

# Things to Know for Calculus

## TRIGONOMETRY

### Trig Functions

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

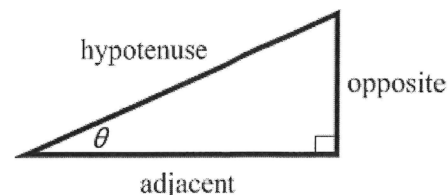
### Reciprocal Functions

$$\csc \theta = \frac{1}{\sin \theta} = \frac{\text{hyp}}{\text{opp}}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{\text{hyp}}{\text{adj}}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{\text{adj}}{\text{opp}}$$

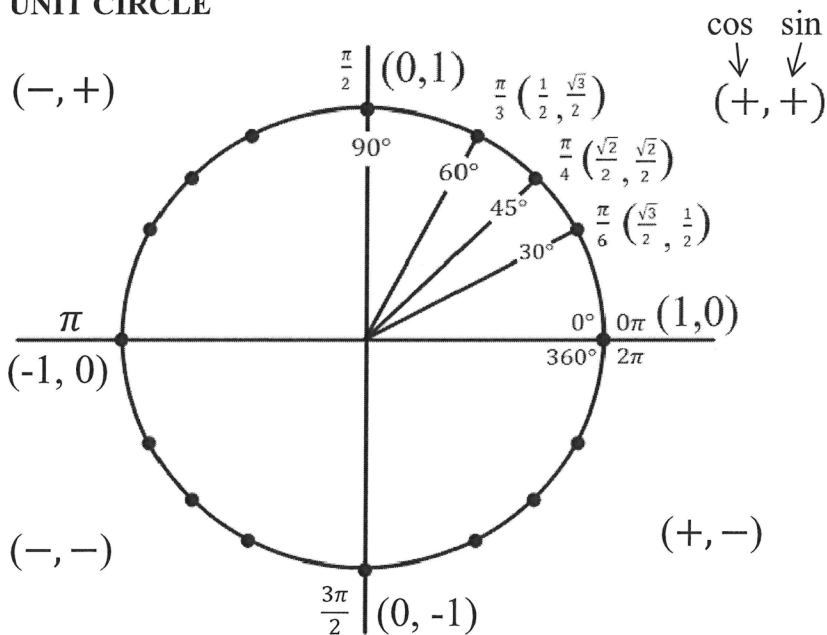
SOH-CAH-TOA



### TEST ONLY USES RADIANS!

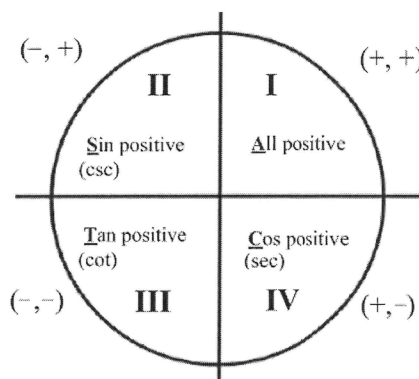
Must know trig values of special angles  $0\pi, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$  using Unit Circle or Special Right Triangles.

### UNIT CIRCLE



To help remember the signs in each quadrant

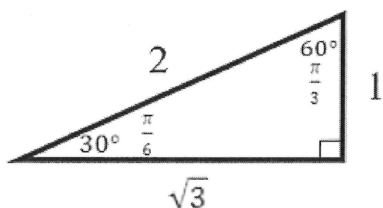
All Students Take Calculus



### SPECIAL RIGHT TRIANGLES

$30^\circ - 60^\circ - 90^\circ$  Triangles

Which are  $\frac{\pi}{6} - \frac{\pi}{3} - \frac{\pi}{2}$  Triangles

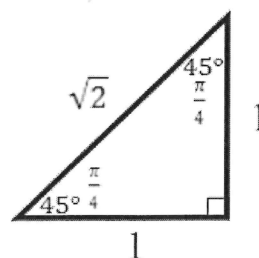


Find  $\tan\left(\frac{\pi}{6}\right)$

$$\tan\left(\frac{\pi}{6}\right) = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}} \text{ simplify to } \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$45^\circ - 45^\circ - 90^\circ$  Triangles

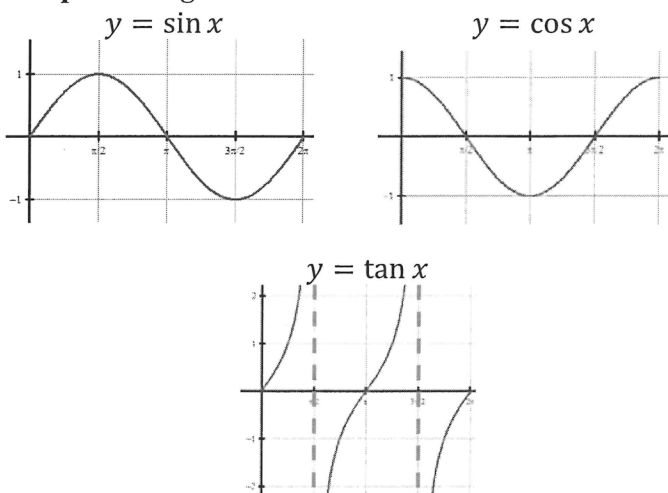
Which are  $\frac{\pi}{4} - \frac{\pi}{4} - \frac{\pi}{2}$  Triangles



Find  $\sin\left(\frac{\pi}{4}\right)$

$$\sin\left(\frac{\pi}{4}\right) = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{2}} \text{ simplify to } \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

## Graphs of trig functions



## Inverse Trig Function

$\sin^{-1}\theta$  is the same as  $\arcsin \theta$

$\sin^{-1}\theta = \left(\frac{\sqrt{3}}{2}\right)$  means what angle has a sine value of  $\frac{\sqrt{3}}{2}$

that means  $\theta = \frac{\pi}{3} \pm 2\pi n$  or  $\frac{2\pi}{3} \pm 2\pi n$

Since  $\theta$  has infinite answers then it isn't a function. Bummer. To make it a function we define inverses like:

$\sin/\csc$  and  $\tan/\cot$  use quadrant I and IV for inverses  
 $\cos/\sec$  use quadrant I and II for inverses

So...  $\theta = \frac{\pi}{3}$  because it is in the first quadrant

## Trig Identities

There are a bunch, but you really only need to know Pythagorean Identity.  **$\sin^2 x + \cos^2 x = 1$**

Subtract  $\sin^2 x$  to get  $\cos^2 x = 1 - \sin^2 x$  or subtract  $\cos^2 x$  to get  $\sin^2 x = 1 - \cos^2 x$

Divide by  $\sin^2 x$  to get  $1 + \cot^2 x = \csc^2 x$  or divide by  $\cos^2 x$  to get  $\tan^2 x + 1 = \sec^2 x$

## GEOMETRY

### FORMULAS

#### AREA

$$\text{Triangle} = \frac{1}{2}bh$$

$$\text{Circle} = \pi r^2$$

$$\text{Trapezoid} = \frac{1}{2}(b_1 + b_2)h$$

#### CIRCUMFERENCE

$$\text{Circle} = 2\pi r$$

#### SURFACE AREA

$$\text{Sphere} = 4\pi r^2$$

#### LATERAL AREA

$$\text{Cylinder} = 2\pi rh$$

#### VOLUME

$$\text{Sphere} = \frac{4}{3}\pi r^3$$

$$\text{Cylinder} = \pi r^2 h$$

$$\text{Cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Prism} = Bh$$

$$\text{Pyramid} = \frac{1}{3}Bh$$

$B$  is the area of the base

#### DISTANCE FORMULA

The distance between two points  $(x_1, y_1)$  and  $(x_2, y_2)$  is  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

## ALGEBRA

### Linear Functions

Slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

y-intercept Form

(slope-intercept Form)

$$y = mx + b$$

Point Slope Form

$$y - y_1 = m(x - x_1)$$

Parallel Lines

Have the same slope

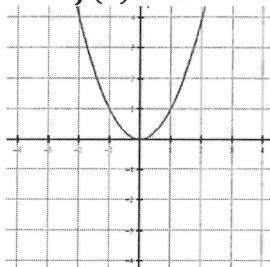
Perpendicular Lines

Have the opposite reciprocal slopes

## Functions

Quadratic Function

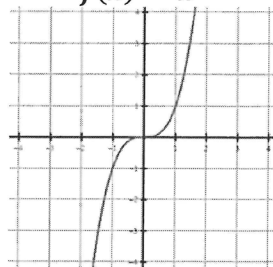
$$f(x) = x^2$$



$$y = a(x - h)^2 + k$$

Cubic Function

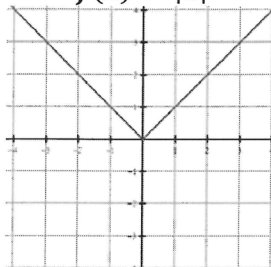
$$f(x) = x^3$$



$$y = a(x - h)^3 + k$$

Absolute Value

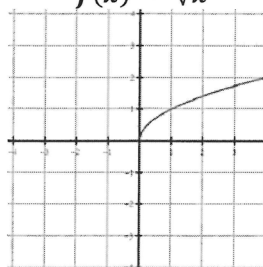
$$f(x) = |x|$$



$$y = a|x - h| + k$$

Square Root Function

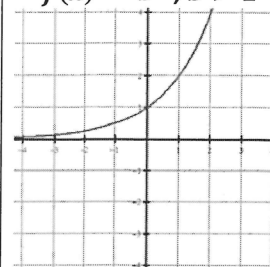
$$f(x) = \sqrt{x}$$



$$y = a\sqrt{x - h} + k$$

Exponential Function

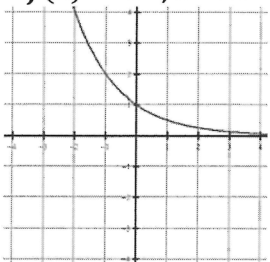
$$f(x) = b^x, b > 1$$



$$y = a \cdot b^{(x-h)} + k$$

Exponential Function

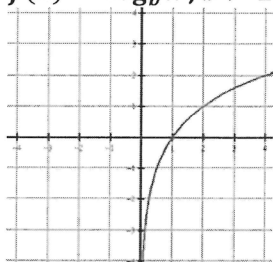
$$f(x) = b^x, b < 1$$



$$y = a \cdot b^{(x-h)} + k$$

Logarithmic Function

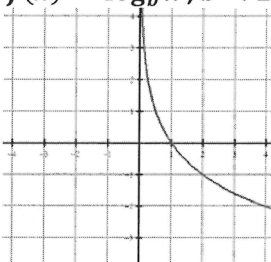
$$f(x) = \log_b x, b > 1$$



$$y = a \log_b(x - h) + k$$

Logarithmic Function

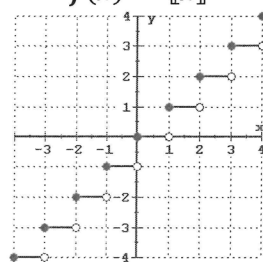
$$f(x) = \log_b x, b < 1$$



$$y = a \log_b(x - h) + k$$

Greatest Integer

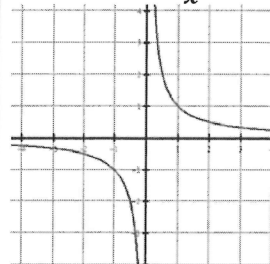
$$f(x) = \llbracket x \rrbracket$$



$$y = a\llbracket x - h \rrbracket + k$$

Rational Function

$$f(x) = \frac{1}{x}$$



$$y = \frac{a}{x - h} + k$$

## Translations

All functions move the same way!

Given the parent function  $y = x^2$

Move up 4

$$y = x^2 + 4$$

Move down 3

$$y = x^2 - 3$$

Move left 2

$$y = (x + 2)^2$$

Move right 1

$$y = (x - 1)^2$$

Move left 2 and down 3

$$y = (x + 2)^2 - 3$$

To flip (reflect) the function vertically  $y = -x^2$

To flip (reflect) the function horizontally  $y = (-x)^2$

So  $f(x) = -\sqrt{x - 3} + 1$  is a square root function reflected vertically, shifted right 3 and up 1

## Notation

Notice open parenthesis ( ) versus closed [ ]

Inequality

$$-3 < x \leq 5$$



Interval

$$(-3, 5]$$

$$-3 \leq x \leq 5$$



$$[-3, 5]$$

$$-3 < x < 5$$



$$(-3, 5)$$

$$-3 \leq x < 5$$



$$[-3, 5)$$

Infinity is always open parenthesis

Inequality

$$x < 3$$



Interval

$$(-\infty, 3)$$

$$x \leq 3 \text{ or } x > 5$$



$$(-\infty, 3] \cup (5, \infty)$$

$$x \neq 3$$



$$(-\infty, 3) \cup (3, \infty)$$

all Real numbers



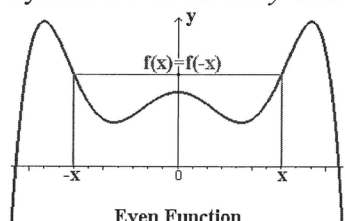
$$(-\infty, \infty)$$

## Even and Odd Functions

EVEN

$$f(-x) = f(x)$$

Symmetric about the y-axis

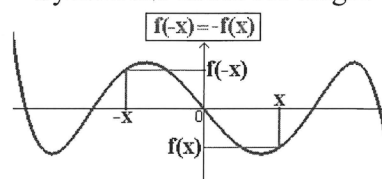


Even Function

ODD

$$f(-x) = -f(x)$$

Symmetric about the origin



Odd Function

## Domain and Range

Domain = all possible  $x$  values

Range = all possible  $y$  values

Algebraically  
You can't divide by zero  
You can't square root a negative

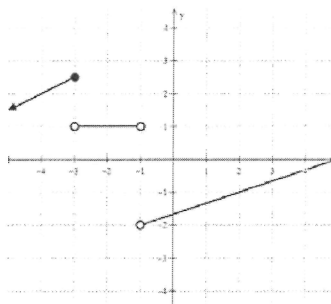
$$y = \sqrt{2x + 5}$$

$$D: [-\frac{5}{2}, \infty)$$

$$y = \frac{x^2 - 1}{x^2 + 7x + 12}$$

$$D: (-\infty, -4)(-4, -3)(-3, \infty)$$

Graphically  
Just look at it



$$D: (-\infty, -1)(-1, 5]$$

$$R: (-\infty, 2.5]$$

## Finding zeros

Must be able to factor and use the quadratic formula:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

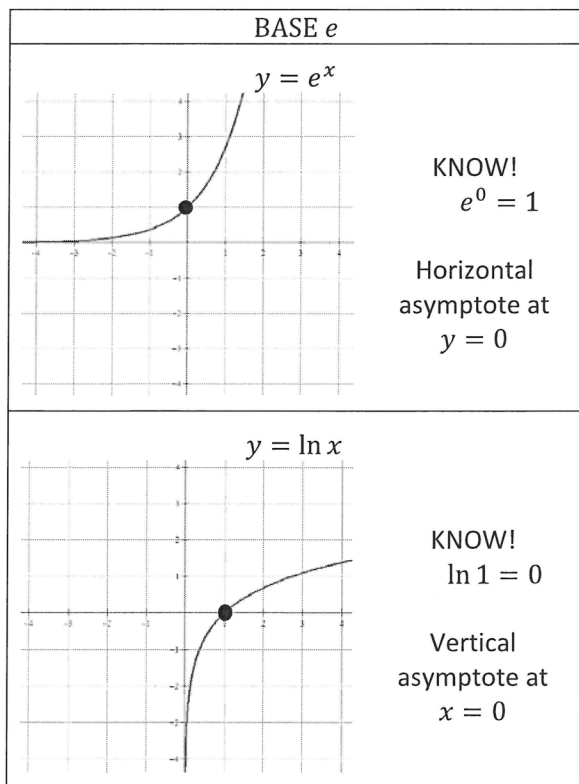
## Special products

Sum of cubes:  $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Difference of cubes:  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

## Exponential and Logarithmic Properties

The exponential function  $b^x$  of base  $b$  is one-to-one which means it has an inverse which is called the logarithmic function of base  $b$  or logarithm of base  $b$  which is denoted  $\log_b x$  which reads "the logarithm of base  $b$  of  $x$ " or "log base  $b$  of  $x$ ". So...



$$y = \log_b x \longleftrightarrow x = b^y$$

Exponential		Logarithmic
$b^x b^y = b^{x+y}$	Product Rule	$\log_b xy = \log_b x + \log_b y$
$\frac{b^x}{b^y} = b^{x-y}$	Quotient Rule	$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$
$(b^x)^y = b^{xy}$	Power Rule	$\log_b x^y = y \log_b x$
$b^{-x} = \frac{1}{b^x}$		$\log_b \left(\frac{1}{x}\right) = -\log_b x$
$b^0 = 1$		$\log_b 1 = 0$
$b^1 = b$		$\log_b b = 1$
	Change of Base	$\log_b x = \frac{\log_c x}{\log_c b}$
	Natural Log	$\log_e x = \ln x$
	Common Log	$\log_{10} x = \log x$



# Calculus - SUMMER PACKET

NAME: \_\_\_\_\_

Summer + Math = (Best Summer Ever)<sup>2</sup>

**NO CALCULATOR!!!**

Given  $f(x) = x^2 - 2x + 5$ , find the following.

1.  $f(-2) =$

2.  $f(x + 2) =$

3.  $f(x + h) =$

Use the graph  $f(x)$  to answer the following.

4.  $f(0) =$

$f(4) =$

$f(-1) =$

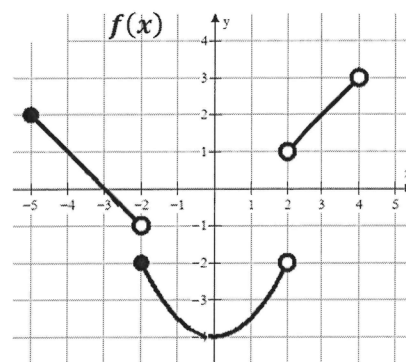
$f(-2) =$

$f(2) =$

$f(3) =$

$f(x) = 2$  when  $x = ?$

$f(x) = -3$  when  $x = ?$



Write the equation of the line meets the following conditions. Use point-slope form.

$y - y_1 = m(x - x_1)$

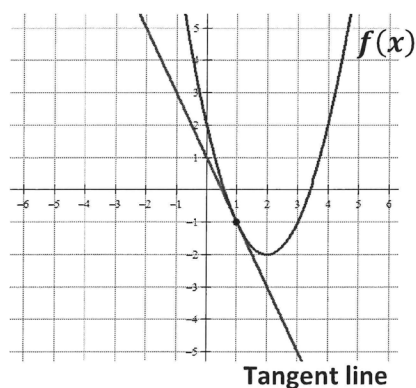
5. slope = 3 and  $(4, -2)$

6.  $m = -\frac{3}{2}$  and  $f(-5) = 7$

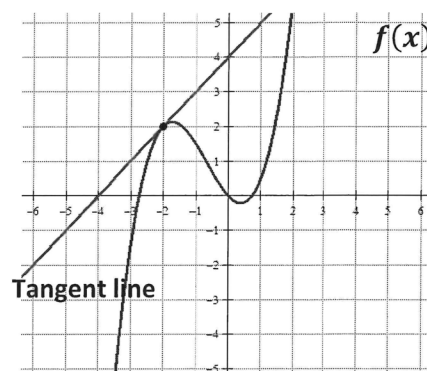
7.  $f(4) = -8$  and  $f(-3) = 12$

Write the equation of the tangent line in point slope form.  $y - y_1 = m(x - x_1)$

8. The line tangent to  $f(x)$  at  $x = 1$



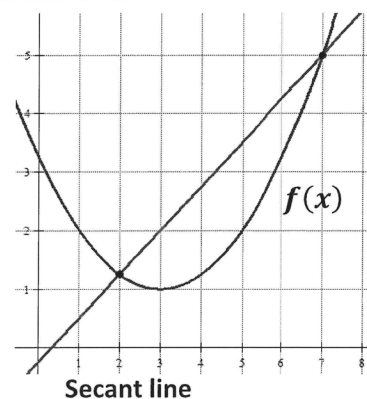
9. The line tangent to  $f(x)$  at  $x = -2$



**MULTIPLE CHOICE!** Remember slope  $= \frac{y_2 - y_1}{x_2 - x_1}$

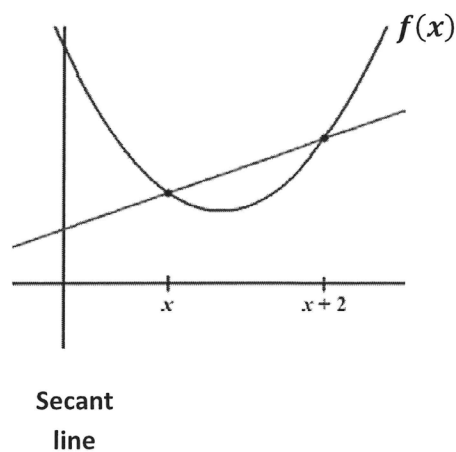
10. Which choice represents the slope of the secant line shown?

- A)  $\frac{7-2}{f(7)-f(2)}$     B)  $\frac{f(7)-2}{7-f(2)}$     C)  $\frac{7-f(2)}{f(7)-2}$     D)  $\frac{f(7)-f(2)}{7-2}$



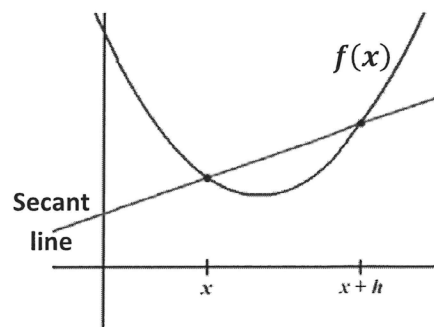
11. Which choice represents the slope of the secant line shown?

- A)  $\frac{f(x)-f(x+2)}{x+2-x}$     B)  $\frac{f(x+2)-f(x)}{x+2-x}$     C)  $\frac{f(x+2)-f(x)}{x-(x+2)}$
- D)  $\frac{x+2-x}{f(x)-f(x+2)}$



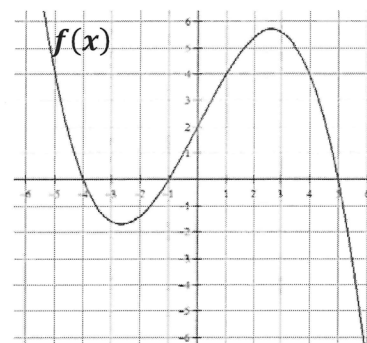
12. Which choice represents the slope of the secant line shown?

- A)  $\frac{f(x+h)-f(x)}{x-(x+h)}$     B)  $\frac{x-(x+h)}{f(x+h)-f(x)}$     C)  $\frac{f(x+h)-f(x)}{x+h-x}$
- D)  $\frac{f(x)-f(x+h)}{x+h-x}$



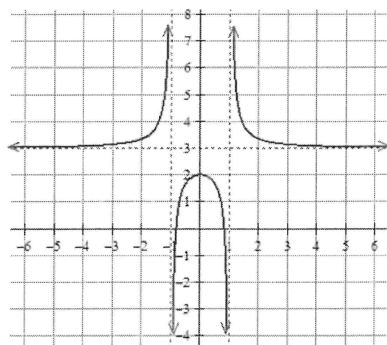
13. Which of the following statements about the function  $f(x)$  is true?

- I.  $f(2) = 0$   
 II.  $(x + 4)$  is a factor of  $f(x)$   
 III.  $f(5) = f(-1)$
- (A) I only  
 (B) II only  
 (C) III only  
 (D) I and III only  
 (E) II and III only



**Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.**

14.



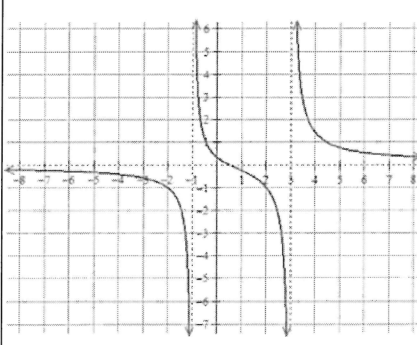
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

15.



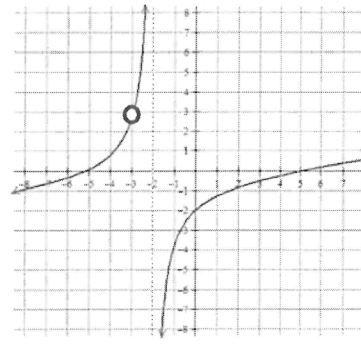
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

16.



Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

**MULTIPLE CHOICE!**

17. Which of the following functions has a vertical asymptote at  $x = 4$  ?

(A)  $\frac{x+5}{x^2-4}$

(B)  $\frac{x^2-16}{x-4}$

(C)  $\frac{4x}{x+1}$

(D)  $\frac{x+6}{x^2-7x+12}$

(E) None of the above

18. Consider the function:  $f(x) = \frac{x^2-5x+6}{x^2-4}$ . Which of the following statements is true?

I.  $f(x)$  has a vertical asymptote of  $x = 2$

II.  $f(x)$  has a vertical asymptote of  $x = -2$

III.  $f(x)$  has a horizontal asymptote of  $y = 1$

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II and III

**Rewrite the following using rational exponents. Example:  $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$**

19.  $\sqrt[5]{x^3} + \sqrt[5]{2x}$

20.  $\sqrt{x+1}$

21.  $\frac{1}{\sqrt{x+1}}$

22.  $\frac{1}{\sqrt{x}} - \frac{2}{x}$

23.  $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$

24.  $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$

**Write each expression in radical form and positive exponents. Example:  $x^{-\frac{2}{3}} + x^{-2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x^2}$**

25.  $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$

26.  $\frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$

27.  $3x^{-\frac{1}{2}}$

28.  $(x+4)^{-\frac{1}{2}}$

29.  $x^{-2} + x^{\frac{1}{2}}$

30.  $2x^{-2} + \frac{3}{2}x^{-1}$

**Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.**

31. $\sin \frac{\pi}{6}$	32. $\cos \frac{\pi}{4}$	33. $\sin 2\pi$
34. $\tan \pi$	35. $\sec \frac{\pi}{2}$	36. $\cos \frac{\pi}{6}$
37. $\sin \frac{\pi}{3}$	38. $\sin \frac{3\pi}{2}$	39. $\tan \frac{\pi}{4}$
40. $\csc \frac{\pi}{2}$	41. $\sin \pi$	42. $\cos \frac{\pi}{3}$
43. Find $x$ where $0 \leq x \leq 2\pi$ , $\sin x = \frac{1}{2}$	44. Find $x$ where $0 \leq x \leq 2\pi$ , $\tan x = 0$	45. Find $x$ where $0 \leq x \leq 2\pi$ , $\cos x = -1$

**Solve the following equations. Remember  $e^0 = 1$  and  $\ln 1 = 0$ .**

46. $e^x + 1 = 2$	47. $3e^x + 5 = 8$	48. $e^{2x} = 1$
49. $\ln x = 0$	50. $3 - \ln x = 3$	51. $\ln(3x) = 0$
52. $x^2 - 3x = 0$	53. $e^x + xe^x = 0$	54. $e^{2x} - e^x = 0$

Solve the following trig equations where  $0 \leq x \leq 2\pi$ .

55. $\sin x = \frac{1}{2}$	56. $\cos x = -1$	57. $\cos x = \frac{\sqrt{3}}{2}$
58. $2\sin x = -1$	59. $\cos x = \frac{\sqrt{2}}{2}$	60. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$
61. $\tan x = 0$	62. $\sin(2x) = 1$	63. $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$

For each function, determine its domain and range.

<u>Function</u>	<u>Domain</u>	<u>Range</u>
64. $y = \sqrt{x - 4}$		
65. $y = (x - 3)^2$		
66. $y = \ln x$		
67. $y = e^x$		
68. $y = \sqrt{4 - x^2}$		

Simplify.

69. $\frac{\sqrt{x}}{x}$	70. $e^{\ln x}$	71. $e^{1+\ln x}$
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72. $\ln 1$	73. $\ln e^7$	74. $\log_3 \frac{1}{3}$
75. $\log_{1/2} 8$	76. $\ln \frac{1}{2}$	77. $27^{\frac{2}{3}}$
78. $(5a^{2/3})(4a^{3/2})$	79. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$	80. $(4a^{5/3})^{3/2}$
<b>If <math>f(x) = \{(3, 5), (2, 4), (1, 7)\}</math>      <math>g(x) = \sqrt{x-3}</math>  <math>h(x) = \{(3, 2), (4, 3), (1, 6)\}</math>      <math>k(x) = x^2 + 5</math> , then determine each of the following.</b>		
81. $(f+h)(1)$	82. $(k-g)(5)$	83. $f(h(3))$
84. $g(k(7))$	85. $h(3)$	86. $g(g(9))$
87. $f^{-1}(4)$	88. $k^{-1}(x)$	
89. $k(g(x))$	90. $g(f(2))$	

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

# Review

## 1 Review – Limits

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 1.

### 1.1 Limits Graphically:

What is a limit?

The **y-value** a function approaches at a given **x-value**.

Give the value of each statement. If the value does not exist, write "does not exist" or "undefined."

1.  $\lim_{x \rightarrow 3} f(x) =$

5.  $\lim_{x \rightarrow 2} f(x) =$

2.  $\lim_{x \rightarrow 1} f(x) =$

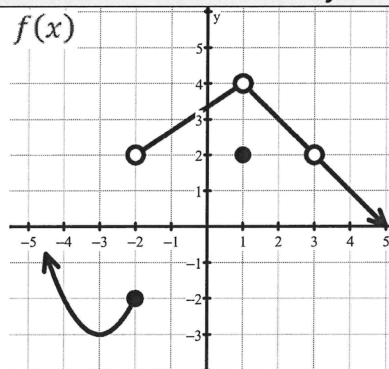
6.  $\lim_{x \rightarrow -2^+} f(x) =$

3.  $f(3) =$

7.  $f(1) =$

4.  $f(-2) =$

8.  $\lim_{x \rightarrow -2^-} f(x) =$



### 1.2 Limits Analytically:

#### Finding a limit:

1. Direct Substitution.
2. Simplify and then try direct substitution.
  - a. Factor and Cancel.
  - b. Rationalize if you see square roots.
3. L'Hôpital's Rule (for indeterminate forms  $\frac{0}{0}$  or  $\frac{\infty}{\infty}$ )

#### Special Trig Limits:

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = \quad \text{or} \quad \lim_{x \rightarrow 0} \frac{x}{\sin x} =$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = \quad \text{or} \quad \lim_{x \rightarrow 0} \frac{\cos x - 1}{x} =$$

Evaluate each limit.

9.  $\lim_{x \rightarrow -4} (2x^2 + 3x - 2)$

10.  $\lim_{x \rightarrow 1} \sqrt{7x + 42}$

11.  $\lim_{x \rightarrow 13} 2$

12.  $\lim_{x \rightarrow 10} \frac{x^2 - 5x - 50}{x - 10}$



$$13. \lim_{x \rightarrow 0} \frac{\sqrt{x+19} - \sqrt{19}}{x}$$

$$14. \lim_{x \rightarrow 0} \frac{\frac{1}{x+1} - 1}{x}$$

$$15. \lim_{x \rightarrow 0} \frac{\sin(7x)}{11x}$$

$$16. \lim_{x \rightarrow 0} \frac{\sin^2(3x)}{\sin^2(5x)}$$

### 1.3 Asymptotes:

#### Vertical Asymptotes:

If the denominator equals 0, then there is a hole or a vertical asymptote. If the factor does not cancel, then it's a vertical asymptote.

One-sided limits at vertical asymptotes approach  $-\infty$  or  $\infty$ .

#### Horizontal asymptotes:

$\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)}$  will produce a horizontal asymptote at

- $y = 0$  if  $g$  increases faster than  $f$ .
- $y = \frac{a}{b}$  if  $g$  and  $f$  are increasing at the relative same amount where  $a$  and  $b$  are the coefficients of the fastest growing terms.

Don't forget to check the left and right sides when looking for horizontal asymptotes.

Evaluate each limit.			Find all horizontal asymptotes.
17. $\lim_{x \rightarrow \infty} \frac{4x^5 - 2x^2 + 3}{3x^2 + 2x^5 - x^4}$	18. $\lim_{x \rightarrow \infty} x^5 3^{-x}$	19. $\lim_{x \rightarrow \infty} \sin \frac{x + 3\pi x^2}{2x^2}$	20. $f(x) = \frac{\sqrt{16x^6 + x^3 + 5x}}{5x^3 - 8x}$

### 1.4 Continuity:

#### Types of Discontinuities:

1. Removable (hole).
2. Discontinuity due to vertical asymptote.
3. Jump discontinuity.

#### Finding Domain:

Restrictions occur with two scenarios:

1. Denominators can't be zero.
2. Even radicals can't be negative.

Don't forget the Intermediate Value Theorem (for continuous functions)! What is it and what does it tell us?

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

# Review

## 1 Review – Limits

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 1.

### 1.1 Limits Graphically:

What is a limit?

The **y-value** a function approaches at a given **x-value**.

Give the value of each statement. If the value does not exist, write "does not exist" or "undefined."

1.  $\lim_{x \rightarrow 3} f(x) =$

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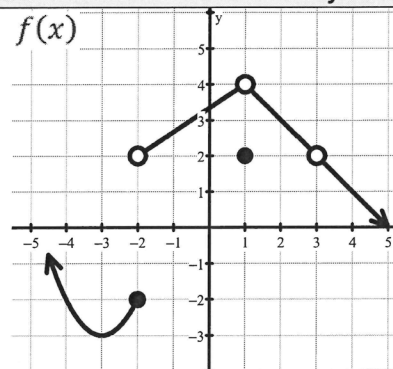
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