Gateway Regional School District

Atlas

Unit Plan

6.2 Introducing Ratios

Gateway Regional Middle School / Grade 6 / Mathematics

Week 6 - Week 9 | 6 Curriculum Developers | Last Updated: Mar 26, 2024 by LeBlanc, Deanna

Style Guide

What is the purpose of the unit? What are the major take-aways?

Standards

MA: Mathematics (2017)

MA: Grade 6

Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

- [x] 7. Look for and make use of structure. Show Details
- [x] 8. Look for and express regularity in repeated reasoning. Show Details
- [x] 5. Use appropriate tools strategically. Show Details
- [x] 6. Attend to precision. Show Details
- [x] 2. Reason abstractly and quantitatively. Show Details

Ratios & Proportional Relationships

6.RP Understand ratio and rate concepts and use ratio and rate reasoning to solve problems.

- [x] 3a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
- [x] 3b. Solve unit rate problems including those involving unit pricing and constant speed. Show Details
- [x] 3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- [x] 1. Understand the concept of a ratio including the distinctions between part:part and part:whole and the value of a ratio; part/part and part/whole. Use ratio language to describe a ratio relationship between two quantities. Show Details
- [x] 2. Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship, including the use of units. Show Details

Enduring Understandings

1. Understanding the Structure of Ratios:
Students will recognize that ratios are an inherent part of the structure of various mathematical and real-world contexts. Ratios compare two quantities and can be used to express the relationship between parts of a whole or between two distinct but related quantities.

Essential Questions

Standard 7: Look for and make use of structure.
1. How can recognizing patterns help us understand ratios and their properties?
2. How does the structure of a ratio help us to compare different quantities?
3. What structures do you notice when you create equivalent ratios?

2. Recognizing Patterns and Regularity in Ratios:
Students will understand that ratios often reveal patterns and regularities, which can be utilized to predict outcomes, find equivalent relationships, and simplify complex problems. Through repeated use and examination of ratios, students will develop the ability to spot and apply these patterns in various contexts.

3. Strategic Use of Tools in Ratios:
Students will learn to strategically select and apply appropriate mathematical tools, such as ratio tables, tape diagrams, double number lines, and scales, to investigate and understand ratios. These tools will be instrumental in modeling, comparing, and solving problems involving ratios.

4. Precision in Articulating Ratios:
Students will appreciate the importance of precision when expressing ratios, ensuring a clear and exact understanding of the relationship between quantities. Precision includes correctly stating ratios in their simplest form, selecting the appropriate units, and using accurate language to communicate ratio relationships.

5. Abstract and Quantitative Reasoning with Ratios:
Students will become proficient at reasoning about ratios both abstractly and quantitatively. This includes interpreting and constructing ratio statements from concrete situations, representing ratios using various forms (e.g., a to b, a:b, a/b), and translating between these representations to solve problems and analyze information.

Standard 8: Look for and express regularity in repeated reasoning.
1. How can identifying and using regularity in numerical patterns help in solving problems involving ratios?
2. When comparing ratios, what repetitive reasoning can we use to ensure the ratios are equivalent?
3. How does repeated reasoning help us predict the outcomes of scaled quantities?

Standard 5: Use appropriate tools strategically.
1. What tools can we use to represent ratios visually and numerically?
2. How can we use tables, graphs, or diagrams to better understand and solve ratio problems?
3. When is it appropriate to use technology, such as a calculator, to compute ratios or equivalent ratios?

Standard 6: Attend to precision.
1. How accurate do our ratio comparisons need to be in different real-world contexts?
2. In what ways is precision important when measuring and calculating ratios?
3. How can we use precise mathematical language to describe the relationships between ratios?

Standard 2: Reason abstractly and quantitatively.
1. How can we represent a ratio abstractly and how does this help us understand real-world quantitative relationships?
2. What does it mean to reason quantitatively with ratios when faced with a practical problem?
3. When solving ratio problems, how can we move between the concrete and the abstract to better understand the concept?

Content

Work with ratios in grade 6 draws on earlier work with numbers and operations. In elementary school, students worked to understand, represent, and solve arithmetic problems involving quantities with the same units. In grade 4, students began to use two-column tables, e.g., to record conversions between measurements in inches and yards. In grade 5, they began to plot points on the coordinate plane, building on their work with length and area. These early experiences were a brief introduction to two key representations used to study relationships between quantities, a major focus of work that begins in grade 6 with the study of ratios.
Starting in grade 3, students worked with relationships that can be expressed in terms of ratios and rates (e.g., conversions between measurements in inches and in yards), however, they did not use these terms. In grade 4, students studied multiplicative comparison. In grade 5, they began to interpret multiplication as scaling, preparing them to think about simultaneously scaling two quantities by the same factor. They learned what it means to divide one whole number by another, so they are well equipped to consider the quotients ab/ and ba/ associated with a ratio a:b for non-zero whole numbers a and b.
In this unit, students learn that a ratio is an association between two quantities, e.g., “1 teaspoon of drink mix to 2 cups of water.” Students analyze contexts that are often expressed in terms of ratios,

Skills

Student-facing Learning Targets:

- I can write or say a sentence that describes a ratio. (Lesson 1)
- I know how to say words and numbers in the correct order to accurately describe the ratio. (Lesson 1)
- I can draw a diagram that represents a ratio and explain what the diagram means. (Lesson 2)
- I include labels when I draw a diagram representing a ratio, so that the meaning of the diagram is clear. (Lesson 2)
- I can explain the meaning of equivalent ratios using a recipe as an example. (Lesson 3)
- I can use a diagram to represent a recipe, a double batch, and a triple batch of a recipe. (Lesson 3)
- I know what it means to double or triple a recipe. (Lesson 3)
- I can explain the meaning of equivalent ratios using a color mixture as an example. (Lesson 4)
- I can use a diagram to represent a single batch, a double batch, and a triple batch of a color mixture. (Lesson 4)
- I know what it means to double or triple a color mixture. (Lesson 4)
- If I have a ratio, I can create a new ratio that is equivalent to it. (Lesson 5)
- If I have two ratios, I can decide whether they are equivalent to each other. (Lesson 5)

such as recipes, mixtures of different paint colors, constant speed (an association of time measurements with distance measurements), and uniform pricing (an association of item amounts with prices).
One of the principles that guided the development of these materials is that students should encounter examples of a mathematical concept in various contexts before the concept is named and studied as an object in its own right. The development of ratios, equivalent ratios, and unit rates in this unit and the next unit is in accordance with that principle. In this unit, equivalent ratios are first encountered in terms of multiple batches of a recipe and "equivalent" is first used to describe a perceivable sameness of two ratios, for example, two mixtures of drink mix and water taste the same or two mixtures of red and blue paint are the same shade of purple. Building on these experiences, students analyze situations involving both discrete and continuous quantities, and involving ratios of quantities with units that are the same and that are different. Several lessons later, *equivalent* acquires a more precise meaning (MP6): All ratios that are equivalent to a:b can be made by multiplying both a and b by the same non-zero number (note that students are not yet considering negative numbers).
This unit introduces *discrete diagrams* and *double number line diagrams*, representations that students use to support thinking about equivalent ratios before their work with tables of equivalent ratios.

Initially, discrete diagrams are used because they are similar to the kinds of diagrams students might have used to represent multiplication in earlier grades. Next come double number line diagrams. These can be drawn more quickly than discrete diagrams, but are more similar to tables while allowing reasoning based on the lengths of intervals on the number lines. After some work with double number line diagrams, students use tables to represent equivalent ratios. Because equivalent pairs of ratios can be written in any order in a table and there is no need to attend to the distance between values, tables are the most flexible and concise of the three representations for equivalent ratios, but they are also the most abstract. Use of tables to represent equivalent ratios is an important stepping stone toward use of tables to represent linear and other functional relationships in grade 8 and beyond. Because of this, students should learn to use tables to solve all kinds of ratio problems, but they should always have the option of using discrete diagrams and double number line diagrams to support their thinking.
When a ratio involves two quantities with the same units, we can ask and answer questions about ratios of each quantity and the total of the two. Such ratios are sometimes called "part-part-whole" ratios and are often used to introduce ratio work. However, students often struggle with them so, in this unit, the study of part-part-whole ratios occurs at the end. (Note that tape diagrams are reserved for ratios in which all quantities have the same units.) The major use of part-part-whole ratios occurs with certain kinds of percentage problems, which comes in the next unit.
On using the terms ratio, rate, and proportion. In these materials, a *quantity* is a measurement that is or can be specified by a number and a unit, e.g., 4 oranges, 4 centimeters, "my height in feet," or "my height" (with the understanding that a unit of measurement will need to be chosen). The term *ratio* is used to mean an association between two or more quantities and the fractions a/b and b/a are never called ratios. Ratios of the form 1:b/a or a/b:1 (which are equivalent to a:b) are highlighted as useful but a/b and b/a are not identified as *unit rates* for the ratio a:b until the next unit. However, the meanings of these fractions in contexts is very carefully developed. The word

- I can label a double number line diagram to represent batches of a recipe or color mixture. (Lesson 6)
- When I have a double number line that represents a situation, I can explain what it means. (Lesson 6)
- I can create a double number line diagram and correctly place and label tick marks to represent equivalent ratios. (Lesson 7)
- I can explain what the word per means. (Lesson 7)
- I can choose and create diagrams to help me reason about prices. (Lesson 8)
- I can explain what the phrase "at this rate" means, using prices as an example. (Lesson 8)
- If I know the price of multiple things, I can find the price per thing. (Lesson 8)
- I can choose and create diagrams to help me reason about constant speed. (Lesson 9)
- If I know an object is moving at a constant speed, and I know two of these things: the distance it travels, the amount of time it takes, and its speed, I can find the other thing. (Lesson 9)
- I can decide whether or not two situations are happening at the same rate. (Lesson 10)
- I can explain what it means when two situations happen at the same rate. (Lesson 10)
- I know some examples of situations where things can happen at the same rate. (Lesson 10)
- If I am looking at a table of values, I know where the rows are and where the columns are. (Lesson 11)
- When I see a table representing a set of equivalent ratios, I can come up with numbers to make a new row. (Lesson 11)
- When I see a table representing a set of equivalent ratios, I can explain what the numbers mean.(Lesson 11)
- I can solve problems about situations happening at the same rate by using a table and finding a "1" row. (Lesson 12)
- I can use a table of equivalent ratios to solve problems about unit price. (Lesson 12)
- I can create a table that represents a set of equivalent ratios. (Lesson 13)
- I can explain why sometimes a table is easier to use than a double number line to solve problems involving equivalent ratios. (Lesson 13)
- I include column labels when I create a table, so that the meaning of the numbers is clear. (Lesson 13)
- I can decide what information I need to know to be able to solve problems about situations happening at the same rate. (Lesson 14)
- I can explain my reasoning using diagrams that I choose. (Lesson 14)
- I can create tape diagrams to help me reason about problems involving a ratio and a total amount. (Lesson 15)
- I can solve problems when I know a ratio and a total amount. (Lesson 15)
- I can choose and create diagrams to help think through my solution. (Lesson 16)
- I can solve all kinds of problems about equivalent ratios. (Lesson 16)
- I can use diagrams to help someone else understand why my solution makes sense. (Lesson 16)
- I can apply what I have learned about ratios and rates to solve a more complicated problem. (Lesson 17)
- I can decide what information I need to know to be able to solve a real-world problem about ratios and rates. (Lesson

"per" is used with students in interpreting a unit rate in context, as in "$3 per ounce," and "at the same rate" is used to signify a situation characterized by equivalent ratios.
In the next unit, students learn the term "unit rate" and that if two ratios a:b and c:d are equivalent, then the unit rates a/b and c/d are equal.
The terms *proportion* and *proportional relationship* are not used anywhere in the grade 6 materials. A proportional relationship is a collection of equivalent ratios, and such collections are objects of study in grade 7. In high school—after their study of ratios, rates, and proportional relationships—students discard the term "unit rate," referring to a to b, a:b, and a/b as "ratios."

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How will you gauge student learning?

Assessments

6.2 End-of-Unit Assessment | Summative | Written Test

Grade6-2-End-of-Unit-Assessment-assessment.pdf

4 State Standards Assessed

How will students learn?

Learning Activities

What are Ratios?	**Lesson 1** • Comprehend the word “ratio” (in written and spoken language) and the notation a:b (in written language) to refer to an association between quantities. • Describe (orally and in writing) associations between quantities using the language “For every a of these, there are b of those” and “The ratio of these to those is a:b (or a to b).” **Lesson 2** • Coordinate discrete diagrams and multiple written sentences describing the same ratios. • Draw and label discrete diagrams to represent situations involving ratios. • Practice reading and writing sentences describing ratios.
Equivalent Ratios	**Lesson 3** • Draw and label a discrete diagram with circled groups to represent multiple batches of a recipe. • Explain equivalent ratios (orally and in writing) in terms of different sized batches of the same recipe having the same taste. • Understand that doubling or tripling a recipe involves multiplying the amount of each ingredient by the same number, yielding something that tastes the same. **Lesson 4** • Comprehend and respond (orally and in writing) to questions asking whether two ratios are equivalent, in the context of color mixtures. • Draw and label a discrete diagram with circled groups to represent multiple batches of a color mixture. • Explain equivalent ratios (orally and in writing) in terms of the amounts of each color in a mixture being multiplied by the same number to create another mixture that is the same shade. **Lesson 5** • Generate equivalent ratios and justify that they are equivalent. • Present (in words and through other representations) a definition of equivalent ratios, including examples and non-examples.
Representing Equivalent Ratios	**Lesson 6** • Compare and contrast (orally and in writing) discrete diagrams and double number line diagrams representing the same situation. • Explain (orally) how to use a double number line diagram to find equivalent ratios. • Label and interpret a double number line diagram that represents a familiar context. **Lesson 7** • Comprehend and use the word “per” (orally and in writing) to mean “for each.” • Draw and label a double number line diagram from scratch, with parallel lines and equally-spaced tick marks. • Use double number line diagrams to find a wider range of equivalent ratios. **Lesson 8** • Calculate equivalent ratios between prices and quantities and present the solution method (using words and other representations). • Calculate unit price and express it using the word “per” (orally and in writing). • Understand the phrase “at this rate” indicates that equivalent ratios are involved. **Lesson 9** • Calculate the distance an object travels in 1 unit of time and express it using a phrase like “meters per second” (orally and in writing). • For an object moving at a constant speed, use a double number line diagram to represent equivalent ratios between the distance traveled and elapsed time. • Justify (orally and in writing) which of two objects is moving faster, by identifying that it travels more distance in the same amount of time or that it travels the same distance in less time. **Lesson 10** • Choose and create diagrams to help compare two situations and explain whether they happen at the same rate. • Justify that two situations do not happen at the same rate by finding a ratio to describe each situation where the two ratios share one value but not the other. • Recognize that a question asking whether two situations happen “at the same rate” is asking whether the ratios are equivalent.

Solving Ratio and Rate Problems	**Lesson 11** • Comprehend the words “row” and “column” (in written and spoken language) as they are used to describe a table of equivalent ratios. • Explain (orally and in writing) how to find a missing value in a table of equivalent ratios. • Interpret a table of equivalent ratios that represents different sized batches of a recipe. **Lesson 12** • Choose multipliers strategically while solving multi-step problems involving equivalent ratios. • Describe (orally and in writing) how a table of equivalent ratios was used to solve a problem about prices and quantities. • Remember that dividing by a whole number is the same as multiplying by an associated unit fraction. **Lesson 13** • Compare and contrast (orally) double number line diagrams and tables representing the same situation. • Draw and label a table of equivalent ratios from scratch to solve problems about constant speed. **Lesson 14** • Determine what information is needed to solve a problem involving equivalent ratios. Ask questions to elicit that information. • Understand the structure of a what-why info gap activity.
Part-part-whole Ratios	**Lesson 15** • Comprehend the word “parts” as an unspecified unit in sentences (written and spoken) describing ratios. • Draw and label a tape diagram to solve problems involving ratios and the total amount. Explain (orally) the solution method. **Lesson 16** • Choose and create diagrams to help solve problems involving ratios and the total amount. • Compare and contrast (orally) different representations of and solution methods for the same problem.
Let's Put it to Work	**Lesson 17** • Apply reasoning developed throughout this unit to an unfamiliar problem. • Decide what information is needed to solve a real-world problem. • Make simplifying assumptions about a real-world situation.
Section	**Teacher-facing learning goals**

Differentiated Instruction

Technology Integration

21st Century Skills

Positive Behavior

CASEL

Collaborative for Academic, Social, and Emotional Learning

Resources

Teacher Notes and Reflections