Gateway Regional School District

Atlas

Unit Plan

6.3 Unit Rates and Percentages

Gateway Regional Middle School / Grade 6 / Mathematics

Week 10 - Week 13 | 6 Curriculum Developers | Last Updated: Mar 26, 2024 by LeBlanc, Deanna

Style Guide

What is the purpose of the unit? What are the major take-aways?

Standards

MA: Mathematics (2017)

MA: Grade 6

Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

- [x] 7. Look for and make use of structure. Show Details
- [x] 8. Look for and express regularity in repeated reasoning. Show Details
- [x] 4. Model with mathematics. Show Details
- [x] 6. Attend to precision. Show Details
- [x] 1. Make sense of problems and persevere in solving them. Show Details
- [x] 2. Reason abstractly and quantitatively. Show Details
- [x] 3. Construct viable arguments and critique the reasoning of others. Show Details

Ratios & Proportional Relationships

6.RP Understand ratio and rate concepts and use ratio and rate reasoning to solve problems.

- [x] 3c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
- [x] 3a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
- [x] 3d. Use ratio reasoning to convert measurement units within and between measurement systems; manipulate and transform units appropriately when multiplying or dividing quantities. Show Details
- [x] 3b. Solve unit rate problems including those involving unit pricing and constant speed. Show Details
- [x] 3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- [x] 1. Understand the concept of a ratio including the distinctions between part:part and part:whole and the value of a ratio; part/part and part/whole. Use ratio language to describe a ratio relationship between two quantities. Show Details
- [x] 2. Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship, including the use of units. Show Details
- [x] 3.e. Solve problems that relate the mass of an object to its volume.

Geometry

6.G Solve real-world and mathematical problems involving area, surface area, and volume.

- [x] 1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
- [x] 3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
- [x] 4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.
- [x] 2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

Enduring Understandings

1. Understanding the Concept of Unit Rates:
Students will understand that a unit rate is a comparative rate that indicates the quantity of one item in relation to a single unit of another item. They will recognize the importance of determining and using unit rates to make comparisons and informed decisions in real-world situations. For example, finding the cost per ounce of different brands of the same product allows for a precise and fair comparison.

2. Applying Unit Rates to Solve Problems:
Students will comprehend that applying unit rates allows them to solve a variety of real-world problems with proficiency, demonstrating their ability to use mathematics as a tool for understanding and dealing with everyday situations, such as calculating the speed of a vehicle (miles per hour) or determining the efficiency of appliances (energy per use).

3. Grasping the Relevance of Percentages:
Students will gain the understanding that percentages are a critical component of mathematics, representing parts-per-hundred, which is essential in describing probability, discounts, interest rates, and population growth, among other things. They will learn to convert between percentages, decimals, and fractions to compute and compare quantities precisely.

4. Employing Precision in Calculations and Interpretations:
Students will understand the necessity of accuracy and attention to detail when computing and interpreting unit rates and percentages. They will learn to express their findings with clarity and precision to ensure their results are reliable and their communications are understood, whether they are analyzing data or making financial decisions.

5. Solving Problems Involving Rates and Percentages:
Students will develop the ability to make sense of problems involving unit rates and percentages and persevere in finding solutions. They will understand that breaking down complex problems into smaller, more manageable parts is a strategic approach to problem-solving and that estimation can serve as a powerful tool in checking the reasonableness of answers.

6. Quantitative Reasoning and Abstraction:
Students will understand that being able to reason abstractly and

Essential Questions

For Standard 4: Model with mathematics:
1. How can we use unit rates and percentages to model real-life situations like shopping, traveling, or cooking?
2. What are some strategies to accurately represent everyday problems involving unit rates or percentages mathematically?
3. How can the concept of unit rates and percentages help us make predictions and informed decisions in practical situations?

For Standard 6: Attend to precision:
1. Why is it important to be precise when calculating unit rates and percentages?
2. How can precision in our calculations of unit rates and percentages impact the outcome of a problem?
3. What tools or methods can we use to ensure that our work with unit rates and percentages is accurate and precise?

For Standard 1: Make sense of problems and persevere in solving them:
1. What steps can you take to understand a problem that involves unit rates or percentages before attempting to solve it?
2. How can you remain persistent when faced with a challenging problem that includes unit rates or percentages?
3. What strategies can be applied to monitor and adjust your approach when solving complex problems involving unit rates or percentages?

For Standard 2: Reason abstractly and quantitatively:
1. How can we use unit rates and percentages to make abstract concepts more concrete and understandable?
2. What is the relationship between the numbers in a problem involving unit rates or percentages and the context of the situation?
3. In what ways can reasoning quantitatively help us understand and interpret the meaning of unit rates and percentages in everyday life?

For Standard 3: Construct viable arguments and critique the reasoning of others:
1. How can you explain the reasoning behind your approach to solving a problem involving unit rates or percentages?
2. What evidence can you use to support your solutions to problems with unit rates or percentages?

quantitatively is crucial when working with unit rates and percentages. They will learn to decontextualize a problem by extracting the numerical data and to contextualize the solution by applying it back to the real-world scenario, thus demonstrating a deep comprehension of the concepts involved.

7. Constructing Viable Arguments:
Students will appreciate that constructing viable arguments and critiquing the reasoning of others are fundamental skills in mathematics and beyond. When discussing rates and percentages, they will understand how to present logical, well-reasoned explanations for their thinking and how to evaluate the arguments of others for precision and validity. This process helps in developing a deeper understanding of mathematical concepts and encourages continuous improvement in problem-solving techniques.

3. How can we respectfully critique the reasoning of others when discussing our methods and solutions involving unit rates and percentages?

Content

In the previous unit, students began to develop an understanding of ratios and rates. They started to describe situations using terms such as “ratio,” “rate,” “equivalent ratios,” “per,” “constant speed,” and “constant rate” (MP6). They understood specific instances of the idea that a:b is equivalent to every other ratio of the form sa:sb, where s is a positive number. They learned that “at this rate” or “at the same rate” signals a situation that is characterized by equivalent ratios. Although the usefulness of ratios of the form a/b:1 and 1:b/a was highlighted, the term “unit rate” was not introduced.
In this unit, students find the two values a/b and b/a that are associated with the ratio a:b, and interpret them as rates per 1. For example, if a person walks 13 meters in 10 seconds at a constant rate, that means they walked at a speed of 10/13 meters per 1 second and a pace of 10/13 seconds per 1 meter.
Students learn that one of the two values (a/b or b/a) may be more useful than the other in reasoning about a given situation. They find and use rates per 1 to solve problems set in contexts (MP2), attending to units and specifying units in their answers. For example, given item amounts and their costs, which is the better deal? Or given distances and times, which object is moving faster? Measurement conversions provide other opportunities to use rates.
Students observe that if two ratios a:b and c:d are equivalent, then a/b=c/d. The values a/b and c/d are called *unit rates* because they can be interpreted in the context from which they arose as rates per unit. Students note that in a table of equivalent ratios, the entries in one column are produced by multiplying a unit rate by the corresponding entries in the other column. Students learn that “percent” means “per 100” and indicates a rate. Just as a unit rate can be interpreted in context as a rate per 1, a percentage can be interpreted in the context from which it arose as a rate per 100. For example, suppose a beverage is made by mixing 1 cup of juice with 9 cups of water. The *percentage* of juice in 20 cups of the beverage is 2 cups and 10 *percent* of the beverage is juice. Interpreting the 10 as a rate: “there are 10 cups of juice per 100 cups of beverage” or, more generally, “there are 10 units of juice per 100 units of beverage.” The percentage—and the rate—indicate equivalent ratios of juice to beverage, e.g., 2 cups to 20 cups and 10 cups to 100 cups.
In this unit, tables and double number line diagrams are intended to help students connect percentages with equivalent ratios, and reinforce an understanding of percentages as rates per 100.
Students should internalize the meaning of important benchmark

Skills

Student-facing Learning Targets:

- I can see that thinking about “how much for 1” is useful for solving different types of problems. (Lesson 1)
- I can name common objects that are about as long as 1 inch, foot, yard, mile, millimeter, centimeter, meter, or kilometer. (Lesson 2 - *Optional)*
- I can name common objects that weigh about 1 ounce, pound, ton, gram, or kilogram, or that hold about 1 cup, quart, gallon, milliliter, or liter. (Lesson 2 - *Optional)*
- When I read or hear a unit of measurement, I know whether it is used to measure length, weight, or volume. (Lesson 2 - *Optional)*
- When I know a measurement in one unit, I can decide whether it takes more or less of a different unit to measure the same quantity. (Lesson 3)
- I can convert measurements from one unit to another, using double number lines, tables, or by thinking about “how much for 1.” (Lesson 4)
- I know that when we measure things in two different units, the pairs of measurements are equivalent ratios. (Lesson 4)
- I understand that if two ratios have the same rate per 1, they are equivalent ratios. (Lesson 5)
- When measurements are expressed in different units, I can decide who is traveling faster or which item is the better deal by comparing “how much for 1” of the same unit. (Lesson 5)
- I can choose which unit rate to use based on how I plan to solve the problem. (Lesson 6)
- When I have a ratio, I can calculate its two unit rates and explain what each of the means in the situation. (Lesson 6)
- I can give an example of two equivalent ratios and show that they have the same unit rates. (Lesson 7)
- I can multiply or divide by the unit rate to calculate missing values in a table of equivalent ratios. (Lesson 7)
- I can solve more complicated problems about constant speed situations. (Lesson 8)
- I can choose how to use unit rates to solve problems. (Lesson 9)
- I can create a double number line with percentages on one line and dollar amounts on the other line. (Lesson 10)
- I can explain the meaning of percentages using dollars and cents as an example. (Lesson 10)

percentages, for example, they should connect “75% of a number” with “3434 times a number” and “0.75 times a number.” Note that 75% (“seventy-five per hundred”) does not represent a fraction or decimal (which are numbers), but that “75% of a number” is calculated as a *fraction of* or a *decimal times* the number.
Work done in grades 4 and 5 supports learning about the concept of a percentage. In grade 5, students understand why multiplying a given number by a fraction less than 1 results in a product that is less than the original number, and why multiplying a given number by a fraction greater than 1 results in a product that is greater than the original number. This understanding of multiplication as scaling comes into play as students interpret,

- I can use double number line diagrams to solve different problems like “What is 40% of 60?” or “60 is 40% of what number?” (Lesson 11)
- I can use tape diagrams to solve different problems like “What is 40% of 60?” or “60 is 40% of what number?” (Lesson 12)
- When I read or hear that something is 10%, 25%, 50%, or 75% of an amount, I know what fraction of that amount they are referring to. (Lesson 13)
- I can choose and create diagrams to help me solve problems about percentages. (Lesson 14)
- I can solve different problems like “What is 40% of 60?” by dividing and multiplying. (Lesson 15)
- I can solve different problems like “60 is what percentage of 40?” by dividing and multiplying. (Lesson 16)
- I can apply what I have learned about unit rates and percentages to predict how long it will take and how much it will cost to paint all the walls in a room. (Lesson 17)

How will you gauge student learning?

Assessments

6.3 End-of-Unit Assessment | Summative | Written Test

Grade6-3-End-of-Unit-Assessment-assessment.pdf

6 State Standards Assessed

How will students learn?

Learning Activities

Units of Measurement	**Lesson 1** • Evaluate (orally) the usefulness of calculating a rate per 1 when solving problems involving unfamiliar rates. • Explain (orally, in writing, and through other representations) how to solve a problem involving rates in a less familiar context, e.g., minutes per window.
Unit Conversion	**Lesson 2 - Optional** • Compare (orally) the relative size of different units of measure for one attribute, i.e., length, volume, weight or mass. • Comprehend the approximate size of 1 “inch,” “foot,” “yard,” “mile,” “millimeter,” “centimeter,” “meter,” “kilometer,” “ounce,” “pound,” “ton,” “gram,” “kilogram,” “cup,” “quart,” “gallon,” “milliliter,” and “liter.” • Identify which unit is closest to the length, volume, weight, or mass of a given object, and explain (orally) the reasoning. **Lesson 3** • Generalize (orally and in writing) that it takes more of a smaller unit or fewer of a larger unit to measure the same quantity. • Given a measurement in one unit, estimate what would be the same amount expressed in a different unit, and explain (orally) the reasoning. **Lesson 4** • Choose and create a double number line diagram or table to solve problems involving unit conversion. • Explain (orally) how to use a “rate per 1” to solve problems involving unit conversion. • Recognize that when we measure things in two different units, the pairs of measurements are equivalent ratios.
Rates	**Lesson 5** • Explain (orally and in writing) that if two ratios have the same rate per 1, they are equivalent ratios. • Justify (orally and in writing) comparisons of speeds or prices. • Recognize that calculating how much for 1 of the same unit is a useful strategy for comparing rates. Express these rates (in spoken and written language) using the word “per” and specifying the unit. **Lesson 6** • Calculate and interpret the two unit rates associated with a ratio, i.e., ab�/ and ba/ for the ratio a:b:. • Choose which unit rate to use to solve a given problem and explain the choice (orally and in writing). • Comprehend the term “unit rate” (in spoken and written language) refers to a rate per 1. **Lesson 7** • Apply reasoning about unit rates to complete a table of equivalent ratios, and explain (orally and in writing) the solution method. • Explain (orally) that if two ratios are equivalent, they have the same rate per 1. • Generalize that the unit rate is the factor that takes you from one column to the other column in a table of equivalent ratios. **Lesson 8** • Calculate unit rates that represent speed or pace, use them to determine unknown distances or elapsed times, and explain (orally) the solution method. • Interpret a verbal (written) description of a situation involving two objects moving at constant speeds, and create a diagram or table to represent the situation. **Lesson 9** • Apply reasoning about ratios and rates to convert and compare (in writing) distances expressed in different units. • Apply reasoning about ratios and rates to justify (orally) whether a given price is a good deal. • Practice grade 5 arithmetic with fractions and decimals.

Percentages	**Lesson 10** • Comprehend the word “percentage” (in written and spoken language) and the symbol “%” (in written language) to mean a rate per 100. • Draw and label a double number line diagram to represent percentages of a dollar and to find corresponding monetary values or percentages. **Lesson 11** • Comprehend a phrase like “A% of B” (in written and spoken language) to refer to the value that makes a ratio with B that is equivalent to A : 100. • Explain (orally) how to use a double number line diagram or table to solve problems such as A% of B is ? and A% of ? is C. • State explicitly what one is finding the percentage of. **Lesson 12** • Choose and create diagrams to solve problems such as A% of B is ? and A% of ? is C. • Draw and label a tape diagram to represent a situation involving percentages. • Interpret tape diagrams that represent multiplicative comparisons and express such comparisons using fractions and percentages. **Lesson 13** • Explain (orally and in writing) how to solve problems involving the percentages 10%, 25%, 50%, and 75% by reasoning about the fractions 110110, 1414, 1212, and 3434. • Generalize (orally) processes for calculating 10%, 25%, 50%, and 75% of a quantity. **Lesson 14** • Choose and create a tape diagram, double number line diagram, or table to solve problems involving percentages and explain (orally) the solution method. • Determine what information is needed to solve a problem involving percentages. Ask questions to elicit that information. **Lesson 15** • Choose and create diagrams to calculate A% of B, and explain (orally) the solution method. • Generalize a process for finding A% of B and justify (orally) why this can be abstracted as A100·B◆100·◆. • Identify equivalent expressions that could be used to find A% of B and justify (orally) that they are equivalent. **Lesson 16** • Critique or justify (orally) statements about percentages and equivalent numerical expressions. • Generalize a process for finding the percentage that C is of B and justify (orally) why this can be abstracted as CB·100◆◆·100.
Let's Put it to Work	**Lesson 17** • Apply rates and percentages to calculate how long it will take and how much it will cost to complete a painting project, and explain (orally) the reasoning. • Make simplifying assumptions and determine what information is needed to solve a problem about painting a room.
Section	Teacher-facing learning goals

Differentiated Instruction

Technology Integration

21st Century Skills

Positive Behavior

CASEL

Collaborative for Academic, Social, and Emotional Learning

Resources

Teacher Notes and Reflections