Gateway Regional School District

Atlas

Unit Plan

6.5 Arithmetic in Base Ten

Gateway Regional Middle School / Grade 6 / Mathematics

Week 18 - Week 20 | 6 Curriculum Developers | Last Updated: Apr 8, 2024 by LeBlanc, Deanna

Style Guide

What is the purpose of the unit? What are the major take-aways?

Standards

MA: Mathematics (2017)

MA: Grade 6

Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

- [x] 7. Look for and make use of structure. Show Details
- [x] 8. Look for and express regularity in repeated reasoning. Show Details
- [x] 4. Model with mathematics. Show Details
- [x] 2. Reason abstractly and quantitatively. Show Details
- [x] 3. Construct viable arguments and critique the reasoning of others. Show Details

Ratios & Proportional Relationships

6.RP Understand ratio and rate concepts and use ratio and rate reasoning to solve problems.

- [x] 3b. Solve unit rate problems including those involving unit pricing and constant speed. Show Details

The Number System

6.NS Compute fluently with multi-digit numbers and find common factors and multiples.

- [x] Use prime factorization to find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two relatively prime numbers. Show Details
- [x] 2. Fluently divide multi-digit numbers using the standard algorithm.
- [x] 3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Expressions & Equations

6.EE Apply and extend previous understandings of arithmetic to algebraic expressions.

- [x] 4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). Show Details
- [x] 3. Apply the properties of operations to generate equivalent expressions. Show Details
- [x] 2. Write, read, and evaluate expressions in which letters stand for numbers.
- [x] 1. Write and evaluate numerical expressions involving whole-number exponents.

Enduring Understandings

- Students will understand that the base ten system is a structured way of representing numbers, utilizing the place value concept to comprehend and perform arithmetic operations effectively.
- Students will recognize patterns and regularities in the base ten system that facilitate easier calculation and prediction of outcomes when performing arithmetic operations.
- Through the application of the base ten system, students will model real-world situations mathematically, demonstrating its utility beyond theoretical exercises.
- Students will develop the ability to reason abstractly and quantitatively by translating problems from real-world contexts into numerical expressions and vice versa, using the base ten system.
- Students will engage in constructing arguments to justify their mathematical reasoning and critique the reasoning of their peers to deepen their understanding of arithmetic operations within the base ten system.

Essential Questions

- How do we understand and apply the place value system to perform arithmetic operations including addition, subtraction, multiplication, and division with multi-digit whole numbers?
- How does understanding the base-ten system help us in solving real-world problems?
- What strategies can we use to estimate and mentally calculate sums, differences, products, and quotients?
- How do we use properties of operations to generate equivalent numerical expressions and solve problems?
- Why is it important to understand and apply the standard algorithm for multi-digit arithmetic operations?
- In what ways can we extend our understanding of arithmetic operations from base ten to other bases?

Content

By the end of grade 5, students learn to use efficient algorithms to fluently calculate sums, differences, and products of multi-digit whole numbers. They calculate quotients of multi-digit whole numbers with up to four-digit dividends and two-digit divisors. These calculations use strategies based on place value, the properties of operations, and the relationship between multiplication and division. Grade 5 students illustrate and explain these calculations with equations, rectangular arrays, and area diagrams.
In grade 5, students also calculate sums, differences, products, and quotients of decimals to hundredths, using concrete representations or drawings, and strategies based on place value, properties of operations, and the relationship between addition and subtraction. They connect their strategies to written methods and explain their reasoning.

In this unit, students learn an efficient algorithm for division and extend their use of other base-ten algorithms to decimals of arbitrary length. Because these algorithms rely on the structure of the base-ten system, students build on the understanding of place value and the properties of operations developed during earlier grades (MP7).

The unit begins with a lesson that revisits sums and differences of decimals to hundredths, and products of a decimal and whole number. The tasks are set in the context of shopping and budgeting, allowing students to be reminded of appropriate magnitudes for results of calculations with decimals.
The next section focuses on extending algorithms for addition, subtraction, and multiplication, which students used with whole numbers in earlier grades, to decimals of arbitrary length.

Students begin by using “base-ten diagrams,” diagrams analogous to base-ten blocks for ones, tens, and hundreds. These diagrams show, for example, ones as large squares, tenths as rectangles, hundredths as medium squares, thousandths as small rectangles, and ten-

Skills

Student-Facing Learning Targets:

- I can use decimals to make estimates and calculations about money. (Lesson 1)
- I can use diagrams to represent and reason about addition and subtraction of decimals. (Lesson 2 - *Optional)*
- I can use place value to explain addition and subtraction of decimals. (Lesson 2 - *Optional*)
- I can use vertical calculations to represent and reason about addition and subtraction of decimals. (Lesson 2 - *Optional*)
- I can tell whether writing or removing a zero in a decimal will change its value. (Lesson 3)
- I know how to solve subtraction problems with decimals that require “unbundling” or “decomposing.” (Lesson 3)
- I can solve problems that involve addition and subtraction of decimals. (Lesson 4)
- I can use place value and fractions to reason about multiplication of decimals. (Lesson 5)
- I can use area diagrams to represent and reason about multiplication of decimals. (Lesson 6)
- I know and can explain more than one way to multiply decimals using fractions and place value. (Lesson 6)
- I can use area diagrams and partial products to represent and find products of decimals. (Lesson 7)
- I can describe and apply a method for multiplying decimals. (Lesson 8)
- I know how to use a product of whole numbers to find a product of decimals. (Lesson 8)
- I can use the partial quotients method to find a quotient of two whole numbers when the quotient is a whole number. (Lesson 9)
- I can use long division to find a quotient of two whole numbers when the quotient is a whole number. (Lesson 10)
- I can use long division to find the quotient of two whole numbers when the quotient is not a whole number. (Lesson 11)
- I can divide a decimal by a whole number. (Lesson 12)

thousandths as small squares. These are designed so that the area of a figure that represents a base-ten unit is one tenth of the area of the figure that represents the base-ten unit of next highest value. Thus, a group of 10 figures that represent 10 like base-ten units can be replaced by a figure whose area is the sum of the areas of the 10 figures.
Students first calculate sums of two decimals by representing each number as a base-ten diagram, combining representations of like base-ten units and replacing representations of 10 like units by a representation of the unit of next highest value, e.g., 10 rectangles compose 1 large square. Next, they examine "vertical calculations," representations of calculations with symbols that show one summand above the other, with the sum written below. They check each vertical calculation by representing it with base-ten diagrams. This is followed by a similar lesson on subtraction of decimals. The section concludes with a lesson designed to illustrate efficient algorithms and their advantages, and to promote their use.

The third section, multiplication of decimals, begins by asking students to estimate products of a whole number and a decimal, allowing students to be reminded of appropriate magnitudes for results of calculations with decimals. In this section, students extend their use of efficient algorithms for multiplication from whole numbers to decimals. They begin by writing products of decimals as products of fractions, calculating the product of the fractions, then writing the product as a decimal. They discuss the effect of multiplying by powers of 0.1, noting that multiplying by 0.1 has the same effect as dividing by 10. Students use area diagrams to represent products of decimals. The efficient multiplication algorithms are introduced and students use them, initially supported by area diagrams.
In the fourth section, students learn long division. They begin with quotients of whole numbers, first representing these quotients with base-ten diagrams, then proceeding to efficient algorithms, initially supporting their use with base-ten diagrams. Students then tackle quotients of whole numbers that result in decimals, quotients of decimals and whole numbers, and finally quotients of decimals.

The unit ends with two lessons in which students use calculations with decimals to solve problems set in real-world contexts. These require students to interpret diagrams, and to interpret results of calculations in the contexts from which they arose (MP2). The second lesson draws on work with geometry and ratios from previous units. Students fold papers of different sizes to make origami boxes of different dimensions, then compare the lengths, widths, heights, and surface areas of the boxes.

- I can explain the division of a decimal by a whole number in terms of equal-sized groups. (Lesson 12)
- I know how multiplying both the dividend and the divisor by the same factor affects the quotient. (Lesson 12)
- I can explain how multiplying dividend and divisor by the same power of 10 can help me find a quotient of two decimals. (Lesson 13)
- I can find the quotient of two decimals. (Lesson 13)
- I can use addition, subtraction, multiplication, and division on decimals to solve problems. (Lesson 14)
- I can use the four operations on decimals to find surface areas and reason about real-world problems. (Lesson 15 - *Optional*)

How will you gauge student learning?

Assessments

6.5 End-of-Unit Assessment | Summative | Written Test

Grade6-5-End-of-Unit-Assessment-assessment.pdf

3 State Standards Assessed

How will students learn?

Learning Activities

Warming Up to Decimals	**Lesson 1** • Calculate sums and products of decimals in the context of money, and explain (orally and in writing) the calculation strategy • Estimate sums, differences, products, and quotients of decimals in the context of money, and explain (orally) the estimation strategy.
Adding and Subtracting Decimals	**Lesson 2 - Optional** • Compare and contrast (orally and in writing) vertical calculations and base-ten diagrams that represent adding and subtracting decimals. • Explain (in words and through other representations) that adding and subtracting decimals requires combining digits that represent like base-ten units. • Interpret and create diagrams that represent 10 like base-ten units being composed into 1 unit of higher place value, e.g., 10 tenths as 1 one, and comprehend the word “bundle” to refer to this concept. **Lesson 3** • Add or subtract decimals, and explain the reasoning (using words and other representations). • Comprehend the term “unbundle” means to decompose a larger base-ten unit into 10 units of lower place value (e.g., 1 tenth as 10 hundredths). • Recognize and explain (orally) that writing additional zeros or removing zeros after the last non-zero digit in a decimal does not change its value. **Lesson 4** • Add or subtract decimals with multiple non-zero digits, and explain (orally) the solution method. • Interpret a description (in written language) of a real-world situation involving decimals, and write an addition or subtraction problem to represent it. • Recognize and explain (orally) that vertical calculation is an efficient strategy for adding and subtracting decimals, especially decimals with multiple non-zero digits.
Multiplying Decimals	**Lesson 5** • Generalize (orally and in writing) that the number of decimal places in a product is related to the number of decimal places in the factors. • Justify (orally) the product of two decimals, which each have only one non-zero digit, by multiplying equivalent fractions that have a power of ten in the denominator. **Lesson 6** • Interpret different methods for computing the product of decimals, and evaluate (orally) their usefulness. • Justify (orally, in writing, and through other representations) where to place the decimal point in the product of two decimals with multiple non-zero digits. **Lesson 7** • Comprehend how the phrase “partial products” (in spoken and written language) refers to decomposing a multiplication problem. • Coordinate area diagrams and vertical calculations that represent the same decimal multiplication problem. • Use an area diagram to represent and justify (orally and in writing) how to find the product of two decimals. **Lesson 8** • Draw and label a diagram to check the answer to a decimal multiplication problem. • Interpret a description (in written language) of a real-world situation involving multiplication of decimals, and write a multiplication problem to represent it. • Use an algorithm to calculate the product of two decimals, and explain (orally) the solution method.

Dividing Decimals	Lesson 9 • Comprehend that the phrase “partial quotients” (in spoken and written language) refers to decomposing a division problem. • Divide whole numbers that result in a whole-number quotient, and explain the reasoning (using words and other representations). • Interpret different methods for computing the quotient of whole numbers, i.e., base-ten diagrams and partial quotients, and evaluate (orally) their usefulness. Lesson 10 • Interpret the long division method, and compare and contrast it (orally) with other methods for computing the quotient of whole numbers. • Recognize and explain (orally) that long division is an efficient strategy for dividing numbers, especially with multi-digit dividends. • Use long division to divide whole numbers that result in a whole-number quotient, and multiply the quotient by the divisor to check the answer. Lesson 11 • Interpret different methods for computing a quotient that is not a whole number, and express it (orally and in writing) in terms of “unbundling.” • Use long division to divide whole numbers that result in a quotient with a decimal, and explain (orally) the solution method. Lesson 12 • Compare and contrast (orally and using other representations) division problems with whole-number and decimal dividends • Divide decimals by whole numbers, and explain the reasoning (orally and using other representations). • Generalize (orally and in writing) that multiplying both the dividend and the divisor by the same factor does not change the quotient. Lesson 13 • Compare and contrast (orally and using other representations) division problems with whole-number and decimal divisors. • Divide whole numbers or decimals by decimals, and explain the reasoning (orally and using other representations), including choosing to divide a different expression that gets the same quotient. • Generate another division expression that has the same value as a given expression, and justify (orally) that they are equal.
Let's Put It to Work	Lesson 14 • Apply operations with decimals to solve problems about the dimensions of a sports field or court, and explain (orally, in writing, and using other representations) the solution method. • Choose whether an exact answer or an estimate is appropriate for a given problem. • Interpret a verbal description or diagram that represents the dimensions of a sports field or court. Lesson 15 - Optional • Apply operations with decimals to calculate the surface area of paper boxes. • Describe (orally) sources of measurement error, and justify an appropriate level of precision for reporting the answer. • Measure and compare (orally and in writing) the dimensions of paper boxes.
Section	Teacher-facing learning goals

Differentiated Instruction

Technology Integration

21st Century Skills

Positive Behavior

CASEL

Collaborative for Academic, Social, and Emotional Learning

Resources

Teacher Notes and Reflections