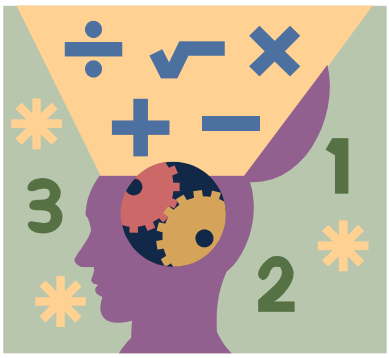
Name_____

For students entering AP Calculus AB/BC in September 2025



Summer is a time to relax and have fun, and we hope that you enjoy it! We also would like to give you the opportunity to review and reinforce your math skills to avoid "brain drain" this summer. We have created this work packet so that you focus your review in the most important areas of mathematics from the course you completed this year. Please set aside some time each week to keep your skills sharp. This packet will be completed and reviewed as a portion of the initial homework assignments given in September.

Name : Solutions

AP Calculus AB/BC preparation packet

Directions: Place all answers on the answer sheet attached. All support work **must be attached** and clearly labeled with the problem number in chronological order.

SKETCHING:
For each function below, make an accurate sketch WITHOUT the use of a graphing calculator. Be sure to use at least three key points and state the domain, range, asymptotes (if any), and zeros. Grids are attached at the end of the packet.

(see pages 5 and 6)

1. $y = 4x - x^2$
2. $f(x) = 2^{-x} + 3$
3. $f(x) = \dfrac{x+3}{x^2-9}$
4. $f(s) = \sqrt{s-4}$
5. $f(t) = (t-4)^{\frac{2}{3}}$
6. $f(x) = \begin{cases} |x|, & x \le 2 \\ 3-x, & x > 2 \end{cases}$
7. $f(x) = \ln(x+1)$
8. $f(x) = e^x$
9. $f(x) = \arctan x$
10. $f(\theta) = -\cos 2\theta$

SOLVE ALGEBRAICALLY:
For problems 11 – 16, ALGEBRAICALLY solve each equation for the missing variable.

(see page 4)

11. $e^{2x} - 4e^x + 3 = 0$
12. $\log_2 x + \log_2(x-7) = 3$
13. $3\ln(2x) = 12$
14. $3^x + 1 = 5$
15. $2\sin x \cos x = -\sin x$ for x in the domain $[0, 2\pi)$.
16. $|3x-4| \le 2$

No calculator!!

SIMPLIFY COMPLETELY:
For problems 17 – 23, simplify COMPLETELY. Justify ALL answers.

17. $\dfrac{\sin^2(15) + \cos^2(15)}{\sin\left(\frac{\pi}{6}\right)\sec\left(\frac{\pi}{3}\right)}$ $= \dfrac{1}{\frac{1}{2}\cdot 2} = 1$
18. $\dfrac{\frac{1}{x} - \frac{1}{x-1}}{1 - \frac{1}{x}}$ $\dfrac{3x+2}{2x+1}$
19. $\dfrac{\frac{x^2-1}{3x+2}}{\frac{x+1}{3x^2-x-2}}$ $= (x-1)^2$
20. $2[3\ln x - \ln(x+1) - \ln(x-1)]$ $\ln\left(\frac{x^3}{x^2-1}\right)^2$
21. $\ln\dfrac{\sqrt{x}y^4}{z^4}$ $\frac{1}{2}\ln x + 4\ln y - 4\ln z$
22. $\dfrac{2x^{\frac{1}{3}}\left(3x^{\frac{1}{3}} - 4x^{\frac{4}{3}}\right)}{2x^{-\frac{1}{3}}}$ $= 3x - 4x^2$
23. a. $\ln\dfrac{1}{\sqrt{e}}$ $-1/2$

b. $e^{2\ln x}$ $e^{\ln x^2}$ x^2

c. $\log_{27} 9$ $27^x = 9$, $3^{3x} = 3^2$, $x = 2/3$

25. Use long division to divide $\frac{x^2}{x^2+1}$.

$$\begin{array}{r} 1 \\ x^2+1 \overline{)\, x^2} \\ -(x^2+1) \\ \hline -1 \end{array}$$

$$1 + \frac{-1}{x^2+1}$$

FACTORING:

For problems 27-31, factor as indicated. ~~Rewrite with positive exponents, if necessary.~~ Record your final solutions on the answer sheet.

27. $81x^4 - 16$ $(9x^2-4)(9x^2+4)$ $(3x-2)(3x+2)(9x^2+4)$

28. $e^{-x} - xe^{-x} + 2x^2e^{-x}$ $e^{-x}(1-x+2x^2)$

29. $(x-3)^3(x+2) - 2(x-3)^2(x+2)^2$ $(x-3)^2(x+2)(x-3-2(x+2))$ $(x-3)^2(x+2)(-x-7)$

30. $6x^2 + 13x - 5$ $(3x-1)(2x+5)$

31. $2\sqrt{x} + 6x^{3/2} = 2\sqrt{x}(\quad 1+3x \quad)$

FUNCTIONS:

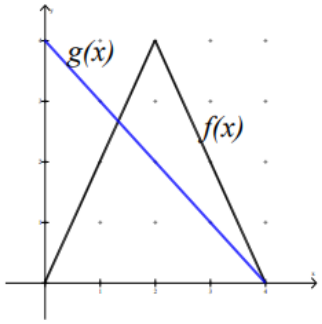
32. If $f(x) = \frac{1}{x}$, find the simplified formula for the difference quotient $\frac{f(x+h)-f(x)}{h}$ $= \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{-1}{x(x+h)}$

For questions 33 – 35, use the given graph of functions $f(x)$ and $g(x)$ to answer the following questions.

33. $f(g(2))$ $= f(2) = 4$

34. $g(f(2))$ $= g(4) \neq 0$

35. $f^{-1}(4) + g^{-1}(3)$ $2 + 1 = 3$



36. Sketch the graph of the rational function below WITHOUT TECHNOLOGY. Be sure to state the domain, range, horizontal and vertical asymptotes, and the coordinates of any hole(s). CAREFULLY sketch and provide a minimum of 4 coordinates clearly labeled on your graph.

$$f(x) = \frac{x^2-1}{x^2+2x-3} = \frac{(x-1)(x+1)}{(x+3)(x-1)} = \frac{x+1}{x+3}$$

hole @ $(1, 1/2)$

V.A. $x = -3$

H.A. $y = 1$

Use the graph of the rational function to determine the following limits.

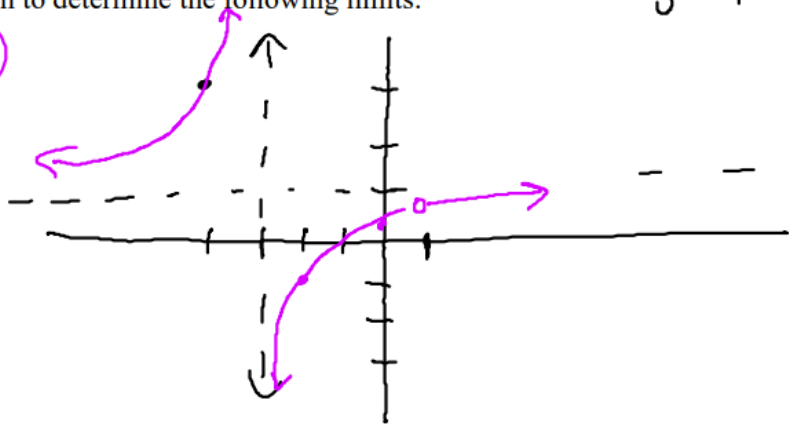
a. $\lim_{x\to 3} f(x) = \frac{3+1}{3+3} = \frac{4}{6} = 2/3$

b. $\lim_{x\to 1} f(x)$ $1/2$

c. $\lim_{x\to -3^-} f(x)$ ∞

d. $\lim_{x\to -3^+} f(x)$ $-\infty$

e. $\lim_{x\to \infty} f(x)$ 1



3

(11) $(e^x-3)(e^x-1)=0$

$e^x-3=0 \quad e^x-1=0$

$e^x=3 \quad e^x=1$

$x=\ln 3, 0$

(12) $\log_2(x(x-7))=3$

$2^3=x^2-7x$

$0=x^2-7x-8$

$0=(x-8)(x+1)$

$x=8, -1$

$x=8$

(13) $e^{\ln(2x)}=e^4$

$x=e^4/2$

(14) $3^x=4$

$\ln 3^x=\ln 4$

$x=\frac{\ln 4}{\ln 3}$

(15) $\sin x(2\cos x+1)=0$

$\sin x=0 \quad \cos x=-1/2$

$x=0, \pi$ $\quad x=2\pi/3, 4\pi/3$

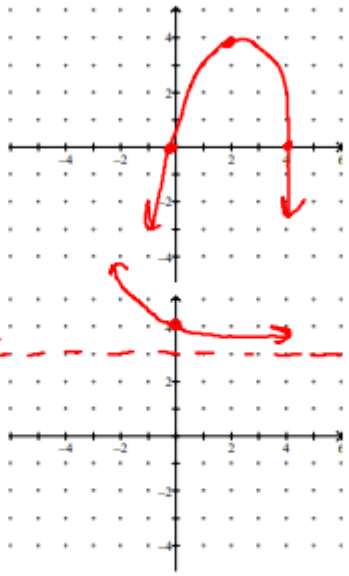
(16) $-2\le 3x-4\le 2$

$2\le 3x\le 6$

$\frac{2}{3}\le x\le 2$

ANSWER SHEET

1. Domain: $(-\infty, \infty)$
   Range: $(-\infty, 4]$
   Zeros: $x = 0, 4$
   Asymptotes: none



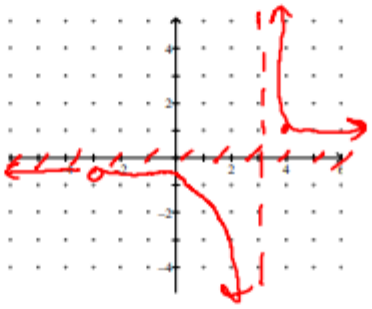
$4x - x^2 = 0$

$x(4 - x) = 0$

$x = 0, 4$

2. Domain: $(-\infty, \infty)$
   Range: $(3, \infty)$
   Zeros: None
   Asymptotes: $y = 3$

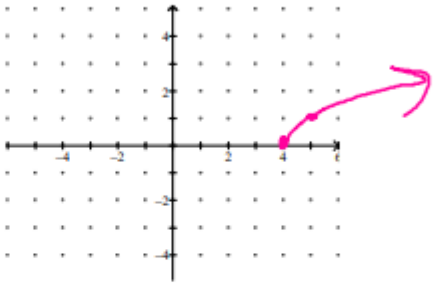
3. Domain: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$
   Range: $(-\infty, -1/6) \cup (-1/6, 0) \cup (0, \infty)$
   Zeros: None
   Asymptotes: $x = 3$, $y = 0$



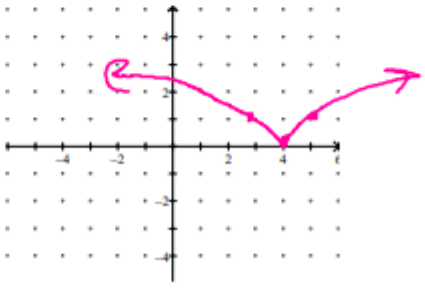
$f(x) = \frac{x+3}{(x+3)(x-3)} = \frac{1}{x-3}$

hole at $(-3, -1/6)$

4. Domain: $[4, \infty)$
   Range: $[0, \infty)$
   Zeros: $s = 4$
   Asymptotes: None



5. Domain: $(-\infty, \infty)$
   Range: $[0, \infty)$
   Zeros: $t = 4$
   Asymptotes: None



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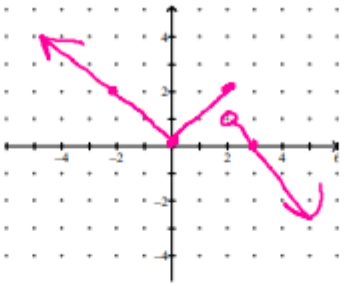
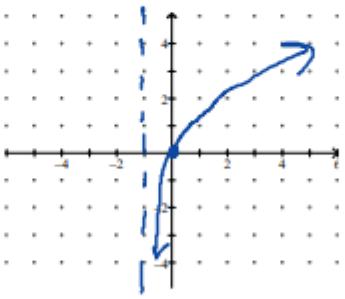
6. Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Zeros: $x = 0, 3$
Asymptotes: None

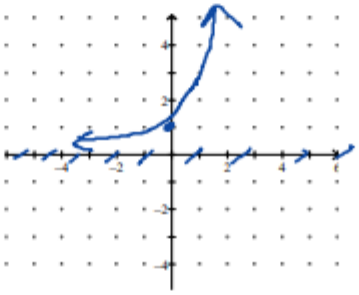


7. Domain: $(-1, \infty)$
Range: $(-\infty, \infty)$
Zeros: $x = 0$
Asymptotes: $x = -1$

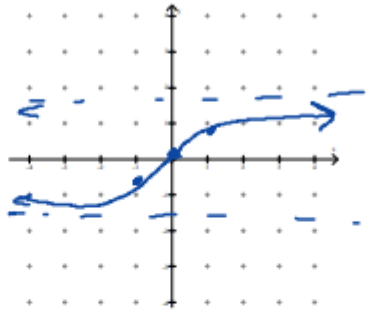


$\ln(x+1)$
$x+1 > 0$
$x > -1$

8. Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Zeros: None
Asymptotes: $y = 0$



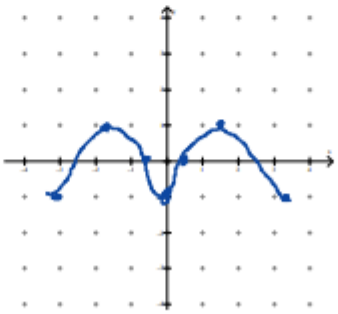
9. Domain: $(-\infty, \infty)$
Range: $(-\pi/2, \pi/2)$
Zeros: $x = 0$
Asymptotes: $y = \pi/2$
$y = -\pi/2$



$\tan^{-1} x$

x	y
-1	$-\pi/4$
1	$\pi/4$
0	0

10. Domain: $(-\infty, \infty)$
Range: $[-1, 1]$
Zeros: $x = \pi/4 + \pi/2 K$
Asymptotes: None



$f(\theta) = -\cos 2\theta$
period $= \pi$
increment $= \pi/4$