that represent the same relationships (MP1).

Skills

Student-facing learning targets

- I can tell whether or not an equation could represent a tape diagram. (Lesson 1)
- I can use a tape diagram to represent a situation. (Lesson 1)
- I can match equations to real life situations they could represent. (Lesson 2)
- I can replace a variable in an equation with a number that makes the equation true, and know that this number is called a solution to the equation. (Lesson 2)
- I can compare doing the same thing to the weights on each side of a balanced hanger to solving equations by subtracting the same amount from each side or dividing each side by the same number. (Lesson 3)
- I can explain what a balanced hanger and a true equation have in common. (Lesson 3)
- I can write equations that could represent the weights on a balanced hanger. (Lesson 3)
- I can explain why different equations can describe the same situation. (Lesson 4)
- I can solve equations that have whole numbers, fractions, and decimals. (Lesson 4)
- I understand the meaning of a fraction made up of fractions or decimals, like 2.10.072.10.07 or 45324532. (Lesson 5)
- When I see an equation, I can make up a story that the equation might represent, explain what the variable represents in the story, and solve the equation. (Lesson 5)
- I can use an expression that represents a situation to find an amount in a story. (Lesson 6)
- I can write an expression with a variable to represent a calculation where I do not know one of the numbers. (Lesson 6)
- I can solve percent problems by writing and solving an equation. (Lesson 7)
- I can explain what it means for two expressions to be equivalent. (Lesson 8)
- I can use a tape diagram to figure out when two expressions are equal. (Lesson 8)
- I can use what I know about operations to decide whether two expressions are equivalent. (Lesson 8)
- I can use a diagram of a rectangle split into two smaller rectangles to write different expressions representing its area. (Lesson 9)
- I can use the distributive property to help do computations in my head. (Lesson 9)
- I can use a diagram of a split rectangle to write different expressions with variables representing its area. (Lesson 10)
- I can use the distributive property to write equivalent expressions with variables. (Lesson 11 - *Optional*)
- I can evaluate expressions with exponents and write expressions with exponents that are equal to a given number. (Lesson 12)
- I understand the meaning of an expression with an exponent like 3535. (Lesson 12)
- I can decide if expressions with exponents are equal by evaluating the expressions or by understanding what exponents mean. (Lesson 13)
- I know how to evaluate expressions that have both an exponent and addition or subtraction. (Lesson 14)
- I know how to evaluate expressions that have both an exponent and multiplication or division. (Lesson 14)
- I can find solutions to equations with exponents in a list of numbers. (Lesson 15)
- I can replace a variable with a number in an expression with exponents and operations and use the correct order to evaluate the expression. (Lesson 15)
- I can create tables and graphs that show the relationship between two amounts in a given ratio. (Lesson 16)
- I can write an equation with variables that shows the relationship between two amounts in a given ratio. (Lesson 16)
- I can create tables and graphs to represent the relationship between distance and time for something moving at a constant speed. (Lesson 17)
- I can write an equation with variables to represent the relationship between distance and time for something moving at a constant speed. (Lesson 17)

- I can create tables and graphs that show different kinds of relationships between amounts. (Lesson 18 - *Optional*)
- I can write equations that describe relationships with area and volume. (Lesson 18 - *Optional*)
- I can create a table and a graph that represent the relationship in a given equation. (Lesson 19)
- I can explain what an equation tells us about the situation. (Lesson 19)

How will you gauge student learning?

Assessments

6.6 End-of-Unit Assessment | Summative | Written Test

10 State Standards Assessed

How will students learn?

Learning Activities

Equations in One Variable	Lesson 1 • Draw tape diagrams to represent equations of the forms x+p=q and px=q. • Interpret (orally and in writing) tape diagrams that represent equations of the form p+x=q or px=q. • Use tape diagrams to find unknown values in equations of the forms x+p=q and px=q and explain (orally) the solution method. Lesson 2 • Comprehend the word “variable” to refer to a letter standing in for a number and recognize that a coefficient next to a variable indicates multiplication (in spoken and written language). • Generate values that make an equation true or false and justify (orally and in writing) whether they are “solutions” to the equation. • Use substitution to determine whether a given number makes an equation true. Lesson 3 • Interpret hanger diagrams (orally and in writing) and write equations that represent relationships between the weights on a balanced hanger diagram. • Use balanced hangers to explain (orally and in writing) how to find solutions to equations of the form x+p=q or px=q. Lesson 4 • Interpret and coordinate sentences, equations, and diagrams that represent the same addition or multiplication situation. • Solve equations of the form x+p=q or px=q and explain (in writing) the solution method. Lesson 5 • Comprehend that the notation a/b can be used to represent division generally, and the numerator and denominator can include fractions, decimals, or variables. • Describe (orally) a situation that could be represented by a given equation of the form x+p=q or px=q. • Express division as a fraction (in writing) when solving equations of the form px=q.
Equal and Equivalent	Lesson 6 • Explain (orally) how to create and solve an equation that represents a situation with an unknown amount. • Write an expression with a variable to generalize the relationship between quantities in a situation. Lesson 7 • State explicitly what the chosen variable represents when creating an equation. • Use equations to solve problems involving percentages and explain (orally) the solution method. • Write equations of the form px=q or equivalent to represent situations where the amount that corresponds to 100% is unknown. Lesson 8 • Draw a diagram to represent the value of an expression for a given value of its variable. • Explain (in writing) that some pairs of expressions are equal for one value of their variable but not for other values. • Justify (orally, in writing, and through other representations) whether two expressions are “equivalent”, i.e., equal to each other for every value of their variable. Lesson 9 • Generate equivalent numerical expressions that are related by the distributive property, and explain (orally or using other representations) the reasoning. • Use an area diagram to make sense of equivalent numerical expressions that are related by the distributive property. Lesson 10 • Generate algebraic expressions that represent the area of a rectangle with an unknown length. • Justify (orally and using other representations) that algebraic expressions that are related by the distributive property are equivalent. Lesson 11 - Optional • Draw a diagram to justify that two expressions that are related by the distributive property are equivalent. • Explain (orally) how to use the distributive property to identify or generate equivalent algebraic expressions. • Use the distributive property to write equivalent algebraic expressions, including where the common factor is a variable.
Expressions with Exponents	Lesson 12 • Describe (orally and in writing) a pattern that could be expressed using repeated multiplication. • Generate and evaluate numerical expressions involving whole-number exponents. • Interpret expressions with exponents larger than 3, and comprehend the phrase “to the power” or “to the” (in spoken language). Lesson 13 • Critique (orally and in writing) arguments that claim two different numerical expressions are equal. • Justify (orally and in writing) whether numerical expressions involving whole-number exponents are equal. Lesson 14 • Evaluate numerical expressions that have an exponent and one other operation, and justify (orally) the process. • Explain (orally and in writing) that the convention is to evaluate the exponent before the other operations in an expression with no grouping symbols. • Interpret expressions with exponents that represent the surface area or volume of a cube. Lesson 15 • Describe (orally) the values that result from evaluating expressions in which a fraction is raised to a power. • Determine whether a given value is a solution to an equation that includes an exponent. • Evaluate expressions that have a variable, an exponent, and one other operation for a given value of the variable, carrying out the operations in the conventional order.

Relationships Between Quantities	Lesson 16 • Compare and contrast (orally) graphs and equations that represent a relationship between the same quantities but have the independent and dependent variables switched. • Comprehend the terms “independent variable” and “dependent variable” (in spoken and written language). • Create a table, graph, and equation to represent the relationship between quantities in a set of equivalent ratios. Lesson 17 • Create a table, graph, and equation to represent the relationship between distance and time for an object moving at a constant speed. • Identify (in writing) the independent and dependent variable in a equation. • Interpret (orally and in writing) an equation that represents the relationship between distance and time for an object moving at a constant speed. Lesson 18 - Optional • Coordinate (orally and in writing) graphs, tables, and equations that represent the same relationship. • Create an equation and a graph to represent the relationship between two variables that are inversely proportional. • Describe and interpret (orally and in writing) a graph that represents a nonlinear relationship between independent and dependent variables.
Let's Put it to Work	Lesson 19 • Create a verbal description and a graph to represent the relationship shown in an equation and table. • Identify tables and equations that represent the same relationship and justify (orally) the match. • Interpret and critique (orally) different representations of the same relationship, i.e. table, equation, graph, and verbal description
Section	Teacher-facing learning goals

Differentiated Instruction

Technology Integration

21st Century Skills

Positive Behavior

CASEL

Collaborative for Academic, Social, and Emotional Learning

Resources

Teacher Notes and Reflections