AP Calculus AB

SUMMER ASSIGNMENT 2024-2025

Due Date: Beginning of Class on the First Day of School

WELCOME TO AP CALCULUS AB!!

Introduction

Welcome to the next level of your math curriculum!

I'm looking forward to working with all of you this Fall! Here's a Summer Assignment for you to complete before the school year begins in September. It has some depth, so this will be your first step in time management for the course. My recommendation is to work on this an hour per week to complete it in time. The purpose of this assignment is to make sure you have mastered all the foundational skills needed to be successful in our AP Calculus AB course.

These are all topics that should have been covered in previous coursework.
I expect you to understand these concepts when you come to class in September.

Assignment

Attached is a summer worksheet with multiple problems. Answer all questions. Show all your work (can be handwritten or typed) and put a box around your final answer. **You will submit all work on the first day of class.**

You are also expected to **review and have a solid understanding of Unit 1 – Limits and Continuity**. We will spend a few days on this Unit and then test your understanding.

You must show all work and answers for each problem in order to get full credit.

Resources

You are not required to purchase a book for this class. We will be using the following book as a reference and there will be come copies in the classroom. You may rent/purchase the book if you would like.

Calculus for AP, 2nd Edition, by Ron Larson and Paul Battaglia, 2020. **ISBN:** 978-0-357-43194-8

We will be utilizing College Board's AP Classroom heavily in this course. You will be given access to the Classroom in September. Here are a few additional resources that will help with Unit 1 – Limits and Continuity, as well as other Units if you choose to look ahead:

- Krista King – Units 1-5 AP Calculus
- The Algebros – Calculus: Unit 1 – Limits and Continuity
- Khan Academy – Introduction to Limits

Name ____

PART I - Functions

1.) If $f(x) = 4x - x^2$, find:

a.) $f(4) - f(-4)$

b.) $\sqrt{f\left(\frac{3}{2}\right)}$

c.) $\frac{f(x+h) - f(x)}{2h}$

2.) If $V(r) = \frac{4}{3}\pi r^3$, find:

a.) $V\left(\frac{3}{4}\right)$

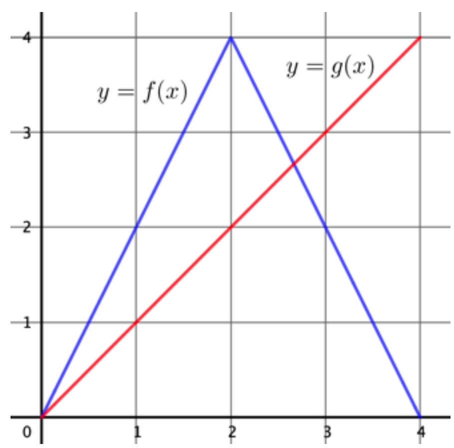
b.) $V(r+1) - V(r-1)$

c.) $\frac{V(2r)}{V(r)}$

3.) If $f(x)$ and $g(x)$ are given in the graph, find:

a.) $(f-g)(3)$

b.) $f(g(3))$



4.) If $f(x) = \begin{cases} -x, & x < 0 \\ x^2 - 1, & 0 \le x < 2 \\ \sqrt{x+2} - 2, & x \ge 2 \end{cases}$, find:

a.) $f(0) - f(2)$

b.) $\sqrt{5 - f(-4)}$

c.) $f(f(3))$

PART II – Domain and Range

Find the domain of the following functions using interval notation:

1.) $f\left(x\right)=3$

2.) $y=x^3-x^2+x$

3.) $y=\dfrac{x^3-x^2+x}{x}$

4.) $y=\dfrac{x-4}{x^2-16}$

5.) $f\left(x\right)=\dfrac{1}{4x^2-4x-3}$

6.) $y=\sqrt{2x-9}$

Find the range of the following functions:

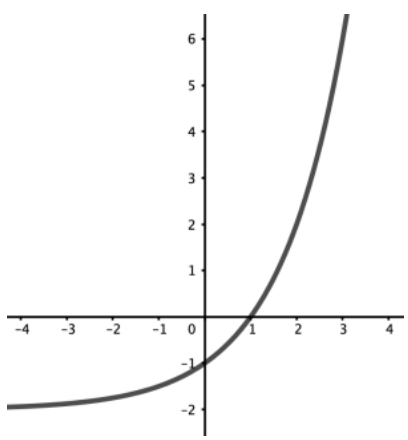
7.) $y=x^4+x^2-1$

8.) $y=100^x$

9.) $y=\sqrt{x^2+1}+1$

Find the domain and range of the following functions using interval notation.

10.)



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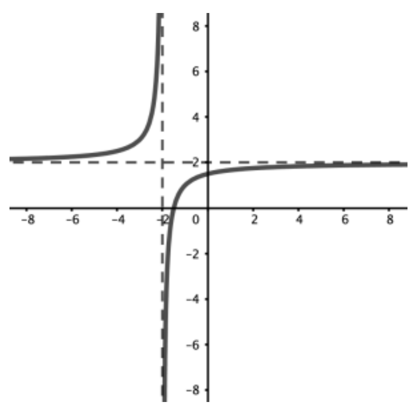

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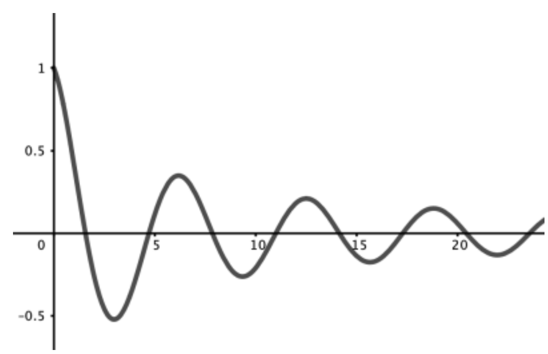
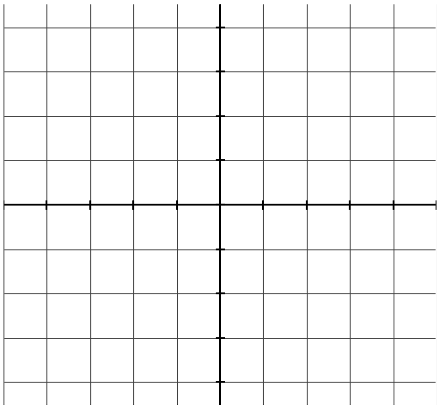


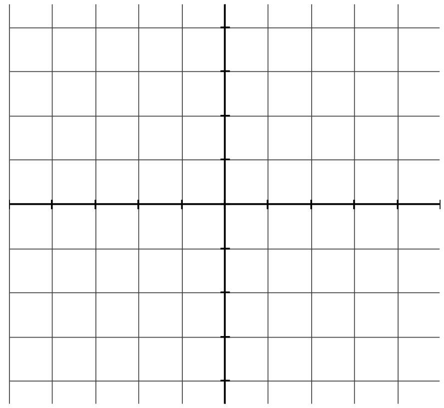
PART III – Graphs of Common Functions

Sketch each of the following as accurately as possible. **You will need to be VERY familiar with each of these graphs throughout the year**. You may use a graphing calculator for some of them if you have access to one over the summer.

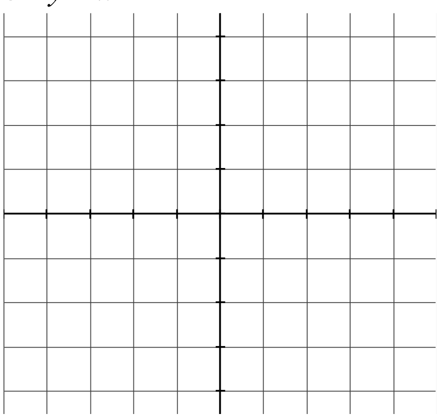
1. $y = x$



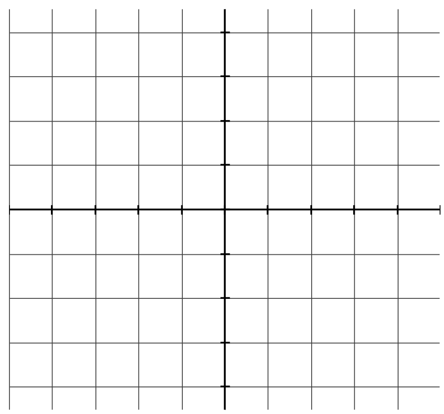
2. $y = x^2$



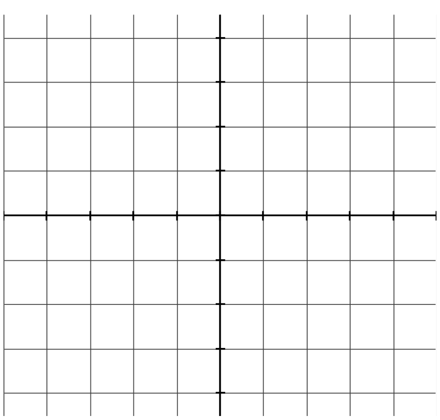
3. $y = x^3$



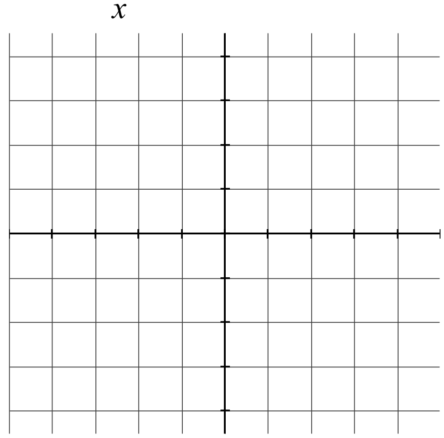
4. $y = \sqrt{x}$



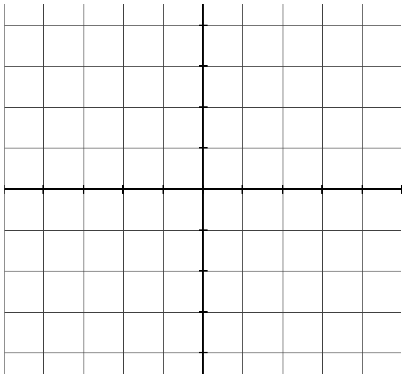
5. $y = |x|$



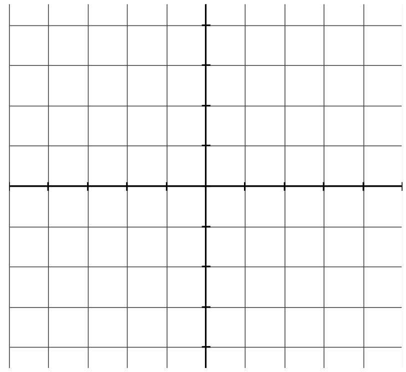
6. $y = \dfrac{|x|}{x}$



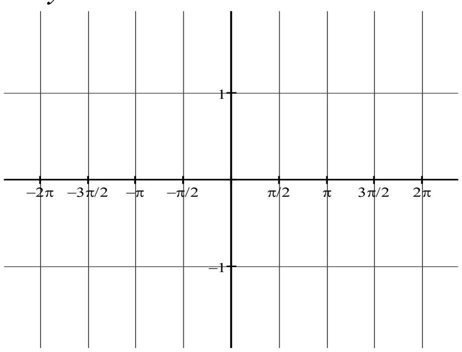
7. $y = x^{1/3}$



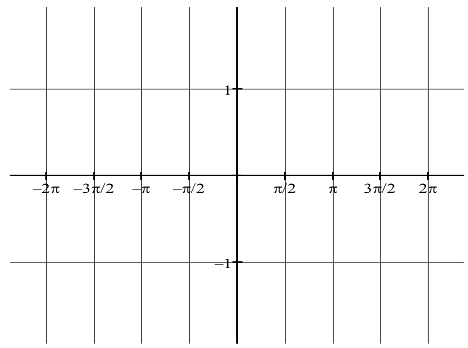
8. $y = x^{2/3}$



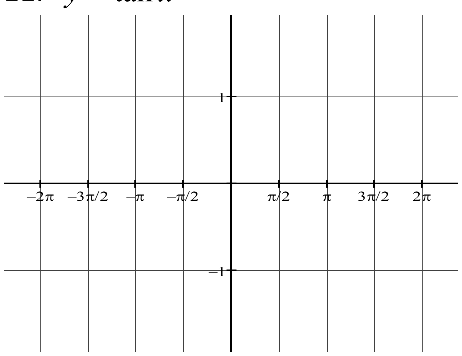
9. $y = \sin x$



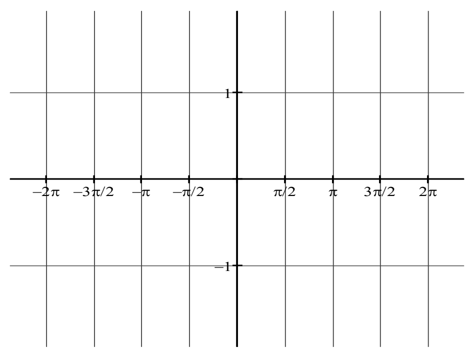
10. $y = \cos x$



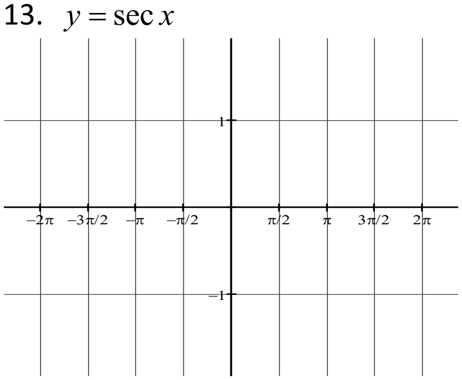
11. $y = \tan x$



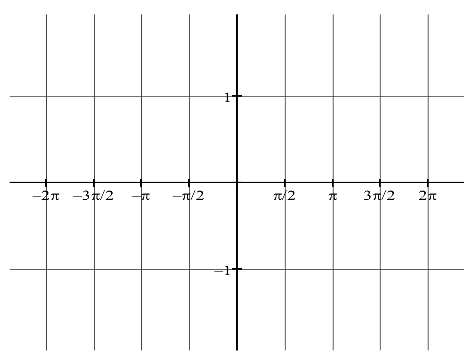
12. $y = \cot x$



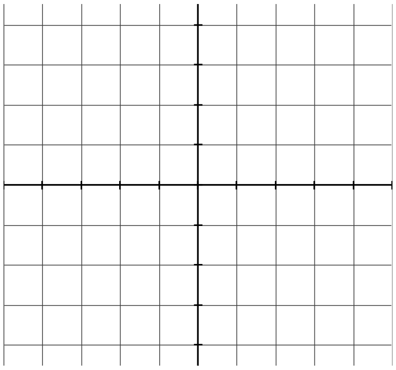
13. $y = \sec x$



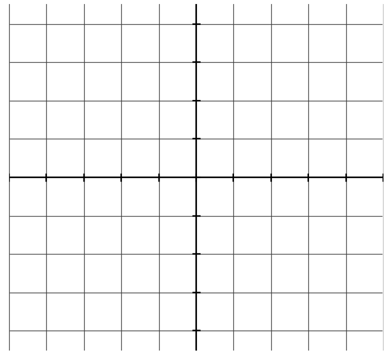
14. $y = \csc x$



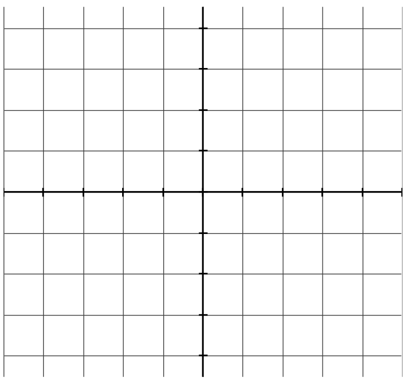
15. $y = e^x$



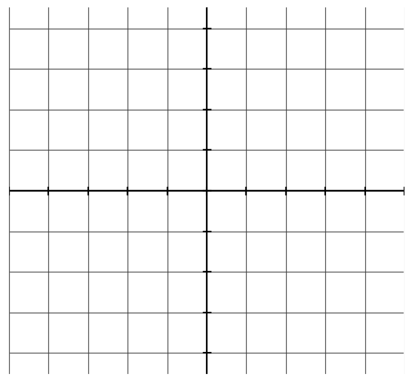
16. $y = \ln x$



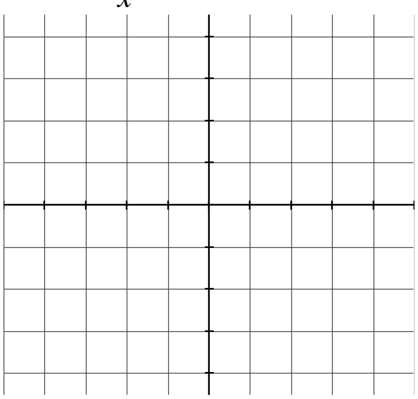
17. $y = \dfrac{1}{x}$



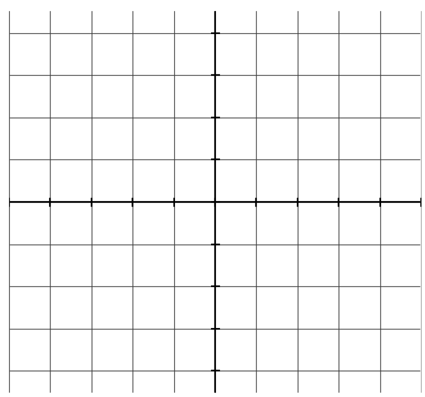
18. $y = [\![x]\!]$



19. $y = \dfrac{1}{x^2}$



20. $y = 2^x$



PART IV – Function Transformations

If $f(x) = x^2 - 1$, describe in words what the following would do to the graph of $f(x)$:

1.) $f(x) - 4$

2.) $f(x-4)$

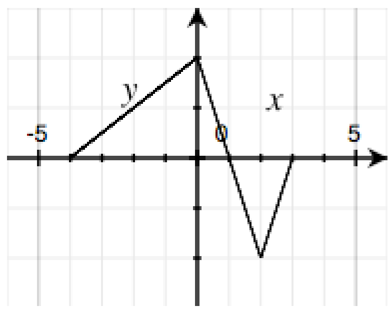
3.) $-f(x+2)$

4.) $5f(x) + 3$

5.) $f(2x)$

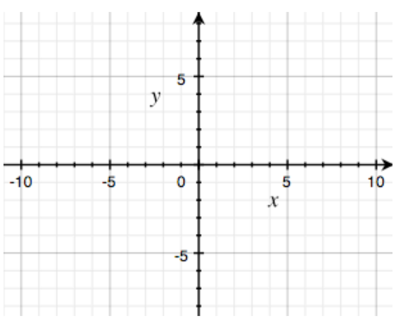
6.) $\left|f(x)\right|$

Here is a graph of $y = f(x)$:

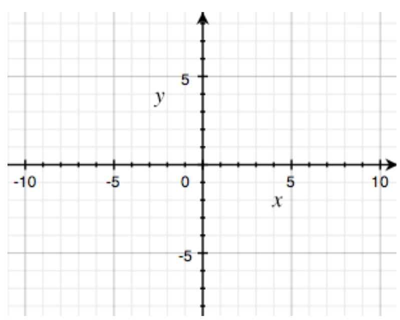


Sketch the following graphs:

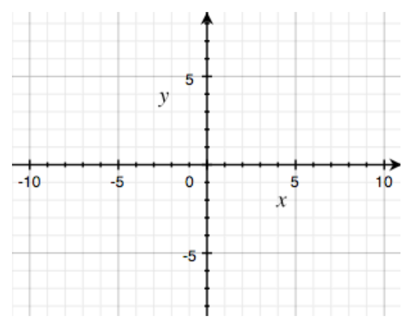
7.) $y = 2f(x)$



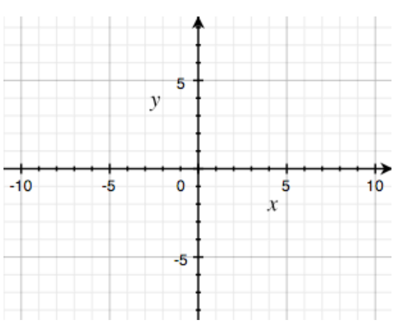
8.) $y = -f(x)$



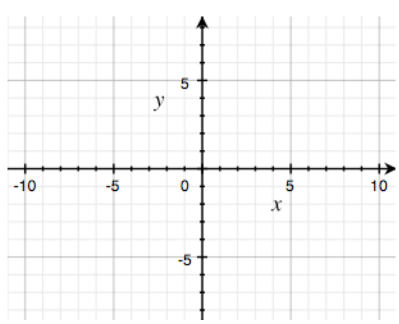
9.) $y = f(x-1)$



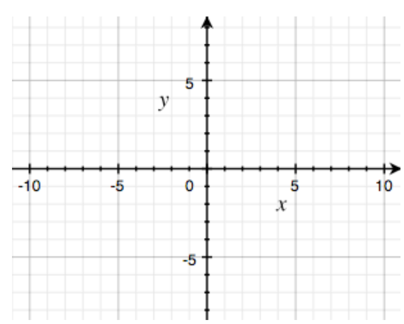
10.) $y = f(x) + 2$



11.) $y = \left|f(x)\right|$



12.) $y = f(|x|)$



PART V – Linear Functions

1.) Find the equation of the line in point-slope form, with the given slope, passing through the given point.

a.) $m = -7,\ (-3,\ -7)$ b.) $m = -\frac{1}{2},\ (2,\ -8)$ c.) $m = \frac{2}{3},\ \left(-6,\ \frac{1}{3}\right)$

2.) Find the equation of the line in point-slope form, passing through the given points.

a.) $(-3,\ 6),\ (-1,\ 2)$ b.) $(-7,\ 1),\ (3,\ -4)$ c.) $\left(-2,\ \frac{2}{3}\right),\ \left(\frac{1}{2},\ 1\right)$

3.) Find the equations of the lines through the given point that are a.) parallel and b.) normal to the given line.

a.) $(5,\ -3),\ x+y=4$ b.) $(-6,\ 2),\ 5x+2y=7$ c.) $(-3,\ -4),\ y=-2$

4.) Find the equation of the line in general form, containing the point $(4,\ -2)$ and parallel to the line containing the points $(-1,\ 4)$ and $(2,\ 3)$.

5.) Find *k* if the lines $3x-5y=9$ and $2x+ky=11$ are a.) parallel and b.) perpendicular.

PART VI - Solving Quadratic and Polynomial Equations

Solve each equation for *x* over the real number system.

1.) $x^2 + 7x - 18 = 0$

2.) $x^2 + x + \frac{1}{4} = 0$

3.) $2x^2 - 72 = 0$

4.) $12x^2 - 5x = 2$

5.) $20x^2 - 56x + 15 = 0$

6.) $81x^2 + 72x + 16 = 0$

7.) $x + \frac{1}{x} = \frac{17}{4}$

8.) $x^3 - 5x^2 + 5x - 25 = 0$

9.) $2x^4 - 15x^3 + 18x^2 = 0$

10.) If $y = x^2 + kx - k$, for what values of *k* will the quadratic have two real solutions?

PART VII: Asymptotes

For each function, find the equations of both the vertical asymptote(s) and horizontal asymptote (if it exists) and the location of any holes.

1.) $y=\dfrac{x-1}{x+5}$

2.) $y=\dfrac{8}{x^2}$

3.) $y=\dfrac{2x+16}{x+8}$

4.) $y=\dfrac{2x^2+6x}{x^2+5x+6}$

5.) $y=\dfrac{x}{x^2-25}$

6.) $y=\dfrac{x^2-5}{2x^2-12}$

7.) $y=\dfrac{x^3}{x^2+4}$

8.) $y=\dfrac{x^3+4x}{x^3-2x^2+4x-8}$

9.) $y=\dfrac{10x+20}{x^3-2x^2-4x+8}$

10.) $y=\dfrac{1}{x}-\dfrac{x}{x+2}$ (Hint: Express with a common denominator)

PART VIII - Negative and Fractional Exponents

Simplify and write with positive exponents.

1.) $-12^2 x^{-5}$

2.) $\left(-12x^5\right)^{-2}$

3.) $\left(4x^{-1}\right)^{-1}$

4.) $\left(\frac{-4}{x^4}\right)^{-3}$

5.) $\left(\frac{5x^3}{y^2}\right)^{-3}$

6.) $\left(x^3-1\right)^{-2}$

7.) $\left(121x^8\right)^{1/2}$

8.) $\left(8x^2\right)^{-4/3}$

9.) $\left(-32x^{-5}\right)^{-3/5}$

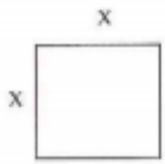
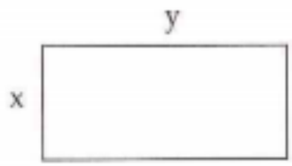
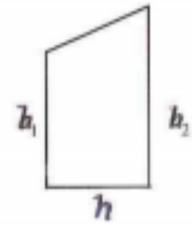
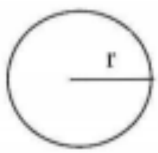
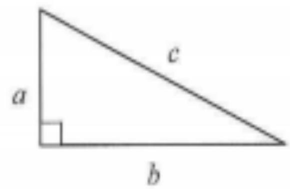
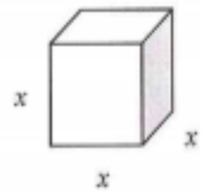
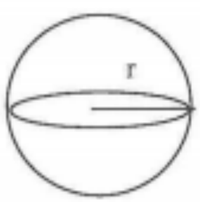
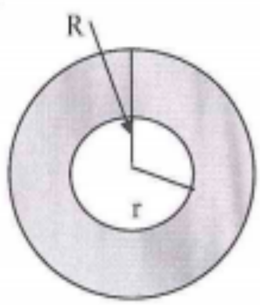
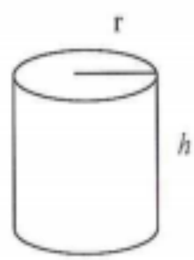
10.) $\frac{1}{4}\left(16x^2\right)^{-3/4}(32x)$

11.) $\frac{\left(x^2-1\right)^{-1/2}}{\left(x^2+1\right)^{1/2}}$

12.) $\left(x^{-2}+2^{-2}\right)^{-1}$

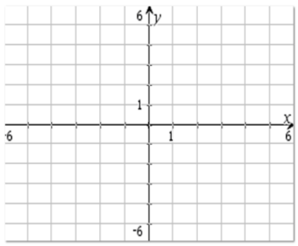
PART IX - Geometry

1.) You will use each of the following formulas in AP Calculus. Complete each of the following.

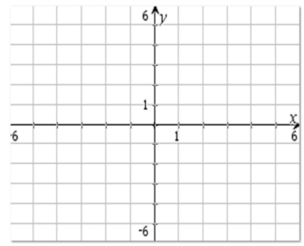
Square  Perimeter = __________ Area = __________	Rectangle  Perimeter = __________ Area = __________	Trapezoid  Area = __________
Circle  Circumference = __________ Area = __________	Triangle  Pythagorean Theorem (only good for right triangles) = ____________________ Area (of any triangle) = ______	Cube  Volume = __________ Surface Area = __________
Sphere  Volume = __________	"Washer"  Area of the shaded region = ____________________	Cylinder  Volume = __________

Find the area between the *x*-axis and *f* (*x*) from *x* = 0 to *x* = 5. Sketch the region to verify.

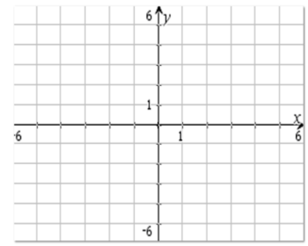
2.) $f(x)=4$



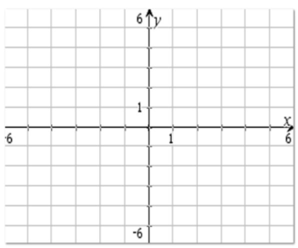
3.) $f(x)=x$



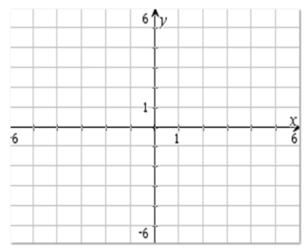
4.) $f(x)=x+3$



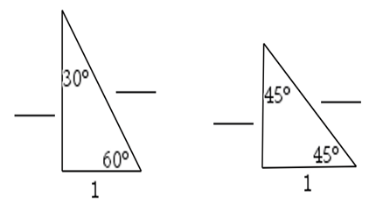
5.) $f(x)=\sqrt{9-x^2}$



6.) $f(x)=\begin{cases} x+1, & x\le 2 \\ 5-x, & x>2 \end{cases}$



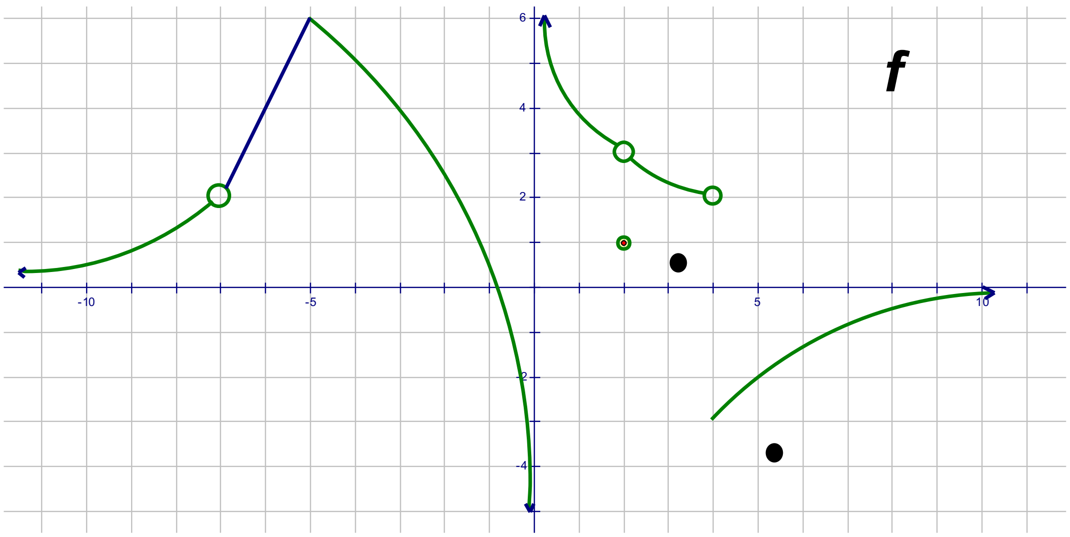
7.) Fill in the four blanks.



PART X

DO NOT USE A CALCULATOR ON ANY PROBLEM IN THIS SECTION. (Problems 1-37)

Consider the graph of function, *f*, shown below.



Answer the following questions about function *f*.

1.) $f(-5) =$

2.) $f(2) =$

3.) $f(4) =$

4.) $\lim_{x \to -7} f(x) =$

5.) $\lim_{x \to -5} f(x) =$

6.) $\lim_{x \to 2} f(x) =$

7.) $\lim_{x \to 4} f(x) =$

8.) $\lim_{x \to 0} f(x) =$

9.) $\lim_{x \to 0^-} f(x) =$

10.) $\lim_{x \to 0^+} f(x) =$

11.) $\lim_{x \to 4^+} f(x) =$

12.) $\lim_{x \to 4^-} f(x) =$

13.) $\lim_{x \to -\infty} f(x) =$

14.) $\lim_{x \to \infty} f(x) =$

15.) Use the definition of a continuous function at a number to answer the following.
Be sure to use reasons based on the definition of continuity at a point that we discussed in class.

a.) f is not continuous at x = -7 because:____

b.) f is not continuous at x = 2 because:____

c.) f is not continuous at x = 4 because:____

DO NOT USE A CALCULATOR

16.) $\lim_{x\to 2}\left(-x^2+4x\right)$	**17.)** $\lim_{x\to 9^-}\frac{\sqrt{x}-3}{x-9}$	**18.)** $\lim_{x\to 0}\frac{x}{\tan x}$
19.) $\lim_{x\to -2^+}\left(\frac{x}{x+2}\right)$	**20.)** $\lim_{x\to 0^-}\left(1+\frac{1}{x}\right)$	**21.)** $\lim_{x\to 1}\left(\sin \pi x\right)$
22.) $\lim_{x\to\infty}\frac{7-6x^5}{x+3}$	**23.)** $\lim_{t\to -\infty}\frac{6-t^3}{7t^3+3}$	**24.)** $\lim_{x\to -\infty}\frac{x-2}{x^2+2x+1}$
25.) $\lim_{y\to -\infty}\frac{2-y}{\sqrt{7+6y^2}}$	**26.)** $\lim_{x\to 2}f\left(x\right)$ when $f\left(x\right)=\begin{cases}x^2-3x+6, & \text{x} < 2\\ -x^2+3x+2, & \text{x} \geq 2\end{cases}$	**27.)** If a $\neq$ 0, then $\lim_{x\to -a}\frac{x^2-a^2}{x^4-a^4}$ is:

28.) Find a ***c*** such that $f(x)$ is continuous on the entire real line.

$$f(x)=\begin{cases} x^2 & \text{when} \quad x \le 4 \\ \dfrac{c}{x} & \text{when} \quad x > 4 \end{cases}$$

29.) Find the *x*-values (if any) at which *f* is discontinuous. Label as removable or non-removable.

$$f(x)=\frac{2x+6}{2x^2-18}$$

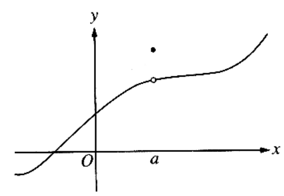
30.) Determine all of the vertical asymptotes of *f*(*x*):

$$f(x)=\frac{x+2}{x^2-4}$$

31.) True or False: If f is undefined at *x* = *c*, then the limit of *f*(*x*) as *x* approaches c does not exist.

32.) True or False: If the $\lim_{x\to c} f(x)=L$ then $f(c)=L$.

33.) The graph of the function *f* is shown to the right. Which of the following statements is false?



a.) $x=a$ is in the domain of f

b.) $\lim_{x\to a^+} f(x)$ is equal to $\lim_{x\to a^-} f(x)$

c.) $\lim_{x\to a} f(x)$ exists

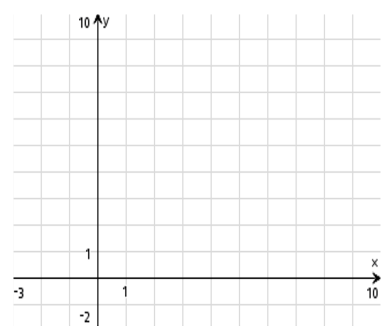
d.) $\lim_{x\to a} f(x)$ is not equal to $f(a)$

e.) f is continuous at $x=a$

34.) $\lim_{\Delta x\to 0}\dfrac{(x+\Delta x)^2-2(x+\Delta x)+1-(x^2-2x+1)}{\Delta x}$

35.) On the graph, draw a function that has the following properties:

- A step (or jump) discontinuity at x = 5
- *f*(5) = 6.



36.) Create a function such that the $\lim_{x\to 5}$ does not exist because it is approaching $+\infty$ from both the left and the right. Show both the function and the graph.

37.) Find a function *f*(*x*) such that *f*(*x*) has a hole at *x* = 7 and a vertical asymptote at *x* = -4.

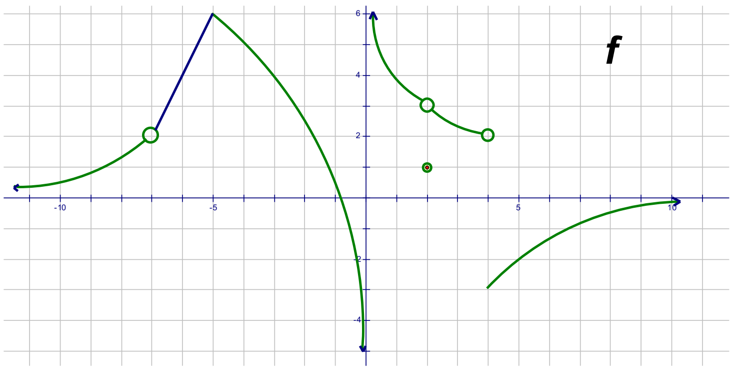
PART XI - CALCULATORS MAY BE USED ON THE FIRST PART OF THIS SECTION.

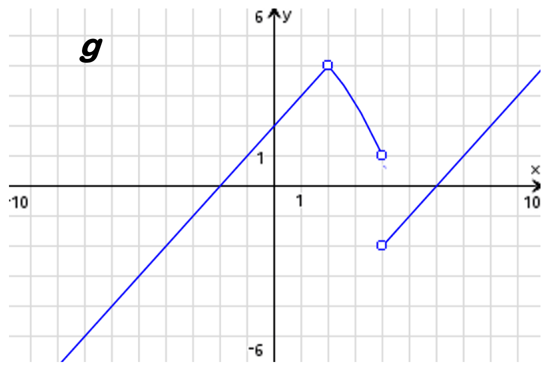
1.) Approximate the limit ***numerically*** by completing the table:

$$\lim_{x \to 2} \frac{x^2}{x-2} = \_\_\_\_\_\_\_\_\_\_$$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$							

2.) Find the limit: $\lim_{x \to 0} \left(\cos \frac{1}{x} \right)$





3.) Find $\lim_{x \to 2} f(g(x))$

4.) $\lim_{x \to 1} f(x-1) \cdot g(x)$

5.) $\lim_{x \to 1^+} \frac{f(x+1)}{g(x+3)}$

6.) No calculator. The piecewise function for $g(x)$ is below. Find the values for *a*, *b*, *c*, and *d* that make $f(x)$ continuous everywhere. Be sure to use the definition of continuity and demonstrate proper notation.

$$f(x) = \begin{cases} \frac{x^2+x-2}{x-1}, & x < 1 \\ a, & x = 1 \\ b(x-c)^2, & 1 < x < 4 \\ d, & x = 4 \\ 2x-8, & x > 4 \end{cases}$$