Gateway Regional School District

Atlas

Unit Plan

6.1 Area and Surface Area

Gateway Regional Middle School / Grade 6 / Mathematics

Week 2 - Week 5 | 6 Curriculum Developers | Last Updated: Mar 26, 2024 by LeBlanc, Deanna

Style Guide

What is the purpose of the unit? What are the major take-aways?

Standards

MA: Mathematics (2017)
MA: Grade 6
Expressions & Equations

6.EE Apply and extend previous understandings of arithmetic to algebraic expressions.

- [x] 2c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). Show Details
- [x] 1. Write and evaluate numerical expressions involving whole-number exponents.
- [x] 2a. Write expressions that record operations with numbers and with letters standing for numbers. Show Details

Geometry

6.G Solve real-world and mathematical problems involving area, surface area, and volume.

- [x] 1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
- [x] 4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Enduring Understandings

1. Understanding of Area:
- Students will understand that the area is a measurement of the surface enclosed within a shape, and it can be calculated for different polygons by breaking them down into simpler shapes such as rectangles and triangles.
- They will recognize that finding areas of composite shapes involves strategic decomposition into shapes with known area formulas or by composing them into a recognizable shape.
- Students will appreciate that the concept of area extends beyond the classroom and is a critical component in various real-world applications including architecture, land development, and interior design.

4. Understanding of Surface Area using Nets:
- Students will understand that three-dimensional figures can be represented as two-dimensional nets consisting of a series of

Essential Questions

1. Essential Questions for "Finding the Area of Shapes":
a. How can you find the area of a triangle, a special quadrilateral, or a more complex polygon by transforming it into simpler shapes like rectangles and other triangles?
b. In what ways can decomposing and composing shapes help us solve problems in the real world, and why is this skill important?
c. How can understanding the properties of shapes and areas assist us in designing and planning practical spaces?

2. Essential Questions for "Surface Area of Three-Dimensional Figures":
a. How do nets help us understand and calculate the surface area of three-dimensional figures, and what real-life objects can be represented by these nets?
b. What steps must we take to ensure we have accurately calculated the surface area when using a net of rectangles and

connected rectangles and triangles.
- They will comprehend that the surface area of a three-dimensional figure is the collective area of the net that represents its faces.
- Students will recognize that calculating the surface area of objects is important in many real-world contexts such as packaging, manufacturing, and construction.

2c & 1. Understanding of Expressions in Context:
- Students will understand that variables can represent numbers in expressions that model real-world and mathematical problems.
- They will recognize the importance of evaluating expressions to find specific values, which may represent quantities like areas or surface areas in practical situations.
- Students will understand the conventions of arithmetic operations and the order of operations, including the use of whole-number exponents, to accurately evaluate expressions.

2a. Understanding of Algebraic Expressions:
- Students will understand how to write expressions that capture the relationship between numbers or represent operations with numbers, along with the conceptual leap that letters can stand for unknown or variable numbers.
- They will comprehend that expressions can encapsulate patterns or rules, and that this form of mathematical shorthand is essential in translating real-world situations into a mathematical context for analysis.

triangles?
c. How can we apply the concept of surface area to resolve challenges in various real-world situations, such as packaging design or construction?

3. Essential Questions for "Evaluating Expressions":
a. How does substituting specific values into algebraic expressions help us solve real-world mathematical problems, and what type of situations might require this skill?
b. Why is it important to understand and apply the order of operations when evaluating expressions, and how can mistakes in this process impact real-world results?
c. In what situations would we write expressions that include whole-number exponents, and how do these expressions relate to the real world?

4. Essential Questions for "Expressions with Variables and Exponents":
a. How can we write expressions that include variables to represent quantities in real-life situations, and why are these expressions useful?
b. What are the rules for writing and evaluating numerical expressions with whole-number exponents, and how do these rules help us understand and model real-life scenarios?

Content

Work with area in grade 6 draws on earlier work with geometry and geometric measurement. Students began to learn about two- and three-dimensional shapes in kindergarten, and continued this work in grades 1 and 2, composing, decomposing, and identifying shapes. Students' work with geometric measurement began with length and continued with area. Students learned to "structure two-dimensional space," that is, to see a rectangle with whole-number side lengths as composed of an array of unit squares or composed of iterated rows or iterated columns of unit squares. In grade 3, students distinguished between perimeter and area. They connected rectangle area with multiplication, understanding why (for whole-number side lengths) multiplying the side lengths of a rectangle yields the number of unit squares that tile the rectangle. They used area diagrams to represent instances of the distributive property. In grade 4, students applied area and perimeter formulas for rectangles to solve real-world and mathematical problems, and learned to use protractors. In grade 5, students extended the formula for the area of rectangles to rectangles with fractional side lengths.

In grade 6, students extend their reasoning about area to include shapes that are not composed of rectangles. Doing this draws on abilities developed in earlier grades to compose and decompose shapes, for example, to see a rectangle as composed of two congruent right triangles. Through activities designed and sequenced to allow students to make sense of problems and persevere in solving them (MP1), students build on these abilities and their knowledge of areas of rectangles to find the areas of polygons by decomposing and rearranging them to make figures whose areas they can determine (MP7). They learn strategies for finding areas of parallelograms and triangles, and use regularity in

Skills

Student Facing Learning Targets:

- I can explain the meaning of area. (Lesson 1)
- I can explain how to find the area of a figure that is composed of other shapes. (Lesson 2)
- I know how to find the area of a figure by decomposing it and rearranging the parts. (Lesson 2)
- I know what it means for two figures to have the same area. (Lesson 2)
- I can use different reasoning strategies to find the area of shapes. (Lesson 3)
- I can use reasoning strategies and what I know about the area of a rectangle to find the area of a parallelogram. (Lesson 4)
- I know how to describe the features of a parallelogram using mathematical vocabulary. (Lesson 4)
- I can identify pairs of base and height of a parallelogram. (Lesson 5)
- I can write and explain the formula for the area of a parallelogram. (Lesson 5)
- I know what the terms "base" and "height" refer to in a parallelogram. (Lesson 5)
- I can use the area formula to find the area of any parallelogram. (Lesson 6)
- I can explain the special relationship between a pair of identical triangles and a parallelogram. (Lesson 7)
- I can use what I know about parallelograms to reason about the area of triangles. (Lesson 8)
- I can use the area formula to find the area of any triangle. (Lesson 9)
- I can write and explain the formula for the area of a triangle. (Lesson 9)

repeated reasoning (MP8) to develop formulas for these areas, using geometric properties to justify the correctness of these formulas. They use these formulas to solve problems. They understand that any polygon can be decomposed into triangles, and use this knowledge to find areas of polygons. Students find the surface areas of polyhedra with triangular and rectangular surfaces. They study, assemble, and draw nets for polyhedra and use nets to determine surface areas. Throughout, they discuss their mathematical ideas and respond to the ideas of others (MP3, MP6).

Many of the lessons in this unit ask students to work on geometric figures that are not set in a real-world context. This design choice respects the significant intellectual work of reasoning about area. Tasks set in real-world contexts that involve areas of polygons are often contrived and hinder rather than help understanding. Moreover, mathematical contexts are legitimate contexts that are worthy of study. Students do have an opportunity at the end of the unit to tackle a real-world application (MP2, MP4).

In grade 6, students are likely to need physical tools in order to check that one figure is an identical copy of another where "identical copy" is defined as follows:
One figure is an *identical copy* of another if one can be placed on top of the other so that they match up exactly.

In grade 8, students will understand "identical copy of" as "congruent to" and understand congruence in terms of rigid motions, that is, motions such as reflection, rotation, and translation. In grade 6, students do not have any way to check for congruence except by inspection, but it is not practical to cut out and stack every pair of figures one sees. Tracing paper is an excellent tool for verifying that figures "match up exactly," and students should have access to this and other tools at all times in this unit. Thus, each lesson plan suggests that each student should have access to a *geometry toolkit*, which contains tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles. Providing students with these toolkits gives opportunities for students to develop abilities to select appropriate tools and use them strategically to solve problems (MP5). Note that even students in a digitally enhanced classroom should have access to such tools; apps and simulations should be considered additions to their toolkits, not replacements for physical tools. In this grade, all figures are drawn and labeled so that figures that look congruent actually are congruent; in later grades when students have the tools to reason about geometric figures more precisely, they will need to learn that visual inspection is not sufficient for determining congruence. Also note that all arguments laid out in this unit can (and should) be made more precise in later grades, as students' geometric understanding deepens.

- I know what the terms "base" and "height" refer to in a triangle. (Lesson 9)
- I can identify pairs of base and corresponding height of any triangle. (Lesson 10)
- When given information about a base of a triangle, I can identify and draw a corresponding height. (Lesson 10)
- I can describe the characteristics of a polygon using mathematical vocabulary. (Lesson 11)
- I can reason about the area of any polygon by decomposing and rearranging it, and by using what I know about rectangles and triangles. (Lesson 11)
- I know what the surface area of a three-dimensional object means. (Lesson 12)
- I can describe the features of a polyhedron using mathematical vocabulary. (Lesson 13)
- I can explain the difference between prisms and pyramids. (Lesson 13)
- I understand the relationship between a polyhedron and its net. (Lesson 13)
- I can match polyhedra to their nets and explain how I know. (Lesson 14)
- When given a net of a prism or a pyramid, I can calculate its surface area. (Lesson 14)
- I can calculate the surface area of prisms and pyramids. (Lesson 15)
- I can draw the nets of prisms and pyramids. (Lesson 15)
- I can explain how it is possible for two polyhedra to have the same surface area but different volumes, or to have different surface areas but the same volume. (Lesson 16 - *Optional*)
- I know how one-, two-, and three-dimensional measurements and units are different. (Lesson 16 - *Optional*)
- I can write and explain the formula for the volume of a cube, including the meaning of the exponent. (Lesson 17)
- When I know the edge length of a cube, I can find the volume and express it using appropriate units. (Lesson 17)
- I can write and explain the formula for the surface area of a cube. (Lesson 18)
- When I know the edge length of a cube, I can find its surface area and express it using appropriate units. (Lesson 18)
- I can apply what I know about the area of polygons to find the surface area of three-dimensional objects. (Lesson 19)
- I can use surface area to reason about real-world objects. (Lesson 19)

How will you gauge student learning?

Assessments

6.1 End-of-Unit Assessment | Summative | Written Test

Written Test

Grade6-1-End-of-Unit-Assessment-assessment.pdf

3 State Standards Assessed

How will students learn?

Learning Activities

Reasoning to Find Area	Lesson 1 • Compare (orally) areas of the shapes that make up a geometric pattern. • Comprehend that the word “area” (orally and in writing) refers to how much of the plane a shape covers. Lesson 2 • Calculate the area of a region by decomposing it and rearranging the pieces, and explain (orally and in writing) the solution method. • Recognize and explain (orally) that if two figures can be placed one on top one other so that they match up exactly, they must have the same area. • Show that area is additive by composing polygons with a given area. Lesson 3 • Calculate the area of a region by decomposing it and rearranging the pieces, and explain (orally and in writing) the solution method. • Recognize and explain (orally) that if two figures can be placed one on top one other so that they match up exactly, they must have the same area. • Show that area is additive by composing polygons with a given area.
Parallelograms	Lesson 4 • Compare and contrast (orally) different strategies for determining the area of a parallelogram. • Describe (orally and in writing) observations about the opposites sides and opposite angles of parallelograms. • Explain (orally and in writing) how to find the area of a parallelogram by rearranging or enclosing it in a rectangle. Lesson 5 • Comprehend the terms “base” and “height” to refer to one side of a parallelogram and the perpendicular distance between that side and the opposite side. • Generalize (orally) a process for finding the area of a parallelogram, using the length of a base and the corresponding height. • Identify a base and the corresponding height for a parallelogram, and understand that there are two different base-height pairs for any parallelogram. Lesson 6 • Apply the formula for area of a parallelogram to find the area, the length of the base, or the height, and explain (orally and in writing) the solution method. • Choose which measurements to use for calculating the area of a parallelogram when more than one base or height measurement is given, and explain (orally and in writing) the choice.
Triangles	Lesson 7 • Describe (orally and in writing) ways in which two identical triangles can be composed, i.e., into a parallelogram or into a rectangle. • Show how any parallelogram can be decomposed into two identical triangles by drawing a diagonal, and generalize (in writing) that this property applies to all parallelograms, but not all quadrilaterals. Lesson 8 • Draw a diagram to show that the area of a triangle is half the area of an associated parallelogram. • Explain (orally and in writing) strategies for using the base and height of an associated parallelogram to determine the area of a triangle. Lesson 9 • Compare, contrast, and critique (orally) different strategies for determining the area of a triangle. • Generalize a process for finding the area of a triangle, and justify (orally and in writing) why this can be abstracted as $12 \cdot b \cdot h12 \cdot \text{\textdiamond} \cdot h$. • Recognize that any side of a triangle can be considered its base, choose a side to use as the base when calculating the area of a triangle, and identify the corresponding height. Lesson 10 • Draw and label the height that corresponds to a given base of a triangle, making sure it is perpendicular to the base and the correct length. • Evaluate (orally) the usefulness of different base-height pairs for finding the area of a given triangle.
Polygons	Lesson 11 • Compare and contrast (orally) different strategies for finding the area of a polygon. • Describe (orally and in writing) the defining characteristics of polygons. • Find the area of a polygon, by decomposing it into rectangles and triangles, and present the solution method (using words and other representations).

Surface Area	Lesson 12 • Calculate the surface area of a rectangular prism and explain (orally and in writing) the solution method. • Comprehend that the term “surface area” (in written and spoken language) refers to how many square units it takes to cover all the faces of a three-dimensional object. Lesson 13 • Compare and contrast (orally and in writing) features of prisms and pyramids. • Comprehend and use the words “face”, “edge”, “vertex”, and “base” to describe polyhedra (in spoken and written language). • Understand that the word “net” refers to a two-dimensional figure that can be assembled into a polyhedron, and create a net for a given polyhedron. Lesson 14 • Match polyhedra with their nets and justify (orally) that they match. • Use a net with gridlines to calculate the surface area of a prism or pyramid and explain (in writing) the solution method. • Visualize and identify the polyhedron that can be assembled from a given net. Lesson 15 • Draw and assemble a net for the prism or pyramid shown in a given drawing. • Interpret (using words and other representations) two-dimensional representations of prisms and pyramids. • Use a net without gridlines to calculate the surface area of a prism or pyramid and explain (in writing) the solution method. Lesson 16 - Optional • Comprehend that surface area and volume are two different attributes of three-dimensional objects and are measured in different units. • Describe (orally and in writing) shapes built out of cubes, including observations about their surface area and volume. • Determine the surface area and volume of shapes made out of cubes.
Squares and Cubes	Lesson 17 • Generalize a process for finding the volume of a cube, and justify (orally) why this can be abstracted as s3�3. • Include appropriate units (orally and in writing) when reporting lengths, areas, and volumes, e.g. cm, cm2,cm3��2,��3. • Interpret and write expressions with exponents 22 and 33 to represent the area of a square or the volume of a cube. Lesson 18 • Generalize a process for finding the surface area of a cube, and justify (orally) why this can be abstracted as 6·s26·�2. • Interpret (orally) expressions that include repeated addition, multiplication, repeated multiplication, or exponents. • Write expressions, with or without exponents, to represent the surface area of a given cube.
Let's put it to work!	Lesson 19 • Apply understanding of surface area to estimate the amount of fabric in a tent, and explain (orally and in writing) the estimation strategy. • Compare and contrast (orally) different tent designs. • Interpret information (presented in writing and through other representations) about tents and sleeping bags.
Section	Teacher-facing learning goals

Differentiated Instruction

Technology Integration

21st Century Skills

Positive Behavior

CASEL

Collaborative for Academic, Social, and Emotional Learning

Resources

Teacher Notes and Reflections