Due Date: Beginning of Class on the First Day of School

WELCOME TO AP CALCULUS BC!!

Introduction

Welcome to the next level of your math curriculum!

I'm looking forward to working with all of you this Fall! Here's a Summer Assignment for you to complete before the school year begins in September. It has some depth, so this will be your first step in time management for the course. My recommendation is to work on this an hour per week to complete it in time. The purpose of this assignment is to make sure you have mastered all the foundational skills needed to be successful in our AP Calculus BC course.

These are all topics that should have been covered in previous coursework.
I expect you to understand these concepts when you come to class in September.

Assignment

Attached is a summer worksheet with multiple problems. Answer all questions. Show all your work (can be handwritten or typed) and put a box around your final answer. **You will submit all work on the first day of class.**

__You must show all work and answers for each problem in order to get full credit.__

Resources

You are not required to purchase a book for this class. We will be using the following book as a reference and there will be come copies in the classroom. You may rent/purchase the book if you would like.

Calculus for AP, 2nd Edition, by Ron Larson and Paul Battaglia, 2020. **ISBN:** 978-0-357-43194-8

Name _____

PART I – NO CALCULATORS MAY BE USED ON THIS SECTION.

1. At time $t = 0$, a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius ($^\circ C$) at time $t = 0$, and the internal temperature of the potato is greater than $27\,^\circ C$ for all times $t > 0$. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\dfrac{dH}{dt} = -\dfrac{1}{4}(H - 27)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 91$.

    a) Write an equation for the line tangent to the graph of H at $t = 0$. Use this equation to approximate the internal temperature of the potato at time $t = 3$.

    b) Use $\dfrac{d^2 H}{dt^2}$ to determine if your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time $t = 3$.

    c) For $t < 10$, an alternate model for the internal temperature of the potato at time t minutes is the function G that satisfies the differential equation $\dfrac{dG}{dt} = -(G - 27)^{2/3}$, where G is measured in degrees Celsius and $G(0) = 91$. Find an expression for $G(t)$. Based on this model, what is the internal temperature of the potato at $t = 3$?

2. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after is is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

b) Find $\dfrac{d^2 B}{dt^2}$ in terms of B. Use $\dfrac{d^2 B}{dt^2}$ to explain why the graph of B cannot resemble the graph below and to the right.
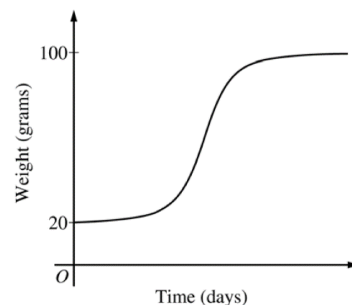
c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.
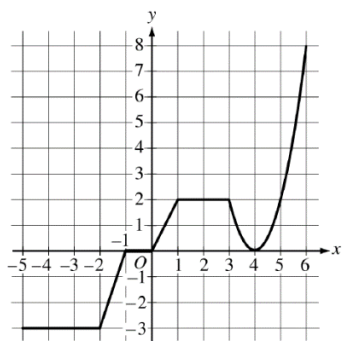
Graph of g

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

(b) Evaluate $\displaystyle\int_{1}^{6} g(x)\,dx.$

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

PART II - CALCULATORS MAY BE USED ON THIS SECTION.

1. Fish enter a lake at a rate modeled by the function E given by $E(t) = 20 + 15\sin\left(\dfrac{\pi t}{6}\right)$. Fish leave the lake at

a rate modeled by the function L given by $L(t) = 4 + 2^{0.1t^2}$. Both $E(t)$ and $L(t)$ are measured in fish per hour, and t is measured in hours since midnight ($t = 0$).

(a) How many fish enter the lake over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$)? Give your answer to the nearest whole number.

(b) What is the average number of fish that leave the lake per hour over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$)?

(c) At what time t, for $0 \le t \le 8$, os the greatest number of fish in the lake? Justify your answer.

(d) Is the rate of change in the number of fish in the lake increasing or decreasing at 5 A.M. ($t = 5$)? Explain your reasoning.

2. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\dfrac{t}{100}\right)^3\left(1-\dfrac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.
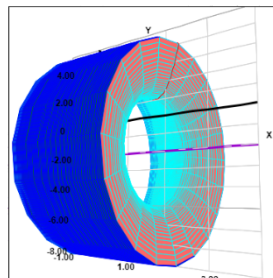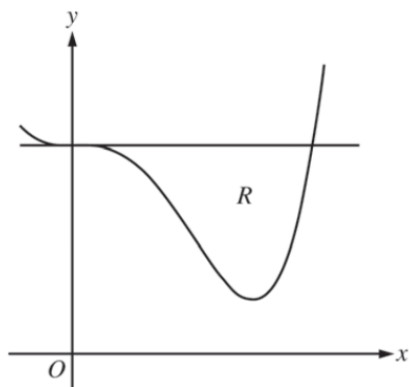
3. Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

(a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

(c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not evaluate, an equation involving integral expressions whose solution gives the value of k.
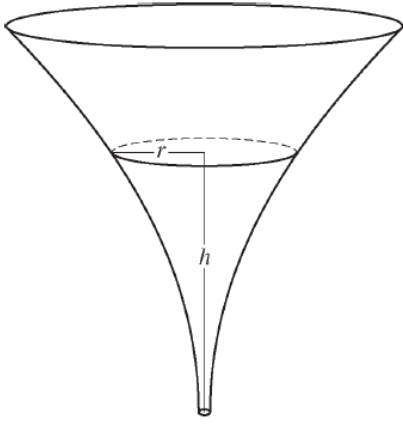
4. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}(3 + h^2)$, where $0 \le h \le 10$. The units of r and h are inches.
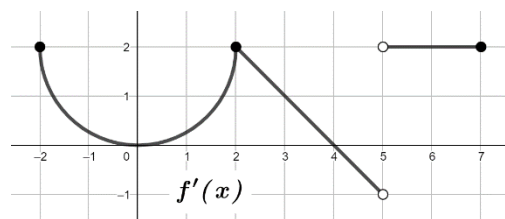
(a) Find the average value of the radius of the funnel.

(b) Find the volume of the funnel.

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time.

PART III

$$h(x) = \begin{cases} \dfrac{\sin(x-1)}{x-1}, & x < 1 \\[2mm] kx^2 - 8x + 6, & 1 \le x \le 3 \\[2mm] 4e^{2x-6} - x^2 + 5, & x > 3 \end{cases}$$

| x	$g''(x)$
-4	13
-1	10
0	8
2	e
3	-1

Consider $f'(x)$, the derivative of the continuous function f, defined on the closed interval $[-6, 7]$ except at $x = 5$. A portion of f' is given in the graph above and consists of a semi-circle and two line segments. The function $h(x)$ is a piecewise defined function given where k is a constant. The function $g(x)$ and its derivatives are differentiable. Selected values for the decreasing function $g''(x)$, the second derivative of g are given in the table above.

(**A**) Find the value of k such that $h(x)$ is continuous at $x = 3$. Show your work.

(**B**) Using the value of k found in part (A), is $h(x)$ continuous at $x = 1$? Justify your answer.

(**C**) Is there a time $c, -4 < c < 3$ such that $g'''(c) = -2$? Give a reason for your answer.

(**D**) For each $x = 2$ and $x = 4$, determine if $f(x)$ has a local minimum, local maximum or neither. Give a reason for your answer.
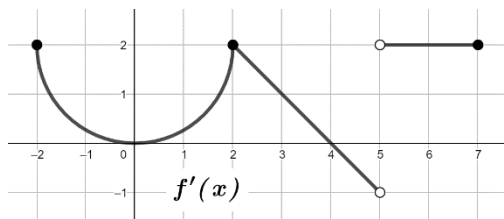
$$h(x) = \begin{cases} \dfrac{\sin(x-1)}{x-1}, & x < 1 \\ kx^2 - 8x + 6, & 1 \le x \le 3 \\ 4e^{2x-6} - x^2 + 5, & x > 3 \end{cases}$$

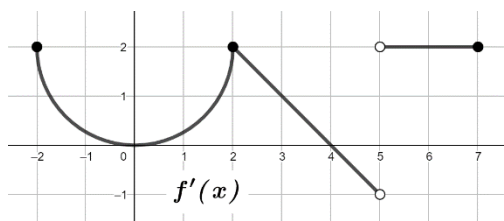| x	$g''(x)$
-4	13
-1	10
0	8
2	e
3	-1

(**E**) Find all x value(s) on the open interval $(-2, 5)$ where $f(x)$ has a point of inflection. Give a reason for your answer.

(**F**) Find the average rate of change of $h(x)$, in terms of k, over the interval [2,5].

(**G**) If $f(3) = 5$, write an equation of the tangent line to $f(x)$ at $x = 3$.

(**H**) Use a right Riemann sum with the four subintervals indicated in the table to approximate $\displaystyle\int_{-4}^{3} g''(x)\,dx$.

Is this approximation an over or under estimate? Give a reason for your answer.

(**I**) Evaluate $\displaystyle\int_{0}^{7} f'(x)\,dx$.

$$h(x) = \begin{cases} \dfrac{\sin(x-1)}{x-1}, & x < 1 \\[2mm] kx^2 - 8x + 6, & 1 \le x \le 3 \\[2mm] 4e^{2x-6} - x^2 + 5, & x > 3 \end{cases}$$

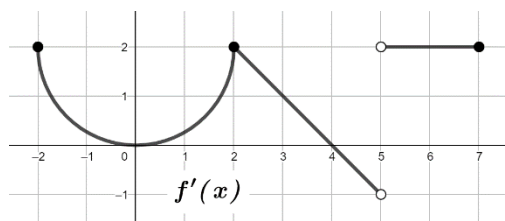| x	$g''(x)$
-4	13
-1	10
0	8
2	e
3	-1

(J) Let $k(x) = x^2 + \int_1^x f'(t)\,dt$. Find the values for $k'(2)$ and $k''(2)$ or state that it does not exist.

(K) Find $h'(4)$.

(L) Let $m(x) = f'(x)g'\left(\frac{x}{2}\right)$. Find $m'(6)$.

(M) Let $p(x) = f(x^2 - 1)$. Find $p'(2)$.

(N) Find the average value of $f'(x)$ over the interval $[2,5]$.

$$h(x) = \begin{cases} \dfrac{\sin(x-1)}{x-1}, & x < 1 \\[2mm] kx^2 - 8x + 6, & 1 \le x \le 3 \\[2mm] 4e^{2x-6} - x^2 + 5, & x > 3 \end{cases}$$

$f'(x)$

x	$g''(x)$
-4	13
-1	10
0	8
2	e
3	-1

(**O**) Evaluate $\displaystyle\int_{-1}^{3} [2g'''(x) + 7]\,dx$
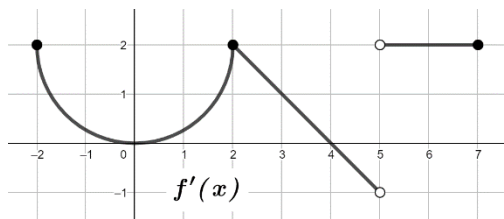
(**P**) If $\displaystyle\int_{-6}^{2} f'(x)\,dx = 5 - 2\pi$, then find $\displaystyle\int_{-2}^{-6} f'(x)\,dx$.

(**Q**) For $0 \le t \le 2.5$, a particle is moving along a horizontal axis with velocity $v(t) = \ln\big(g''(t)\big)$.
Is the particle speeding up or slowing down at time $t = 2$? Give a reason for your answer.

(**R**) Let x be the number of people, in thousands, inside an amusement park. The number of people
inside the park that have contracted a virus can be modeled by $v(x) = \dfrac{h(x)}{3x}$ for $3 < x < 5$.
The number of people in the park is increasing at a constant rate of 0.2 thousands of people per minute.
Using this model, what is the rate that people inside the park are contracting the virus with respect
to time when there are four thousand people in the park?

$$h(x) = \begin{cases} \dfrac{\sin(x-1)}{x-1}, & x < 1 \\[2mm] kx^2 - 8x + 6, & 1 \le x \le 3 \\[2mm] 4e^{2x-6} - x^2 + 5, & x > 3 \end{cases}$$
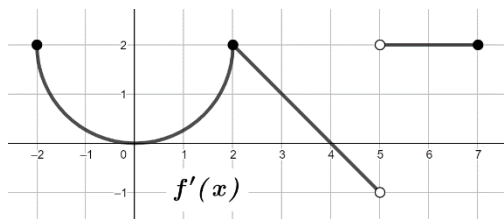
x	$g''(x)$
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3	-1

(S) $\displaystyle \lim_{x \to 2} \frac{\int_4^x f'(t)\,dt + x}{\sin(x^2 - 4)}$

(T) Let $k = 0$, evaluate $\displaystyle \int_2^4 h(x)\,dx$.

(U) Is there a time $c, -4 < c < 3$, such that $g''(c) = 0$? Give a reason for your answer.

(V) Estimate $g'''(-2)$. Show the calculations that lead to your answer.

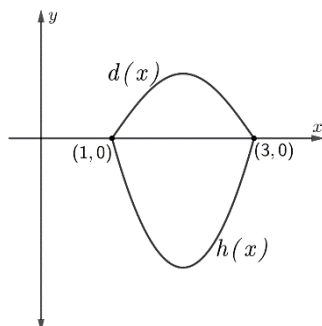(W) For $-6 \le x \le -2, f'(x) = \dfrac{1}{4}(x+4)^3$. If $f(-2) = 0$, find the minimum value of $f(x)$ on $[-6,2]$.

$$h(x) = \begin{cases} \dfrac{\sin(x-1)}{x-1}, & x < 1 \\[2mm] kx^2 - 8x + 6, & 1 \le x \le 3 \\[2mm] 4e^{2x-6} - x^2 + 5, & x > 3 \end{cases}$$

x	$g''(x)$
-4	13
-1	10
0	8
2	e
3	-1

(**X**) Let $y = r(x)$ be the particular solution to the differential equation $\dfrac{dy}{dx} = \dfrac{h(x) + x^2}{y}$ for $x > 3$.

Find the particular solution $y = r(x)$ given the initial condition $(4, -2e)$.

(**Y**) The graphs of $d(x) = -\sin\left(\dfrac{\pi x}{2} + \dfrac{1}{2}\right)$ and $h(x)$ are shown above for $1 \le x \le 3$ when $k = 2$.

Find the area bounded by the graphs of $d(x)$ and $h(x)$.

(**Z**) Set up, but do not evaluate, an expression involving one or more integrals that gives the volume when the region bounded by the graphs above is revolved about the line $y = -5$.