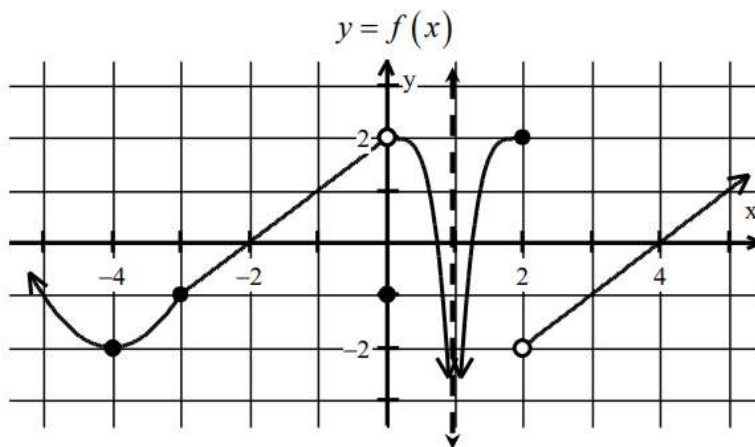


**ALL WORK MUST BE COMPLETED ON A SEPARATE DOCUMENT/FILE.**

**This is a quiz grade due via Canvas on 8/12/24.**

### Assignment 1.1

Use the appearance of the graph shown at the right to find the following limit and function values.



1.  $\lim_{x \rightarrow -4} f(x)$
2.  $\lim_{x \rightarrow -1} f(x)$
3.  $\lim_{x \rightarrow 0^-} f(x)$
4.  $f(0)$
5.  $\lim_{x \rightarrow 0^-} f(x)$
6.  $\lim_{x \rightarrow 1} f(x)$
7.  $f(1)$
8.  $\lim_{x \rightarrow 2} f(x)$
9.  $\lim_{x \rightarrow 2^-} f(x)$
10.  $\lim_{x \rightarrow 2^+} f(x)$
11.  $f(2)$
12.  $\lim_{x \rightarrow 4} f(x)$
13.  $\lim_{x \rightarrow 4^+} f(x)$

Use the function  $g(x) = \begin{cases} 2x-3, & x \leq 0 \\ -x-3, & 0 < x \leq 2 \\ 3x, & x > 2 \end{cases}$  for problems 14-20.

14. Sketch an accurate graph without using a calculator.
15.  $\lim_{x \rightarrow 0} g(x) =$
16.  $\lim_{x \rightarrow 2} g(x) =$
17.  $\lim_{x \rightarrow 2^-} g(x) =$
18.  $\lim_{x \rightarrow 2^+} g(x) =$
19.  $g(2) =$
20.  $\lim_{x \rightarrow -2} g(x) =$

Find each of the following limits without using a calculator. Simplify your answers.

21.  $\lim_{x \rightarrow 0} (2x-5)$
22.  $\lim_{x \rightarrow -3} (x^2 - 5x + 4)$
23.  $\lim_{x \rightarrow 2} \frac{2x-5}{\sqrt{x}+7}$
24.  $\lim_{x \rightarrow -2} |3x+5|$
25.  $\lim_{x \rightarrow 2} \frac{3x-6}{\sqrt{x}+6}$
26.  $\lim_{x \rightarrow \pi} \sin x$
27.  $\lim_{x \rightarrow \frac{\pi}{2}} \cos x$
28.  $\lim_{x \rightarrow \pi} \tan x$
29.  $\lim_{x \rightarrow \frac{\pi}{2}} \cos(2x)$
30.  $\lim_{x \rightarrow 2} \cos \frac{\pi x}{3}$
31.  $\lim_{x \rightarrow 3} \sec \frac{\pi x}{4}$
32.  $\lim_{x \rightarrow 7} \csc \frac{\pi x}{6}$
33.  $\lim_{x \rightarrow \pi} \cot \frac{x}{6}$
34.  $\lim_{x \rightarrow 5\pi} \cos \frac{x}{3}$

Use the function  $f(x) = \begin{cases} 6x-3x^3, & x \leq 2 \\ 4x-x^4, & x > 2 \end{cases}$  for problems 35, 36.

35.  $\lim_{x \rightarrow 2^+} f(x)$
36.  $\lim_{x \rightarrow 2} f(x)$

Use the function  $g(x) = \begin{cases} 2 \sin \frac{3x}{2}, & x \leq \pi \\ \sec \frac{11x}{6}, & x > \pi \end{cases}$  for problems 37-39.

37.  $\lim_{x \rightarrow \pi^+} g(x)$       38.  $\lim_{x \rightarrow \pi^-} g(x)$       39.  $\lim_{x \rightarrow \pi} g(x)$

Use the functions  $g(x) = 3x^2 - 5x$  and  $f(x) = \sqrt[3]{3x+5}$  for problems 40-42.

40.  $\lim_{x \rightarrow 2} g(x)$       41.  $\lim_{x \rightarrow 1} f(x)$       42.  $\lim_{x \rightarrow 3} f(g(x))$

If  $\lim_{x \rightarrow 3} h(x) = 5$  and  $\lim_{x \rightarrow 3} k(x) = 3$  find the following limits.

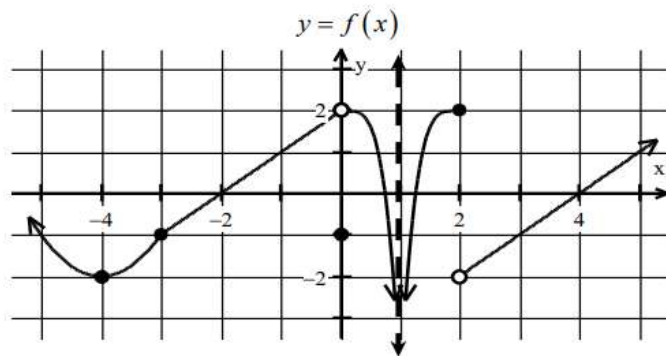
43.  $\lim_{x \rightarrow 3} (h(x) + k(x))$       44.  $\lim_{x \rightarrow 3} (h(x)k(x))$       45.  $\lim_{x \rightarrow 3} \frac{h(x)}{k(x)}$

The symbol  $\lfloor \rfloor$  is used to represent the Greatest Integer Function in the following problems.

Find these limits without using a calculator or state that the limit does not exist.

46.  $\lim_{x \rightarrow 3^-} \lfloor x-1 \rfloor$       47.  $\lim_{x \rightarrow 3^+} \lfloor x-1 \rfloor$       48.  $\lim_{x \rightarrow 3} \lfloor x-1 \rfloor$       49.  $\lim_{x \rightarrow 3} \lfloor \frac{x}{2} - 1 \rfloor$       50.  $\lim_{x \rightarrow 3^-} \lfloor 4x-1 \rfloor$

51. Identify each  $x$ -value at which the function shown appears to be discontinuous and classify each as removable or nonremovable.



Find all discontinuities for the following functions and classify each as removable or nonremovable. Do not use a calculator.

52.  $f(x) = \begin{cases} 3x^3 + 4x, & x \leq -2 \\ x^4 + 16, & x > -2 \end{cases}$

53.  $f(x) = \begin{cases} 3x^3 + 4x, & x \leq 2 \\ x^4 + 16, & x > 2 \end{cases}$

54.  $g(x) = \begin{cases} 2 \sin \frac{\pi x}{2}, & x \leq 1 \\ \cos \frac{\pi x}{3}, & x > 1 \end{cases}$

55.  $g(x) = \begin{cases} \cos x, & x \leq 0 \\ -x + 1, & 0 < x \leq 2 \\ \sin \frac{\pi x}{2}, & x > 2 \end{cases}$

56.  $f(x) = \lfloor x + 5 \rfloor$

57.  $h(x) = \lfloor \frac{x}{3} \rfloor$

### Assignment 1.2

Find the indicated limits without using a calculator. **Show steps using correct limit symbolism!**

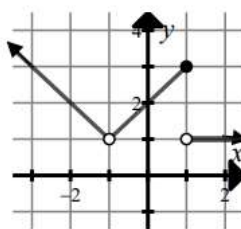
1.  $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$
2.  $\lim_{x \rightarrow -1} \frac{2x^2 - x - 3}{x + 1}$
3.  $\lim_{x \rightarrow -1} \frac{x^2 - 1}{x - 1}$
4.  $\lim_{x \rightarrow -1} \frac{x^3 + 1}{x + 1}$
5.  $\lim_{x \rightarrow -1} \frac{x}{x^2 + 1}$
6.  $\lim_{x \rightarrow 5^+} \frac{x - 5}{x^2 - 25}$
7.  $\lim_{x \rightarrow -5} \frac{x - 5}{x^2 - 25}$
8.  $\lim_{x \rightarrow 2} \frac{2 - x}{x^2 - 4}$
9.  $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - 6x + 9}$
10.  $\lim_{x \rightarrow -2} \sqrt[3]{x^2 + 4}$
11.  $\lim_{x \rightarrow 0} \frac{x}{x - 1}$
12.  $\lim_{x \rightarrow 2^-} \frac{1}{x^2 - 4}$
13.  $\lim_{x \rightarrow 1} \frac{x}{x^2 + 1}$
14.  $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$
15.  $\lim_{x \rightarrow 3} \frac{x - 3}{\sqrt{x + 1} - 2}$
16.  $\lim_{x \rightarrow 0^-} \frac{|x|}{x}$
17.  $\lim_{x \rightarrow 0} \frac{|x|}{x}$
18.  $\lim_{x \rightarrow \frac{\pi}{2}} \sin x$
19.  $\lim_{x \rightarrow \pi} \sec x$
20.  $\lim_{x \rightarrow \frac{\pi}{2}} \cos(3x)$
21.  $\lim_{x \rightarrow 5} \csc \frac{\pi x}{6}$
22.  $\lim_{x \rightarrow 3^+} \lfloor x - 1 \rfloor$
23.  $\lim_{x \rightarrow 3^-} \lfloor x - 1 \rfloor$
24.  $\lim_{x \rightarrow 3} \lfloor x - 1 \rfloor$
25.  $\lim_{x \rightarrow 2} \lfloor x + 6 \rfloor$
26.  $\lim_{x \rightarrow 3} \lfloor \frac{x}{2} \rfloor$
27.  $\lim_{x \rightarrow 5} \lfloor 2x - 3 \rfloor$
28.  $\lim_{x \rightarrow 5} \frac{\frac{1}{x} - \frac{1}{5}}{x - 5}$

$$29. \lim_{x \rightarrow 3} \begin{cases} \frac{1}{2}x + 1, & x \leq 3 \\ \frac{12 - 2x}{3}, & x > 3 \end{cases} \quad 30. \lim_{x \rightarrow 1} \begin{cases} x^2 + 1, & x < 1 \\ x^3 + 1, & x \geq 1 \end{cases} \quad 31. \lim_{x \rightarrow 2} \begin{cases} x - 2, & x \leq 0 \\ x + 2, & x > 0 \end{cases}$$

Use a calculator to find these limits.

$$32. (a) \lim_{x \rightarrow 1} \frac{\sin x}{6x} \quad (b) \lim_{x \rightarrow 0} \frac{\sin x}{6x} \quad 33. \lim_{x \rightarrow -2} \frac{x^3 + 2x^2 - x - 2}{x^3 + 2x^2 + x + 2} \quad 34. \lim_{x \rightarrow 2} \frac{|2 - x|}{25x - 50}$$

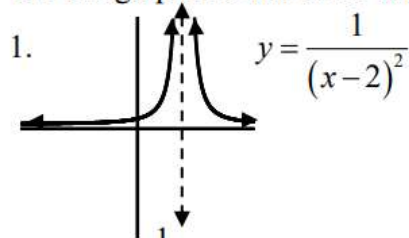
35. a.  $h(-1)$  b.  $h(1)$
- c.  $\lim_{x \rightarrow -1} h(x)$  d.  $\lim_{x \rightarrow 1^-} h(x)$
- e.  $\lim_{x \rightarrow 1^+} h(x)$  f.  $\lim_{x \rightarrow 1} h(x)$
- g. removable discontinuities
- h. nonremovable discontinuities



$$h(x) = \begin{cases} -x, & x < -1 \\ x + 2, & -1 < x \leq 1 \\ 1, & x > 1 \end{cases}$$

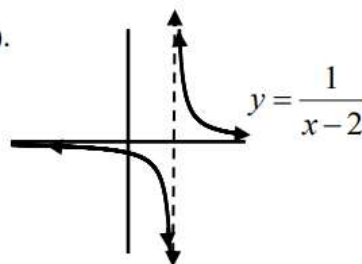
### Assignment 1.3

Use the graphs to find these limits (answer  $\infty$  or  $-\infty$ ).



- a.  $\lim_{x \rightarrow 2^-} \frac{1}{(x - 2)^2}$
- b.  $\lim_{x \rightarrow 2^+} \frac{1}{(x - 2)^2}$

2.



- a.  $\lim_{x \rightarrow 2^-} \frac{1}{x - 2}$
- b.  $\lim_{x \rightarrow 2^+} \frac{1}{x - 2}$



Find the vertical asymptotes, if any, without using a calculator, and classify each of them as even or odd.

3.  $f(x) = \frac{1}{x^2}$

4.  $f(x) = \frac{x}{x(x-1)^2}$

5.  $f(x) = \frac{x}{x^2-4}$

6.  $f(x) = \frac{x}{x^2-x-2}$

7.  $g(x) = \frac{x^3-1}{x-1}$

8.  $g(x) = \csc(\pi x)$

Find these limits without using a calculator. Whenever appropriate answer  $\infty$  or  $-\infty$ .

9.  $\lim_{x \rightarrow 3^-} \frac{x}{x-3}$

10.  $\lim_{x \rightarrow 3} \frac{x}{x-3}$

11.  $\lim_{x \rightarrow 1^+} \frac{x}{x^2-x}$

12.  $\lim_{x \rightarrow 0} \frac{x}{x^2-x}$

13.  $\lim_{x \rightarrow 3} \frac{x+3}{x^2-6x+9}$

14.  $\lim_{x \rightarrow 3} \frac{x-3}{x^2-6x+9}$

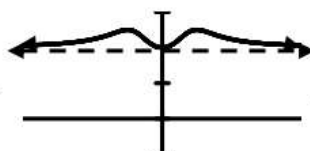
15.  $\lim_{x \rightarrow 0} \frac{x^2-2x}{x^3}$

16.  $\lim_{x \rightarrow 0^-} \left( \frac{1}{x} - 10 \right)$

17.  $\lim_{x \rightarrow \frac{\pi}{2}^+} \frac{3}{\cos x}$

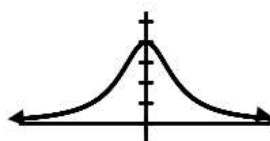
18.  $\lim_{x \rightarrow \pi} \frac{x}{\csc x}$

Find these limits without using a calculator.

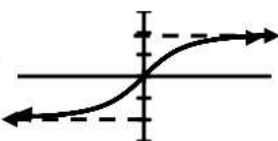
19.   $f(x) = \frac{2x^4 + x^2 + 2}{x^4 + 1}$

a.  $\lim_{x \rightarrow \infty} f(x)$    b.  $\lim_{x \rightarrow -\infty} f(x)$

Find these limits without using a calculator.

20.   $f(x) = \frac{4}{x^2 + 1}$

a.  $\lim_{x \rightarrow \infty} f(x)$    b.  $\lim_{x \rightarrow -\infty} f(x)$

21.   $f(x) = \frac{2x}{\sqrt{x^2 + 2}}$

a.  $\lim_{x \rightarrow \infty} f(x)$    b.  $\lim_{x \rightarrow -\infty} f(x)$

22.  $\lim_{x \rightarrow \infty} \frac{2x+5}{3x-4}$

23.  $\lim_{x \rightarrow -\infty} \frac{1-5x^3}{10x^3-x^2}$

24.  $\lim_{x \rightarrow \infty} \frac{x(2x-1)^2}{3x(x-3)^2}$

25.  $\lim_{x \rightarrow -\infty} \frac{4x^2+3}{2x}$

26.  $\lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2+x}}$

27.  $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2+x}}$

28.  $\lim_{x \rightarrow -\infty} \frac{2-x}{\sqrt{x^2-3}}$

29.  $\lim_{x \rightarrow -\infty} \frac{2x^2-2}{\sqrt{x^4}}$

30.  $\lim_{x \rightarrow \infty} \frac{\sin x}{x+1}$

31.  $\lim_{x \rightarrow -\infty} \left( \frac{4e^x + 2x}{3x} \right)$

32.  $\lim_{x \rightarrow \infty} (x^5 e^x + 2)$

Use a calculator to find the following limits.

33.  $\lim_{x \rightarrow \infty} (x^5 e^{-x} + 2)$

34.  $\lim_{x \rightarrow -\infty} \frac{|2x+5|}{x-7}$

Follow the **Curve Sketching Recipe** to graph each function without using a calculator. List intercepts, asymptotes, holes, end behavior, etc. Show accurate graphs.

$$35. f(x) = (x+2)(x-1)^2 \quad 36. f(x) = \frac{x-2}{x+2} \quad 37. f(x) = \frac{x(x-1)^3}{x^2(x-1)}$$

$$38. f(x) = \frac{1}{\sqrt{x}} \quad 39. f(x) = \frac{-x}{\sqrt{x^2-1}}$$

40. If  $f(x) = \begin{cases} 2ax-6, & x \leq 2 \\ x^2+a, & x > 2 \end{cases}$  is a continuous function, find the value of  $a$ .

Use a calculator to find all discontinuities.

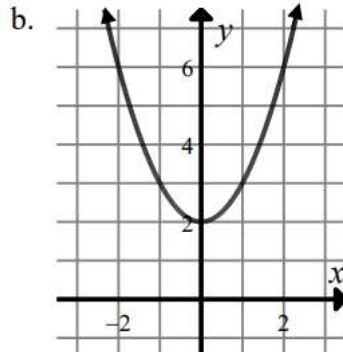
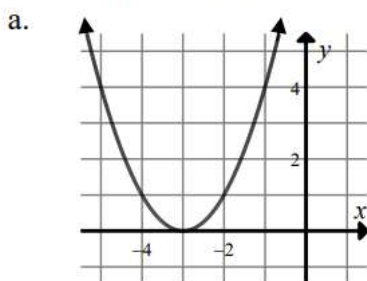
$$41. f(x) = \begin{cases} \frac{\cos x - 1}{x}, & x < 0 \\ 5x, & x > 0 \end{cases} \quad 42. f(x) = \frac{x^2 - 4}{x^3 - 2x^2 - 2x + 4}$$

Does the Intermediate Value Theorem guarantee a value of  $c$  in the given interval? If so, find the  $c$ -value. If not, explain why not.

$$43. f(x) = \frac{x^2 - x}{x}, f(c) = -1 \text{ on } [-2, 2] \quad 44. f(x) = x^2 - x, f(c) = -1 \text{ on } [-2, 2]$$

$$45. f(x) = x^2 - x, f(c) = 5 \text{ on } [-2, 2]$$

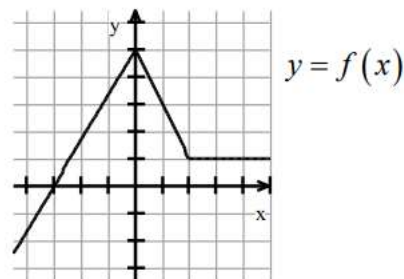
46. Use the parent graph of  $y = x^2$  to determine an equation for each graph.



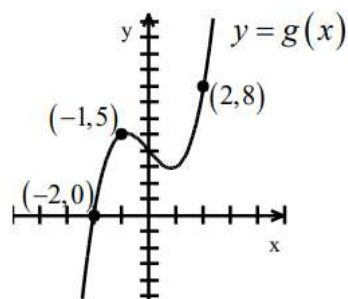
### Assignment 1.4

1. The function  $y = f(x)$  graphed at the right is a piecewise linear function. Find the instantaneous rate of change at each of the following  $x$ -values.

- $x = -1$
- $x = 1$
- $x = 4$



2. The function  $y = g(x)$  is graphed at the right.
- Find the average rate of change on the interval  $[-2, -1]$ .
  - Find the average rate of change on the interval  $[-1, 2]$ .
  - Which of these is a better approximation for the instantaneous rate of change of  $g(x)$  at  $x = -1.5$ ?

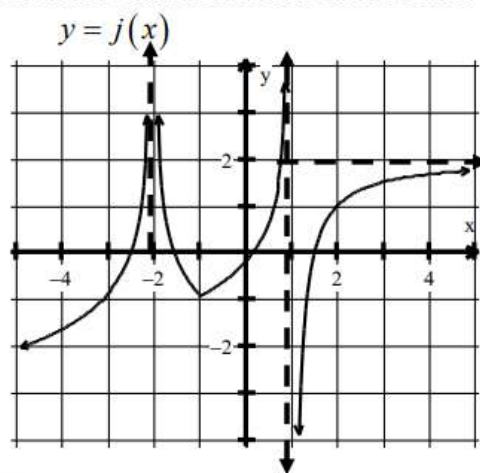
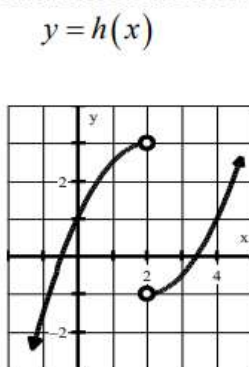
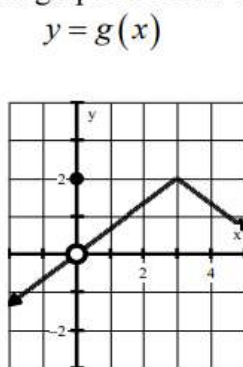
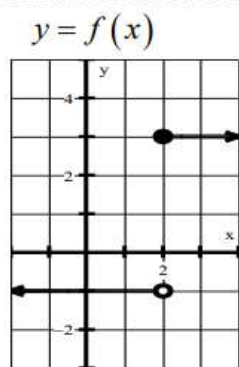


3. Approximate the instantaneous rate of change of  $y = 3e^x + 5\sin x$  at  $x = 3.3$  by finding the average rate of change on the interval  $[3, 4]$  accurate to three decimal places.
4. The data in the table below gives times and distances for a marathoner at selected points in the race.

time in minutes	0	40	55	95	129
miles from start	0	8	12	20	26

- Find the runner's average rate of change (speed in miles per minute) for the 26 miles included in the table.
  - Approximate the instantaneous speed at the half-marathon spot (13.1 miles).
  - Which of the intervals shown in the table was the slowest for the runner?
5. If  $f(x) = \frac{6x-18}{x-3}$  and  $g(x) = \frac{6\sin \frac{\pi x}{6}}{\cos(x-3)}$  and it is known that  $f(x) \leq h(x) \leq g(x)$  on the interval  $[2, 4]$  except at  $x = 3$ . Find  $\lim_{x \rightarrow 3} h(x)$ . Explain your reasoning.
6. Given  $f(x) = \frac{x^2-4}{x+2}$  and  $f(x) \leq h(x) \leq j(x)$  for all  $x$  except  $x = -2$ . If  $\lim_{x \rightarrow -2} h(x)$  can be found by using the Squeeze Theorem what is  $\lim_{x \rightarrow -2} j(x)$ ?

Use the four functions graphed below to find the limits shown or state that the limit does not exist.



- $\lim_{x \rightarrow -2} j(x)$
- $\lim_{x \rightarrow 1} j(x)$
- $\lim_{x \rightarrow -1} \frac{f(x) - 2}{(j(x))^2}$
- $\lim_{x \rightarrow \infty} h(j(x))$
- $\lim_{x \rightarrow -1} g(f(x) + 1)$
- $\lim_{x \rightarrow 0} f(|x| + 2)$
- $\lim_{x \rightarrow 0} (g(x) \cdot f(x + 2))$
- $\lim_{x \rightarrow -2} j(j(x))$



15. Find the equation of the horizontal asymptote for the function  $g(x) = \frac{x^3 + x}{e^x + x}$  without using a calculator.
16. Find  $\lim_{x \rightarrow 0} \frac{\sin x + 2e^x}{\cos x}$  without using a calculator.
18. Use a calculator to find this limit  $\lim_{x \rightarrow 2} \frac{|2-x|}{25x-50}$ .

### Assignment 2.1

Use the limit definition of the derivative to find  $f'(x)$  or  $f'(t)$ . Show correct limit symbolism.

1.  $f(x) = -3x$     2.  $f(x) = x^2 - 1$     3.  $f(x) = \frac{1}{x-1}$     4.  $f(t) = t^3 - 12t$     5.  $f(x) = 3$

Use the alternate form of the limit definition of the derivative to find the indicated derivative.

6.  $f(x) = x^2 - 1$  Find  $f'(2)$ .    7.  $f(x) = x^3 - 2x^2 - 1$  Find  $f'(2)$ .

8.  $f(x) = \frac{1}{x}$  Find  $f'(3)$ .    9.  $f(x) = (x-1)^{\frac{2}{3}}$  Find  $f'(1)$ .

10. If  $y = x^2 - x$ , use the limit definition of the derivative to find  $y'$ .

11. If  $y = x^3 + 1$ , use the limit definition of the derivative to find  $\frac{dy}{dx}$ .

12. If  $f(x) = 2x^2 + 4$ , use the limit definition of the derivative to find  $f'(x)$ . Then find  $f'(4)$ .

13. If  $f(x) = 2x^2 + 4$ , use the alternate form of the limit definition of the derivative to find  $f'(4)$ .

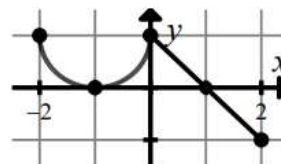
For Problems 14-17, solve for  $\theta$ , where  $0 \leq \theta < 2\pi$ , without using a calculator.

14.  $\sec^2 \theta - 4 = 0$     15.  $\sin^2 \theta = \cos^2 \theta$     16.  $\tan \theta - \sin \theta = 0$     17.  $2 \sin^2 \theta = \cos \theta + 1$

18. Use a calculator to solve for  $x$  on the interval  $[0, 2\pi)$  for  $\tan x = \csc^2 x - 2$ .

Use the graph of  $y = f(x)$  shown to graph the following.

19.  $y = |f(x)|$     20.  $y = f(x-2) + 1$     21.  $y = -2f(x)$



22. Find the domain, vertical asymptotes, holes, intercepts, end behavior, and graph for the function  $y = \frac{x(x-1)}{x^2-1}$ .

Use the graph of  $y = f(x)$  for Problems 23-32.

Find the following limits and function values.

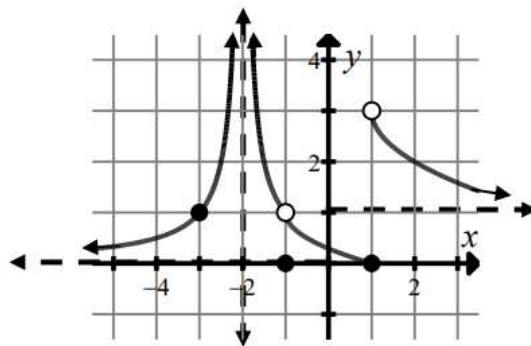
23.  $\lim_{x \rightarrow -1} f(x)$       24.  $f(-1)$       25.  $\lim_{x \rightarrow 1} f(x)$

26.  $\lim_{x \rightarrow 1^+} f(x)$       27.  $\lim_{x \rightarrow -3} f(x)$       28.  $\lim_{x \rightarrow \infty} f(x)$

29.  $\lim_{x \rightarrow -\infty} f(x)$       30.  $\lim_{x \rightarrow -2} f(x)$

31. List all removable discontinuities of  $f(x)$ .

32. List all nonremovable discontinuities of  $f(x)$ .



## Assignment 2.2

Find the derivative. Use correct symbolism.

1.  $y = 2$       2.  $f(x) = x^2$       3.  $g(x) = x^3 + 1$

4.  $y = t + 2$       5.  $f(t) = -2t^2 - 3t + 2$       6.  $f(x) = -\frac{1}{3}x^2 - \frac{2}{5}x + \frac{5}{2}$

Find the value of the derivative of the function at the given point. Show steps with correct symbolism.

7.  $f(x) = 3x^{-2}$  at  $(1, 3)$       8.  $g(x) = x^2 - 2x$  at  $(2, 0)$

9.  $h(x) = x^3 - 1$  at  $(1, 0)$       10.  $f(x) = 2 - x^3$  at  $(2, -6)$

Differentiate each function. Show steps with correct symbolism.

11.  $y = \frac{1}{x}$       12.  $f(x) = x^2 - \frac{4}{x^2}$       13.  $y = (2x - 1)^2$       14.  $g(x) = x(x^2 + 1)$

15.  $y = \frac{\sqrt{x}}{x}$       16.  $y = \sqrt[3]{x} + \sqrt{x^3}$       17.  $f(t) = \frac{t^2 - 2t}{t}$       18.  $f(x) = \frac{1}{\sqrt[3]{x^2}}$

19.  $y = \frac{1}{3x^2}$       20.  $y = \frac{1}{(3x)^2}$       21.  $f(x) = \frac{x^2 - x - 1}{\sqrt{x}}$       22.  $y = (3x^2 - 5)(x + 7)$

Find the indicated value or expression. Show steps with correct symbolism.

23.  $y = 3x^2$ ,  $y'' = ?$       24.  $f(x) = \sqrt{x} + 2$ ,  $f'(4) = ?$       25.  $f(t) = 2 - \frac{2}{t}$ ,  $f''(2) = ?$

26.  $y = x(x - 2)$ ,  $\frac{d^2y}{dx^2} = ?$       27.  $f^{(3)}(x) = 2x - 1$ ,  $f^{(5)}(3) = ?$       28.  $\frac{d}{dx}(x^3 + 5) = ?$

29.  $\frac{d^2}{dx^2}(3x - x^{-1}) = ?$

30. Find the second derivative of  $f(x) = \frac{x^2 - 4x - 6}{2x}$ .



Find an equation of a line with the following characteristics.

31. tangent to the graph of  $f(x) = x^2 - 1$  at the point (2,3)

32. tangent to the graph of  $f(x) = \frac{2}{x}$  when  $x = 1$

33. normal to the graph of  $f(x) = \frac{2}{x}$  when  $x = 1$

34. tangent to the graph of  $y = x^2 - 2x + 3$  when  $x = 1$

35. Find the  $x$ -values of all points where the graph of  $f(x) = 3x^3 + 2x - 2$  has a slope of 11.

36. Find the  $x$ -values of all points where the graph of  $y = x^4 - 3x^2 + 2$  has a horizontal tangent line.

37. Find the point(s) where the graph of  $y = \frac{1}{x}$  has a slope of  $-\frac{1}{4}$ .

38. Find the average rate of change of the function  $f(x) = 3x^3 - 4$  between  $x = 2$  and  $x = 4$ .

39. Find the instantaneous rate of change of the function  $f(x) = 3x^3 - 4$  at  $x = 3$ .

40. Find the average rate of change of  $y = \frac{x}{x+2}$  on the interval  $[1, 4]$ .

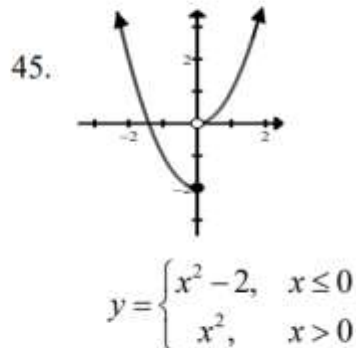
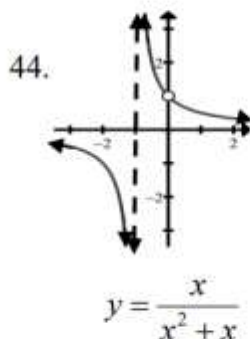
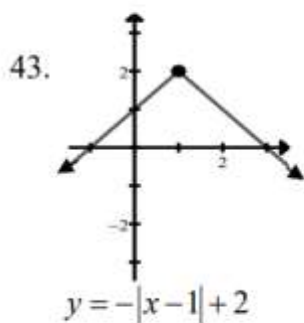
41. Find the rate of change of  $y = \frac{x^2 - x}{x^2}$  at the point (1,0).

42. If  $f(x) = 2x^3 - 3x + 2$  find:

a. the average rate of change on the interval  $[0, 3]$ .

b. the instantaneous rate of change at  $x = 3$ .

Find the  $x$ -values where the function is not differentiable. Give a reason for each value.



Find the  $x$ -values where the function is not differentiable. Give a reason for each value.

46.  $f(x) = x^{\frac{2}{3}}$

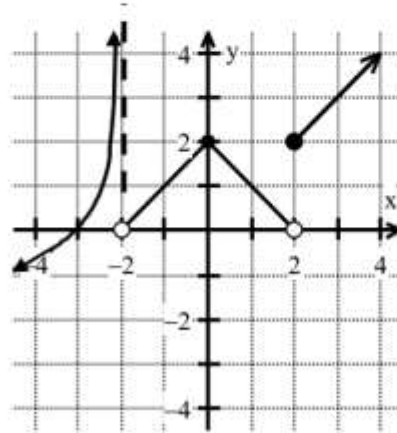
47.  $f(x) = 3x^{\frac{1}{3}}$

For each of the following piecewise functions:

- Find any  $x$ -values at which the function is discontinuous.
- Differentiate the function.
- Find any  $x$ -values at which the function is not differentiable.

$$48. f(x) = \begin{cases} 3x^2 - x, & x \leq 1 \\ 5x - 3, & x > 1 \end{cases} \quad 49. f(x) = \begin{cases} 3x^2 - x, & x \leq 1 \\ 5x - 2, & x > 1 \end{cases} \quad 50. f(x) = \begin{cases} 3x^2 - x, & x \leq 1 \\ 4x - 2, & x > 1 \end{cases}$$

- Identify any  $x$ -values at which the function shown is not continuous.
- Identify any  $x$ -values at which the function shown is not differentiable.



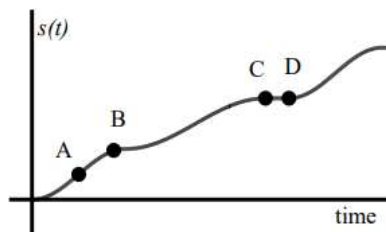
### Assignment 2.3

You may use a calculator for these questions.

- The position, in meters, of a particle moving in a straight line is given by  $x(t) = 4t^3 + 6t + 2.5$  (where  $t$  is measured in seconds).
  - Find the velocity function.
  - Find the velocity at time  $t = 2$  seconds.
  - Find the acceleration function.
  - Find the acceleration at time 3 seconds.
  - When is the velocity of the particle 18 meters per second?
  - Find the velocity when the position of the particle is 25 meters.
  - Find the initial position.
  - Find the particle's displacement from 0 to 1.5 seconds.
- A helium balloon rises so that its height (position) is given by  $s(t) = t^2 + 3t + 5$  (where height is measured in feet and time is measured in seconds). Assume  $t \geq 0$ .
  - When is the balloon 45 feet high?
  - How fast is the balloon rising at time 1 second?
  - How fast is the balloon rising at time 4 seconds?
  - What is the balloon's velocity when it is 45 feet high?

3. A ball rolls on an inclined plane with position function  $s(t) = 2t^3 + 3t^2 + 5$  (where position is measured in centimeters and time is measured in seconds).
- Find the ball's velocity at time 2 seconds.
  - When is the velocity of the ball 30 centimeters per second?

4. The graph at the right shows the position function of a car. Answer these questions and explain each answer.
- What was the car's initial position?
  - Was the car going faster at A or at B?
  - Was the car speeding up or slowing down at B?
  - What happened between C and D?



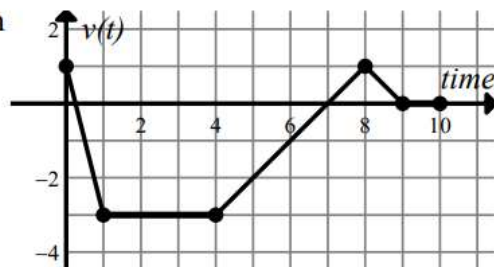
5. A particle moves along a horizontal line with position function  $x(t) = t^3 - 3t^2$  (where position is measured in centimeters and time is measured in minutes).
- Find the particle's displacement between  $t = 0$  minutes and  $t = 5$  minutes.
  - Find the particle's velocity when  $t = 4$  minutes.
  - Find the particle's acceleration when  $t = 4$  minutes.
  - At what time does the particle change direction?
  - What is the total distance traveled by the particle between 0 and 5 minutes?

$$\text{Average Velocity} = \frac{\text{displacement}}{\text{elapsed time}}$$

$$\text{Average Speed} = \frac{\text{total distance}}{\text{elapsed time}}$$

- Find the particle's average velocity (average rate of change of position) between  $t = 0$  and  $t = 5$  minutes.
- Find the particle's average speed between  $t = 0$  and  $t = 5$  minutes.

6. The graph at the right shows the velocity function of a particle moving horizontally.
- When does the particle move left?
  - When is the particle's acceleration positive?
  - When is the speed greatest?
  - When does the particle stop for more than an instant?



7. The position at time  $t$  seconds of a pebble dropped from an initial height of 600 feet is given by  $s(t) = -16t^2 + 600$ .
- At what time will the pebble hit the ground?
  - What is the pebble's velocity when it hits the ground?
  - What is the pebble's speed when it hits the ground?

**Do not use a calculator on problems 8-17.**

Find  $f'(x)$ .

8.  $f(x) = 2x - \frac{3}{x^3}$

9.  $f(x) = (2x + 3)^2$



Evaluate the derivative of  $f(x)$  at the indicated point for Problems 10 and 11.

10.  $f(x) = 2x\sqrt{x}$  at  $(4, 16)$

11.  $f(x) = \sqrt[3]{x^2}$  at  $(-8, 4)$

12. If  $y = x(x-2)$  find  $\frac{d^2y}{dx^2}$ .

13. Find an equation of a line tangent to the graph of  $f(x) = 2x^4 - 3x^3$  when  $x = 1$ .

14. Find a point on the graph of  $f(x) = x^4 + 3$  where a tangent line has a slope of  $-4$ .

15. Use the limit definition of the derivative to find  $f'(x)$  if  $f(x) = 3x^2 - x$ .

16. If  $f(x) = x^3 + 5$ , find the instantaneous rate of change at  $x = 1$ .

17. If  $f(x) = x^3 + 5$ , find the average rate of change between  $x = 0$  and  $x = 2$ .

18. If  $f(x) = 7^x$ , use a calculator to find  $f'(3)$ .

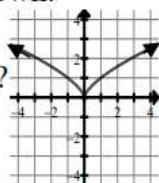
19. If  $g(x) = \sin x^3 + 4x^3$ , find  $g'(2)$ ,  $g'(-4)$ , and  $g''(1)$ .

20. The graph of  $f(x) = x^{\frac{2}{3}}$  is shown.

a. Is  $f$  continuous at  $x = 0$ ?

b. Is  $f$  differentiable at  $x = 0$ ?

If not, why not?

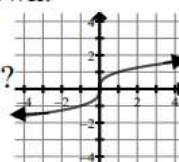


21. The graph of  $f(x) = x^{\frac{1}{3}}$  is shown.

a. Is  $f$  continuous at  $x = 0$ ?

b. Is  $f$  differentiable at  $x = 0$ ?

If not, why not?



22.  $f(x) = \begin{cases} x^2 + 1, & x \leq 0 \\ x^2 + x + 1, & x > 0 \end{cases}$

a. Is  $f$  continuous at  $x = 0$ ?

b. Is  $f$  differentiable at  $x = 0$ ?

If not, why not?

23.  $f(x) = \begin{cases} x^2 + 1, & x \leq 0 \\ -x^2 + 2, & x > 0 \end{cases}$

a. Is  $f$  continuous at  $x = 0$ ?

b. Is  $f$  differentiable at  $x = 0$ ?

If not, why not?

24.  $f(x) = \begin{cases} x^2 + 1, & x \leq 0 \\ -x^2 + 1, & x > 0 \end{cases}$

a. Is  $f$  continuous at  $x = 0$ ?

b. Is  $f$  differentiable at  $x = 0$ ?

If not, why not?

25.  $f(x) = \left\lfloor \frac{x}{2} + 1 \right\rfloor$

a. Is  $f$  continuous at  $x = 0$ ?

b. Is  $f$  differentiable at  $x = 0$ ?

If not, why not?