



UNIVERSITY OF DETROIT JESUIT
HIGH SCHOOL AND ACADEMY

Algebra I Summer Packet

- The following set of problems are designed to reinforce skills and to provide practice with fundamental concepts learned in prior grades.
- This will be collected on the first day of school in the fall.
- Show your work for these problems in the packet.
- Pace yourself working on these throughout the summer. These should not be completed the evening before the first day of school!

Have a good summer and we will see you in August!

Order of Operations

To avoid having different results for the sample problem, mathematicians have agreed on an order of operations when simplifying expressions that contain multiple operations.

1. Perform any operation(s) inside grouping symbols:
parentheses (), brackets [] above or below a fraction bar
2. Simply any term with exponents
3. Multiply and divide in order from left to right
4. Add and subtract in order from left to right

One easy way to remember the order of operations process is to remember the acronym **PEMDAS** or the old saying "Please Excuse My Dear Aunt Sally."

P	Perform operations in grouping symbols like <u>P</u> arentheses
E	Simplify <u>E</u> xponents
M	Perform <u>M</u> ultiplication and <u>D</u> ivision in order from left to right
D	
A	Perform <u>A</u> ddition and <u>S</u> ubtraction in order from left to right
S	

Example 1	Example 2
$2 - 3^2 + (6 + 3 \times 2)$	$-7 + 4 + (2^3 - 8 \div -4)$
$2 - 3^2 + (6 + 6)$	$-7 + 4 + (8 - 8 \div -4)$
$2 - 3^2 + 12$	$-7 + 4 + (8 - -2)$
$2 - 9 + 12$	$-7 + 4 + 10$
$-7 + 12$	$-3 + 10$
= 5	= 7

Order of Operations

Evaluate each expression. Remember your order of operations process (**PEMDAS**).

1. $6 + 4 - 2 \cdot 3 =$	2. $(-2) \cdot 3 + 5 - 7 =$
3. $15 \div 3 \cdot 5 - 4 =$	4. $29 - 3 \cdot 9 + 4 =$
5. $20 - 7 \cdot 4 =$	6. $4 \cdot 9 - 9 + 7 =$

7. $50 - (17 + 8) =$	8. $(12 - 4) \div 8 =$
9. $12 \cdot 5 + 6 \div 6 =$	10. $18 - 4^2 + 7 =$
11. $3(2 + 7) - 9 \cdot 7 =$	12. $3 + 8 \cdot 2^2 - 4 =$
13. $16 \div 2 \cdot 5 \cdot 3 \div 6 =$	14. $12 \div 3 - 6 \cdot 2 - 8 \div 4 =$
15. $10 \cdot (3 - 6^2) + 8 \div 2 =$	16. $6.9 - 3.2 \cdot (10 \div 5) =$
17. $32 \div [16 \div (8 \div 2)] =$	18. $[10 + (2 \cdot 8)] \div 2 =$
19. $180 \div [2 + (12 \div 3)] =$	20. $\frac{1}{4} [3 \cdot 8] + 2 \cdot (-12) =$
21. $\frac{5 + [30 - (8 - 1)^2]}{11 - 2^2} =$	22. $\frac{3 [10 - (27 + 9)]}{4 - 7} =$
23. $5(14 - 39 \div 3) + 4 \cdot \frac{1}{4} =$	24. $[8 \cdot 2 - (3 + 9)] + [8 - 2 \cdot 3] =$
25. $162 + [6(7 - 4)^2] \div 3 =$	

Operations with Signed Numbers

Adding and Subtracting Signed Numbers

Adding Signed Numbers

Like Signs	Different Signs
Add the numbers and carry the sign	Subtract the numbers and carry the sign of the larger number
$(+) + (+) = +$ $(+3) + (+4) = +7$	$(+) + (-) = ?$ $(+3) + (-2) = +1$
$(-) + (-) = -$ $(-2) + (-3) = (-5)$	$(-) + (+) = ?$ $(-5) + (+3) = -2$

Subtracting Signed Numbers

Don't subtract! Change the problem to **addition** and change the sign of the **second** number. Then use the addition rules.

$(+9) - (+12) = (+9) + (-12) = -7$	$(+4) - (-3) = (+4) + (+3) = 7$
$(-5) - (+3) = (-5) + (-3) = -8$	$(-1) - (-5) = (-1) + (+5) = 4$

Simplify. Do not use a calculator for this section.

1. $9 + -4 =$	2. $20 - -6 =$
3. $-8 + 7 =$	4. $7 - 10 =$
5. $-14 - 6 =$	6. $-6 - -7 =$
7. $-30 + -9 =$	8. $5 - 9 =$
9. $14 - 20 =$	10. $-8 - 7 =$
11. $-2 + 11 =$	12. $1 - -12 =$

Multiplying and Dividing Signed Numbers

Like Signs	Different Signs
If the signs are the same, the answer is positive	If the signs are different, the answer is negative
$(+)(+) = +$ $(+3)(+4) = +12$	$(+)(-) = -$ $(+2)(-3) = -6$
$(-)(-) = +$ $(-5)(-3) = +15$	$(-)(+) = -$ $(-7)(+1) = -7$
$(+)/(+) = +$ $(+12) / (+3) = +4$	$(+)/(-) = -$ $(+6)/(-3) = -2$
$(-)/(-) = +$ $(-12)/(-4) = +3$	$(-)/(+) = -$ $(-7)/(+1) = -7$

Simplify. Do not use a calculator for this section.

1. $(-5)(-3) =$	2. $\frac{-7}{-1} =$
3. $\frac{-6}{2} =$	4. $(3)(-4) =$
5. $(2)(4) =$	6. $\frac{8}{-4} =$
7. $\frac{-12}{-4} =$	8. $(-2)(7) =$
9. $(-1)(-5) =$	10. $\frac{-20}{-1} =$
11. $\frac{-16}{8} =$	12. $(2)(-5) =$

Rounding Numbers

Step 1: Underline the place value to which you want to round.

Step 2: Look at the number to the right of that place value to which you want to round.

Step 3: If the number to the right of the place value to which you want to round is LESS than 5, keep the number the same and drop all other numbers.

If the number to the right of the place value to which you want to round is 5 or MORE, round up and drop the rest of the numbers.

Example: Round the following numbers to the tenths place.

Tenths

1. 23.1246

2 is less than 5 so keep the 1 the same

23.1

2. 64.2685

6 is greater than 5 so add 1 to the 2

64.3

3. 83.9721

7 is greater than 5 so add 1 to the 9

$$\begin{array}{r} 1 \\ 83.9721 \\ + 1 \\ \hline 84.0 \end{array}$$

Round the following numbers to the tenths place.

1. 18.6231 _____

2. 0.2658 _____

3. 25.0543 _____

4. 100.9158 _____

5. 3.9215 _____

6. 19.9816 _____

7. 36.9913 _____

8. 17.1083 _____

9. 15.9199 _____

10. 0.6701 _____

Evaluating Expressions

Example

Evaluate the following expression when $x = 5$

Rewrite the expression substituting 5 for the x and simplify.

Expression	Substitute for x and Simplify	Answer
a. $5x =$	$5(5)$	25
b. $-2x =$	$-2(5)$	-10
c. $x + 25 =$	$5 + 25$	30
d. $5x - 15 =$	$5(5) - 15 = 25 - 15$	10
e. $3x + 4 =$	$3(5) + 4 = 15 + 4$	19

Evaluate each expression given that:

$$x = 5 \quad y = -4 \quad z = 6$$

Expression	Substitute values and simplify	Answer
1. $3x$		
2. $2x^2$		
3. $3x^2 + y$		
4. $2(x+z) - y$		
5. $y + 4$		
6. $5z - 6$		
7. $xy + z$		
8. $2x + 3y - z$		

Evaluate each expression given that:

$x = 5 \quad y = -4 \quad z = 6$

Expression	Substitute values and Simplify	Answer
9. $5x - (y + 2z)$		
10. $\frac{xy}{2}$		
11. $x^2 + y^2 + z^2$		
12. $2x(y + z)$		
13. $5z + (y - x)$		
14. $2x^2 + 3$		
15. $4x + 2y - z$		
16. $\frac{yz}{2}$		

Combining Like Terms

What is a <i>term</i> ?	The parts of an algebraic expression that are separated by an addition or subtraction sign are called <i>terms</i> .
What are <i>like terms</i> ?	Terms with the same variable factors are called <i>like terms</i> . 2n and 3n are <i>like terms</i> , but 4x and 3y are <u>NOT like terms</u> because their variable factors x and y are different.

To simplify an expression, you must combine the *like terms*.

Practice. Simplify each expression.

1. $6n + 5n$	2. $25b + 15b$
3. $37z + 4z$	4. $x - 5x$
5. $3n + 1 - 2n + 8$	6. $4f + 5f - 6 + 8$
7. $7t + 9 - 4t + 3$	8. $2k + 4 - 8k - 1$
9. $4r + 3r + 6y - 2y$	10. $8g + 9h - 4g - 5h$
11. $2m + 3n - 4m + 5n$	12. $a + 5b - 2a + 9b$

Graphing

Points in a plane are named using two numbers, called a coordinate pair. The first number is called the x-coordinate. The x-coordinate is positive if the point is to the right of the origin and negative if the point is to the left of the origin. The second number is called the y-coordinate. The y-coordinate is positive if the point is above the origin and negative if the point is below the origin.

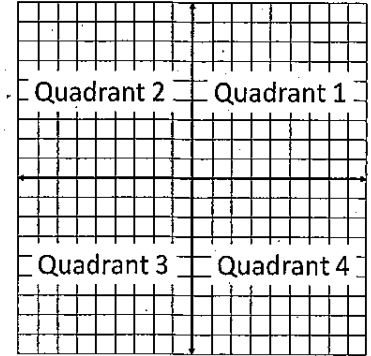
The x-y plane is divided into four quadrants (four sections) as described below.

All points in Quadrant 1 have a positive x coordinate and a positive y coordinate (+x, +y).

All points in Quadrant 2 have a negative x coordinate and a positive y coordinate (-x, +y).

All points in Quadrant 3 have a negative x coordinate and a negative y coordinate (-x, -y).

All points in Quadrant 4 have a positive x coordinate and a negative y coordinate (+x, -y).



Plot each point on the graph below.

Remember, coordinate pairs are labeled (x,y).

Label each point on the graph with the letter given.

1. A (3,4)

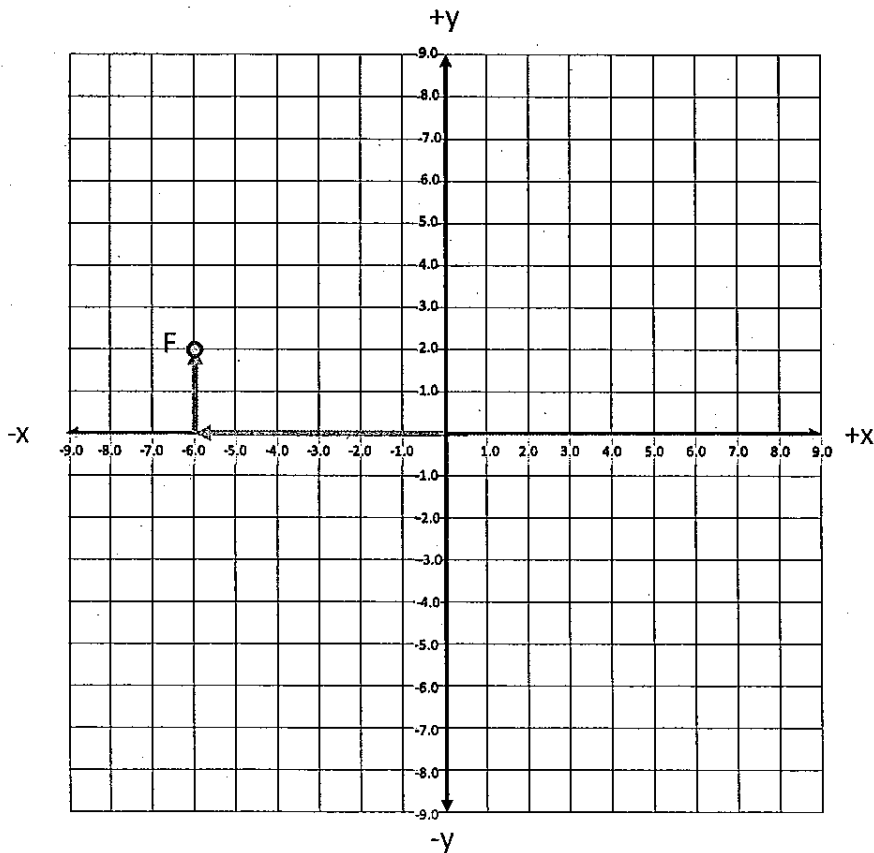
2. B (4,0)

3. C (-4,2)

4. D (-3,-1)

5. E (0,7)

Example. F (-6,2)

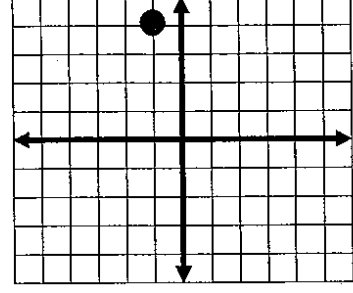
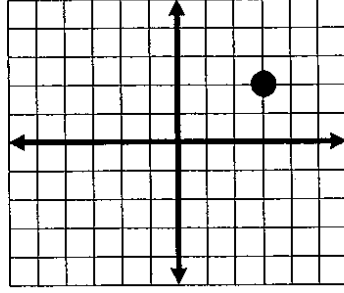
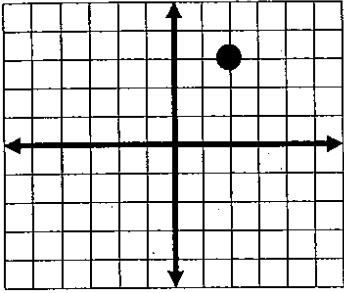


Determine the coordinates for each point below:

Example: (2,3)

6. (,)

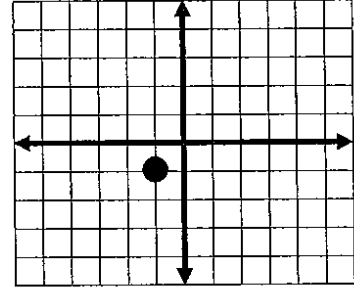
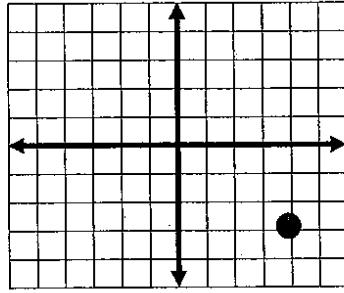
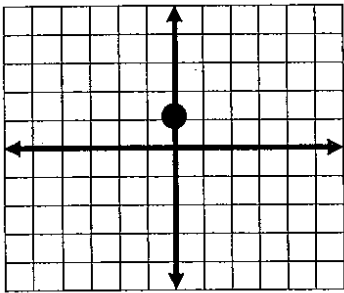
7. (,)



8. (,)

9. (,)

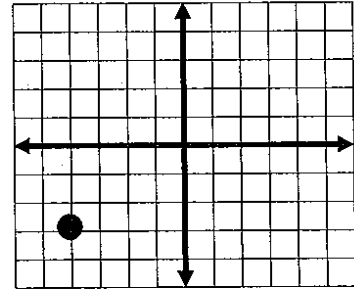
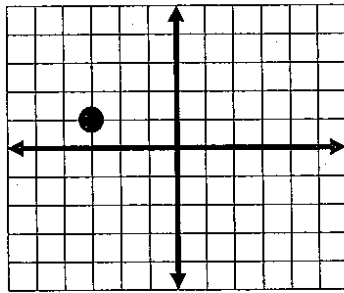
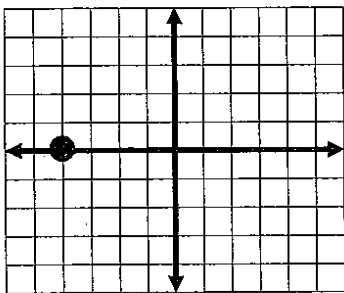
10. (,)



11. (,)

12. (,)

13. (,)



Complete the following tables. Then graph the data on the grid provided.

Problem

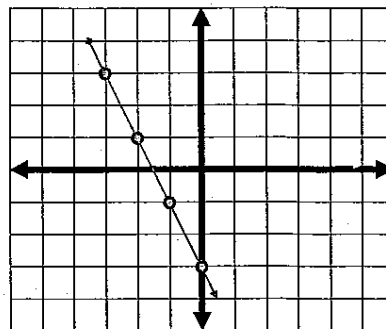
$y = -2x - 3$

X	Y
-3	3
-2	1
-1	-1
0	-3

Work

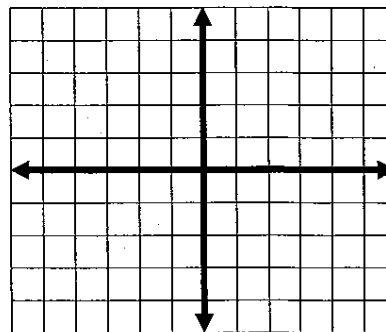
$x = -3$
 $y = -2(-3) - 3 = 6 - 3 = 3$
 therefore $(x,y) = (-3,3)$
 $x = -2$
 $y = -2(-2) - 3 = 4 - 3 = 1$
 therefore $(x,y) = (-2,1)$
 $x = -1$
 $y = -2(-1) - 3 = 2 - 3 = -1$
 therefore $(x,y) = (-1,-1)$
 $x = 0$
 $y = -2(0) - 3 = 0 - 3 = -3$
 therefore $(x,y) = (0,-3)$

Graph



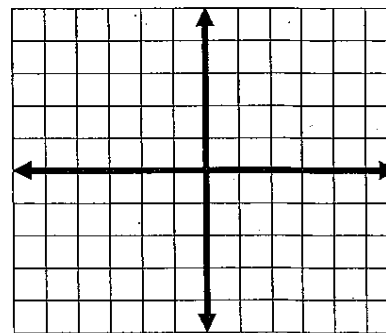
14. $y = x + 2$

X	Y
0	
1	
2	



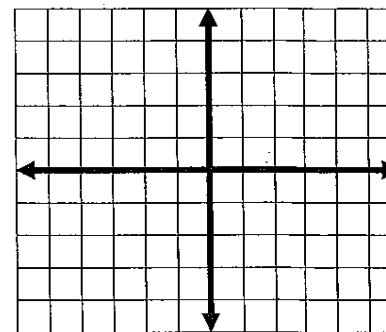
15. $y = 2x$

X	Y
0	
1	
2	
3	



16. $y = -x$

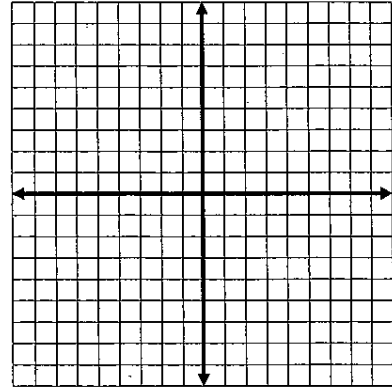
X	Y
-3	
-1	
1	
3	



Problem**Work****Graph**

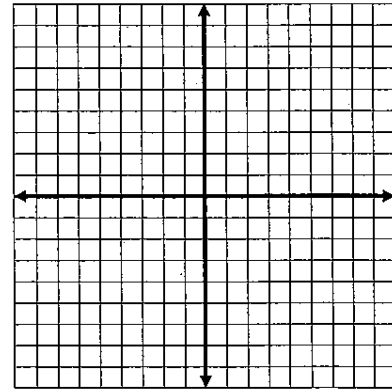
17. $y = 2x - 3$

X	Y
0	
1	
2	
3	



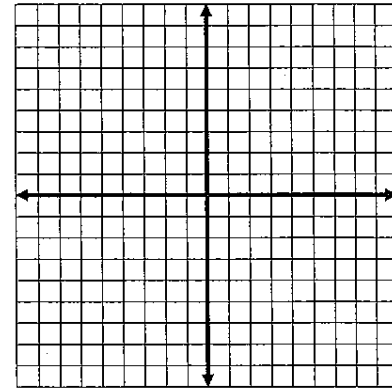
18. $y = \frac{1}{2}x + 1$

X	Y
0	
2	
4	
6	



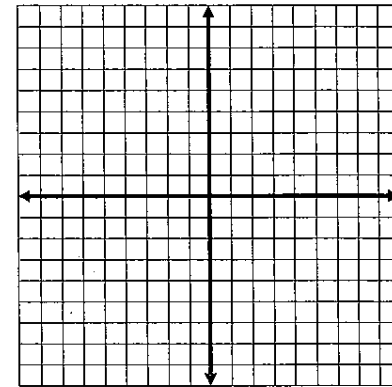
19. $y = \frac{3}{2}x - 1$

X	Y
-2	
0	
2	



20. $y = -\frac{2}{3}x + 1$

X	Y
-3	
0	
3	



Solving Equations

To solve an equation means to **find the value** of the variable. We solve equations by isolating the variable using opposite operations.

Example:

Solve.

$$3x - 2 = 10$$

$$+ 2 \quad + 2$$

Isolate $3x$ by adding 2 to each side

$$\underline{3x} = \underline{12}$$

$$\underline{3} \quad \underline{3}$$

Isolate x by dividing each side by 3

x	$=$	4
-----	-----	-----

Simplify

Opposite Operations:

Addition (+) and Subtraction (-)
Multiplication (x) and Division (÷)

Please remember

To do the same step on each side of
the equation

Check your answer.

$$3(4) - 2 = 10$$

Substitute the value in for the variable

$$12 - 2 = 10$$

Simplify

$$10 = 10$$

Is the equation true? If yes, you solved it correctly!

**Always check your work
by substitution!**

Try these.

1. $x + 3 = 5$	2. $w - 4 = 10$
3. $c - 5 = -8$	4. $3p = 9$
5. $-7k = 14$	6. $-x = -17$
7. $\frac{h}{3} = 5$	8. $\frac{m}{8} = 7$
9. $\frac{4}{5}d = 12$	10. $\frac{3}{8}j = 6$

11. $2x - 5 = 11$	12. $4n + 1 = 9$
13. $5j - 3 = 12$	14. $2x + 11 = 9$
15. $-3x + 4 = -8$	16. $-6x + 3 = -9$
17. $\frac{f}{3} + 10 = 15$	18. $\frac{a}{7} - 4 = 2$
19. $\frac{b+4}{2} = 5$	20. $\frac{x-6}{5} = 3$

Use substitution to determine whether the solution is correct.

21. $4x + 5 = 7$ $x = 3$	22. $-2x + 5 = 13$ $x = 4$
23. $6 - x = 8$ $x = 2$	24. $1 - x = 9$ $x = -8$

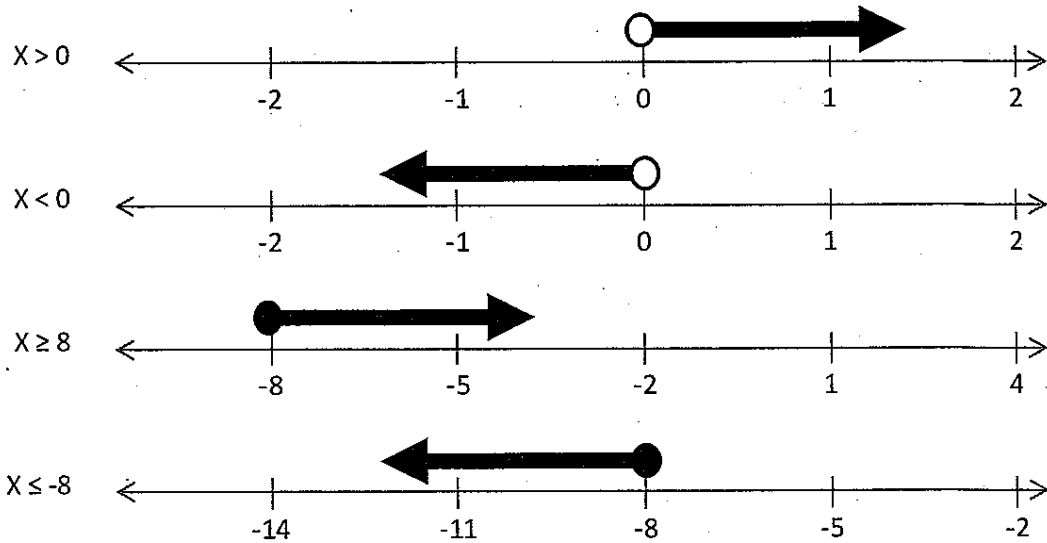
Inequalities

An inequality is a statement containing one of the following symbols:

< is less than < is greater than ≤ is less than or equal to ≥ is greater than or equal to

An inequality has many solutions, and we can represent the solutions of an inequality by a set of numbers on a number line.

Examples:

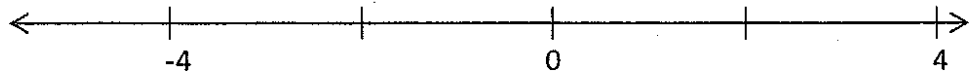


Practice: Write an inequality to represent the solution set that is shown in the graph.

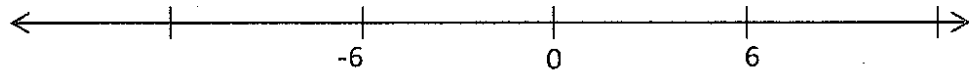
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4.		<hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/>

Graph each of the following inequalities on a number line.

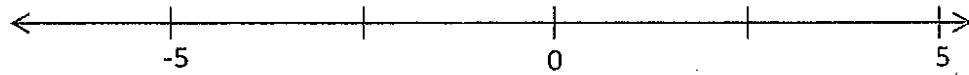
1. $x > 4$



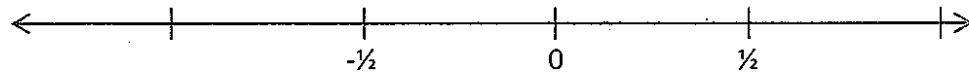
2. $k \leq -6$



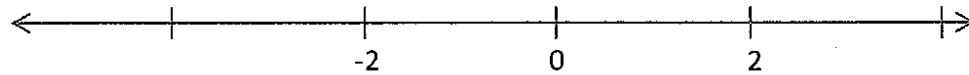
3. $5 > y$



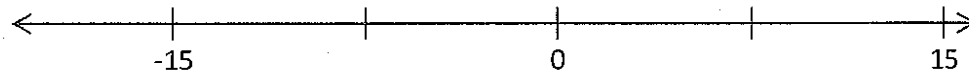
4. $j < \frac{1}{2}$



5. $-2 \leq t$



6. $w \leq 15$



Algebraic Translations

Translating from English to Mathematics

Keywords for Translations

Add	Subtract	Multiply	Divide	Inequalities	Variable	=
Plus	Decreased	Per	One-third	< is less than	A number	Same as
Sum	Smaller	For Every	Quotient	> is greater than	Some number	Equals
Longer than	Less than	For Each	Divided by	≤ is less than or equal to	Quantity	Is
Greater than	Difference	Triple	Each part	≥ is greater than or equal to		Total
Together	Reduced	Multiplied	Half as much			Was
Total	Differ	Of	Split equally			Result
Increased	Fewer	Times				Outcome
More than	Shorter than	Twice				Answer
In all	Minus	Double				
And	Diminished					

Examples:

A) Translate into a mathematical expression: 3 less than 5 times some number

3 **less than** 5 **times** **some number**
 3 to subtract from (multiply) (use a variable)
 translation: **$5n - 3$**

B) Translate into a mathematical expression: 3 less than 5 some number is 22

3 **less than** 5 **times** **some number** **is** 22
 3 (to subtract) (multiply) (use a variable) =
 translation: **$5n - 3 = 22$**

C) Translate into a mathematical expression: the quotient of a number and -4, less 8 is -42

the **quotient** of a **number** and -4 **less** 8 **is** -42
 divide a variable and number subtract =
 translation: **$\frac{n}{-4} - 8 = -42$**

D) Translate into a mathematical expression: four plus 3 times a number is less than or equal to 18

four **plus** 3 **times** a number **is less than or equal to** 18
 add multiply use a variable ≤
 translation: **$4 + 3n \leq 18$**

Practice. Translate each phrase into a mathematical statement

1. Seven plus five times a number is greater than or equal to -9
2. Eight times a number increased by 6 is 62
3. One half of a number is equal to 14
4. 6 less than 8 times some number
5. A number divided by 9
6. p decreased by 5
7. Twice a number decreased by 15 is equal to -27
8. 9 less than 7 times some number is -6
9. The sum of a number and eight is less than 2
10. Eleven increased by a number is -12

Matching. Put the letter of the algebraic expression that best matches the phrase.

- | | | |
|-------|------------------------------|------------------|
| _____ | 1. Two more than a number | a. $2x$ |
| _____ | 2. Two less than a number | b. $x + 2$ |
| _____ | 3. Half of a number | c. $2 - x$ |
| _____ | 4. Twice a number | d. $x - 2$ |
| _____ | 5. Two decreased by a number | e. $\frac{x}{2}$ |

Careful! Pay attention to subtraction. The order makes a difference. Translate to an algebraic expression then reread to check!

Multiplying and Dividing Like and Unlike Fractions

To multiply fractions, multiply the numerators and multiply the denominators. Simplify either before or after multiplying.

Example: $\frac{8}{15} \cdot \frac{5}{9} = \frac{40}{135} = \frac{8}{27}$ or $\frac{8}{15} \cdot \frac{5}{9} = \frac{8}{3} \cdot \frac{1}{9} = \frac{8}{27}$

If the numbers are mixed numbers, write them as improper fractions and then multiply.

Example: $7\frac{1}{2} \cdot 2\frac{2}{3} = \frac{15}{2} \cdot \frac{8}{3}$ simplify first: $\frac{15}{2} \cdot \frac{8}{3} = \frac{5}{1} \cdot \frac{4}{1} = 20$

Practice: Find each product. Show your steps. Write answer in simplest form.

1. $\frac{1}{2} \cdot \frac{3}{5} =$

6. $1\frac{5}{7} \cdot 10\frac{1}{2} =$

2. $\frac{4}{5} \cdot \frac{5}{8} =$

7. $2\frac{1}{8} \cdot 4\frac{4}{7} =$

3. $\frac{7}{9} \cdot \frac{11}{20} =$

8. $4\frac{4}{5} \cdot 1\frac{1}{6} =$

4. $\frac{8}{9} \cdot \frac{5}{16} =$

9. $6 \cdot \frac{2}{3} =$

5. $\frac{2}{5} \cdot 5 =$

10. $1\frac{1}{2} \cdot \frac{4}{15} =$

To divide fractions, multiply by the reciprocal. There are two changes that happen in the set-up. First the division sign becomes multiplication, then we take the reciprocal (flip) the fraction after the division sign.

Any mixed numbers should be changed to improper fractions as the first step. Then change to multiplying by the reciprocal.

****Only simplify after changing to multiplying by the reciprocal. ****

Example: $\frac{3}{4} \div \frac{5}{8} = \frac{3}{4} \cdot \frac{8}{5} = \frac{24}{20} = \frac{6}{5}$

Practice: Find each quotient. Show your steps. Write answer in simplest form.

1. $\frac{1}{2} \div \frac{3}{5} =$

6. $1\frac{5}{7} \div 10\frac{1}{2} =$

2. $\frac{4}{5} \div \frac{3}{10} =$

7. $2\frac{1}{8} \div 4\frac{1}{4} =$

3. $\frac{7}{9} \div \frac{14}{15} =$

8. $4\frac{4}{5} \div 1\frac{1}{5} =$

4. $\frac{8}{9} \div \frac{4}{27} =$

9. $6 \div \frac{2}{3} =$

5. $\frac{2}{5} \div 5 =$

10. $1\frac{1}{2} \div \frac{4}{15} =$

Adding and Subtracting Like and Unlike Fractions

To add or subtract fractions with like denominators, add or subtract the numerators and write the sum or difference over the denominator.

Example $1\frac{2}{9} + 3\frac{4}{9}$ add the whole numbers, add the fractions, combine and simplify

$$1 + 3 = 4 \text{ and } \frac{2}{9} + \frac{4}{9} = \frac{6}{9} = \frac{2}{3} \text{ so the sum is } 4\frac{2}{3}$$

It is often easier when subtracting to write the mixed numbers as improper fractions and then subtract.

$$\text{Example: } 7\frac{1}{3} - 5\frac{2}{3} = \frac{22}{3} - \frac{17}{3} = \frac{5}{3} = 1\frac{2}{3}$$

Practice: Find each sum or difference. Write answer in simplest form.

1. $\frac{11}{12} + \frac{9}{12} =$

5. $\frac{13}{15} + \frac{7}{15} =$

2. $\frac{19}{20} - \frac{17}{20} =$

6. $\frac{23}{25} - \frac{8}{25} =$

3. $3\frac{7}{8} - 4\frac{5}{8} =$

7. $9 + 4\frac{3}{7} =$

4. $9\frac{2}{5} - 6\frac{3}{5} =$

8. $4\frac{11}{12} - 3\frac{7}{12} =$

Adding and subtracting fractions with different denominators. First rename the fractions with a common denominator, then add or subtract and simplify.

Example: $\frac{4}{7} + \frac{1}{3} = \frac{4 \cdot 3}{7 \cdot 3} + \frac{1 \cdot 7}{3 \cdot 7} = \frac{12}{21} + \frac{7}{21} = \frac{19}{21}$

Practice: Find each sum or difference. Write answer in simplest form.

1. $\frac{8}{9} + \frac{2}{5} =$

5. $\frac{7}{15} - \frac{3}{10} =$

2. $3\frac{1}{5} + 2\frac{3}{4} =$

6. $\frac{5}{8} - \frac{1}{4} =$

3. $3\frac{7}{10} - 2\frac{3}{5} =$

7. $9\frac{1}{4} - 4\frac{3}{7} =$

4. $9\frac{2}{3} - 6\frac{3}{4} =$

8. $4\frac{1}{3} - 3\frac{7}{10} =$