Gateway Regional School District

Atlas

Unit Plan

6.4 Dividing Fractions

Gateway Regional Middle School / Grade 6 / Mathematics

Week 14 - Week 17 | 6 Curriculum Developers | Last Updated: Mar 27, 2024 by LeBlanc, Deanna

Style Guide

What is the purpose of the unit? What are the major take-aways?

Standards

MA: Mathematics (2017)

MA: Grade 6

Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

- [x] 7. Look for and make use of structure. Show Details
- [x] 8. Look for and express regularity in repeated reasoning. Show Details
- [x] 4. Model with mathematics. Show Details
- [x] 6. Attend to precision. Show Details
- [x] 1. Make sense of problems and persevere in solving them. Show Details
- [x] 2. Reason abstractly and quantitatively. Show Details
- [x] 3. Construct viable arguments and critique the reasoning of others. Show Details

The Number System

6.NS Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

- [x] 1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. Show Details

Geometry

6.G Solve real-world and mathematical problems involving area, surface area, and volume.

- [x] 1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
- [x] 2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

Enduring Understandings

1. Make sense of problems and persevere in solving them:
- Students will understand that breaking down complex problems involving the division of fractions into simpler steps can lead to

Essential Questions

1. Make sense of problems and persevere in solving them:
- How can understanding the process of dividing fractions help you solve real-world problems?

effective solutions.
- They will recognize that perseverance is crucial when encountering initial difficulties in understanding and applying the process of dividing fractions.

2. Reason abstractly and quantitatively:
- Students will grasp that dividing fractions requires them to think abstractly about numbers and their relationships rather than relying solely on concrete representations.
- They will comprehend that quantitative reasoning is key in choosing and employing the most efficient methods for solving problems with dividing fractions.

3. Construct viable arguments and critique the reasoning of others:
- Students will understand the importance of forming logical arguments when presenting their methods for dividing fractions and be able to explain the reasoning behind each step.
- They will develop the skill to critically analyze and respectfully challenge the approaches and solutions proposed by their peers in dividing fractions.

4. Model with mathematics:
- Students will recognize that using mathematical models, such as visual fraction models and equations, can assist in understanding and representing the division of fractions in real-world contexts.
- They will understand that models are tools for interpreting results and that they can adapt these models to solve diverse problems involving the division of fractions.

6. Attend to precision:
- Students will appreciate the necessity of precision in the process of dividing fractions, including the use of mathematical vocabulary, notation, and the correct placement of numbers and operations.
- They will understand that precision in calculating and stating answers is vital in ensuring the accuracy and reliability of their results.

- What strategies can you use to interpret and solve division problems that involve fractions?

2. Reason abstractly and quantitatively:
- How can you use division of fractions to make sense of situations that involve sharing or partitioning quantities?
- How can we represent the division of fractions using models or diagrams?

3. Construct viable arguments and critique the reasoning of others:
- How can you explain the process of dividing fractions to someone who misunderstands it?
- What common misconceptions might arise when dividing fractions, and how can you address them?

4. Model with mathematics:
- How can you use models to illustrate and understand the division of fractions?
- In what ways do models clarify the concept of finding the reciprocal of a fraction as part of the division process?

6. Attend to precision:
- Why is it important to use precise mathematical language and notation when dividing fractions?
- How does attending to precision affect the accuracy of your solutions when dividing fractions?

Content

Work with fractions in grade 6 draws on earlier work in operations and algebraic thinking, particularly the knowledge of multiplicative situations developed in grades 3 to 5, and making use of the relationship between multiplication and division. Multiplicative situations include three types: equal groups; comparisons of two quantities; dimensions of arrays or rectangles. In the equal groups and comparison situations, there are two subtypes, sometimes called the partitive and the quotitive (or measurement) interpretations of division. Students are not expected to identify the three types of situations or use the terms “partitive” or “quotitive.” However, they should recognize the associated interpretations of division in specific contexts (MP7).

In a comparison situation that involves division, the size of one object may be unknown or the relative sizes of two objects may be unknown.

In situations that involve arrays or rectangles, division can be used to find an unknown factor. In an array situation, the unknown is the number of entries in a row or a column; in a rectangle, the unknown is a length or a width measurement.

Skills

Student-facing Learning Targets:

- When dividing, I know how the size of a divisor affects the quotient. (Lesson 1)
- I can explain how multiplication and division are related. (Lesson 2)
- I can explain two ways of interpreting a division expression such as 27÷327÷3. (Lesson 2)
- When given a division equation, I can write a multiplication equation that represents the same situation. (Lesson 2)
- I can create a diagram or write an equation that represents division and multiplication questions. (Lesson 3)
- I can decide whether a division question is asking “how many groups?” or “how many in each group?”. (Lesson 3)
- I can find how many groups there are when the amount in each group is not a whole number. (Lesson 4)
- I can use diagrams and multiplication and division equations to represent “how many groups?” questions. (Lesson 4)
- I can find how many groups there are when the number of groups and the amount in each group are not whole numbers. (Lesson 5)

At beginning of the unit, students consider how the relative sizes of numerator and denominator affect the size of their quotient. Students first compute quotients of whole numbers, then—without computing—consider the relative magnitudes of quotients that include divisors which are whole numbers, fractions, or decimals.

The second section of the unit focuses on equal groups and comparison situations. It begins with partitive and quotitive situations that involve whole numbers, represented by tape diagrams and equations. Students interpret the numbers in the two situations (MP2) and consider analogous situations that involve one or more fractions, again accompanied by tape diagrams and equations. Students learn to interpret, represent, and describe these situations, using mathematical terminology.

The third section concerns computing quotients of fractions. Students build on their work from the previous section by considering quotients related to products of numbers and unit fractions.

The fourth section returns to interpretations of division in situations that involve fractions. This time, the focus is on using division to find an unknown area or volume measurement. In grade 3, students connected areas of rectangles with multiplication, viewing a rectangle as tiled by an array of unit squares and understanding that, for whole-number side lengths, multiplying the side lengths yields the number of unit squares that tile the rectangle.

In a previous grade 6 unit, students used their familiarity with this formula to develop formulas for areas of triangles and parallelograms. In this unit, they return to this formula, using their understanding of it to extend the formula for the volume of a right rectangular prism (developed in grade 5) to right rectangular prisms with fractional side lengths.

The unit ends with two lessons in which students use what they have learned about working with fractions (including the volume formula) to solve problems set in real-world contexts, including a multi-step problem about calculating shipping costs. These require students to formulate appropriate equations that use the four operations or draw diagrams, and to interpret results of calculations in the contexts from which they arose (MP2).

- I can use a tape diagram to represent equal-sized groups and find the number of groups. (Lesson 6)
- I can tell when a question is asking for the number of groups and that number is less than 1. (Lesson 7)
- I can use diagrams and multiplication and division equations to represent and answer “what fraction of a group?” questions. (Lesson 7)
- I can tell when a question is asking for the amount in one group. (Lesson 8)
- I can use diagrams and multiplication and division equations to represent and answer “how much in each group?” questions. (Lesson 8)
- I can find the amount in one group in different real-world situations. (Lesson 9)
- I can divide a number by a non-unit fraction a/b by reasoning with the numerator and denominator, which are whole numbers. (Lesson 10)
- I can divide a number by a unit fraction 1/b by reasoning with the denominator, which is a whole number. (Lesson 10)
- I can describe and apply a rule to divide numbers by any fraction. (Lesson 11)
- I can use division and multiplication to solve problems involving fractional lengths. (Lesson 12)
- I can use division and multiplication to solve problems involving areas of rectangles with fractional side lengths. (Lesson 13)
- I can explain how to find the volume of a rectangular prism using cubes that have a unit fraction as their edge length. (Lesson 14)
- I can use division and multiplication to solve problems involving areas of triangles with fractional bases and heights. (Lesson 14)
- I know how to find the volume of a rectangular prism even when the edge lengths are not whole numbers. (Lesson 14)
- I can solve volume problems that involve fractions. (Lesson 15)
- I can use mathematical expressions to represent and solve word problems that involve fractions. (Lesson 16)
- I can use multiplication and division of fractions to reason about real-world volume problems. (Lesson 17)

How will you gauge student learning?

Assessments

6.4 End-of-Unit Assessment | Summative | Written Test

Grade6-4-End-of-Unit-Assessment-assessment.pdf

2 State Standards Assessed

How will students learn?

Learning Activities

Making Sense of Division	Lesson 1 • Comprehend the terms “dividend” and “divisor” (in spoken language) to refer to the numbers in a division problem. • Explain (orally) how to estimate quotients, by comparing the size of the dividend and divisor. • Generalize about the size of a quotient, i.e., predicting whether it is a very large number, a very small number, or close to 1. • Lesson 2 • Identify or generate a multiplication equation that represents the same relationship as a division expression, and explain (orally) the reasoning. • Interpret and create tape diagrams that represent situations involving equal-sized groups. • Recognize there are two different ways to interpret a division expression, i.e., asking “how many groups?” or “how many in each group?” Lesson 3 • Create an equation and a diagram to represent a multiplication or division situation involving fractions, and coordinate these representations (orally). • Explain (using words and other representations) how to find the unknown quantity in a multiplication or division situation involving fractions. • Interpret a verbal description of a multiplication situation (in spoken or written language), and identify which quantity is unknown, i.e., the number of groups, the amount in one group, or the total amount.
Meanings of Fraction Division	Lesson 4 • Coordinate multiplication equations and pattern block diagrams in which the yellow hexagon represents one whole. • Create a diagram to represent and solve a problem asking “How many groups?” in which the divisor is a unit fraction, and explain (orally) the solution method. Lesson 5 • Coordinate multiplication and division equations and pattern block diagrams in which the red trapezoid represents one whole. • Create a diagram to represent and solve a problem asking “How many groups?” in which the divisor is a non-unit fraction, and explain (orally) the solution method. • Identify or generate a multiplication or division equation that represents a given situation involving a fractional divisor. Lesson 6 • Explain (orally) how to create a tape diagram to represent and solve a problem asking “How many groups?” • Justify (orally and using other representations) the answer to a problem asking “How many groups?” in which the divisor is a non-unit fraction and the quotient is a fraction greater than 1. Lesson 7 • Comprehend the phrase “What fraction of a group?” (in spoken and written language) as a variation of the question “How many groups?” that is used when the quotient is less than 1. • Create a tape diagram to represent and solve a problem asking “How many groups?” in which the quotient is a fraction less than 1. • Write multiplication and division equations to represent a problem asking “How many times as long?” Lesson 8 • Compare and contrast (orally) strategies for solving problems about “how many groups?” and “how much in 1 group?” • Create a tape diagram to represent and solve a problem asking “How much in 1 group?” where the dividend, divisor, and quotient may be fractions, and explain (orally) the solution method. • Write multiplication and division equations to represent a problem asking “How much in 1 group?” Lesson 9 • Interpret a situation (presented in written language or using other representations) involving equal-sized groups, and generate mathematical questions that could be asked about it. • Solve a problem involving division of fractions, and present the solution method (orally, in writing, and using other representations).

Algorithm for Fraction Division	**Lesson 10** • Interpret and critique explanations (in spoken and written language, as well as in other representations) of how to divide by a fraction. • Use a tape diagram to represent dividing by a non-unit fraction a/b and explain (orally) why this produces the same result as multiplying the number by b and dividing by a. • Use a tape diagram to represent dividing by a unit fraction 1/b and explain (orally and in writing) why this is the same as multiplying by b. **Lesson 11** • Coordinate (orally) different strategies for dividing by a fraction. • Find the quotient of two fractions, and explain (orally, in writing, and using other representations) the solution method. • Generalize a process for dividing a number by a fraction, and justify (orally) why this can be abstracted as n·b/a.
Fractions in Lengths, Areas, and Volumes	**Lesson 12** • Apply dividing by fractions to solve a problem about comparing lengths or measuring with non-standard units, and explain (orally and in writing) the solution method. • Interpret a question (in written language) about multiplicative comparison, e.g., “How many times as long?” and write a division equation to represent it. **Lesson 13** • Apply dividing by fractions to calculate the side length of a rectangle, given its area and the other side length. • Coordinate (orally) diagrams and equations that represent the area of a rectangle with fractional side lengths. • Draw and label a diagram to justify the area of a rectangle with fractional side lengths. **Lesson 14** • Apply dividing by fractions to calculate the base or height of a triangle, given its area and the other measurement. • Determine the volume of a rectangular prism by counting how many 12-inch or 13-inch cubes it takes to build, and explain (orally and in writing) the solution method. • Generalize that the volume of a rectangular prism with fractional edge lengths can be found by multiplying the edge lengths. **Lesson 15** • Apply dividing by fractions to calculate one edge length of a rectangular prism, given its volume and the other two edge lengths. • Explain (orally, in writing, and using other representations) how to solve a problem involving the volume of a rectangular prism with fractional edge lengths. • Generalize that it takes more smaller cubes or fewer larger cubes to fill the same volume.
Let's Put it to Work	**Lesson 16** • Apply operations with fractions to solve problems in a variety of situations, and explain (orally and in writing) the reasoning. • Generate an equation to represent a situation involving fractions, and justify (orally) the operation chosen. **Lesson 17** • Compare and contrast (orally and using other representations) different ways jewelry boxes could be packed inside larger shipping boxes. • Determine which size shipping box is least expensive, and present (orally and in writing) a justification. • Make simplifying assumptions and determine what information is needed to solve a problem about shipping costs.
Section	**Teacher-facing learning goals**

Differentiated Instruction

Technology Integration

21st Century Skills

Positive Behavior

CASEL

Collaborative for Academic, Social, and Emotional Learning

Resources

Teacher Notes and Reflections