"Five to Thrive"

5 Main Topics **to** Review to Help You **Thrive** in 7th Grade Math

Five to Thrive Topics:

Answer Key

1) **Integers**
   - placing on a number line
   - ordering
   - comparing
   - absolute value

2) **Simplifying Expressions**
   - numerical expressions
   - algebraic expressions
     - combining like terms
     - distributive property

3) **Formally Solving and Checking Equations**
   - one-Step Equations (undoing either addition, subtraction, multiplication or division)
   - involving fractions or decimals

4) **Ratios and Rates**
   - write a ratio (3 ways: colon, "to," fraction bar)
   - find equivalent ratios
   - use ratio tables to solve part-to-part and part-to-whole problems
   - unit rate
   - better buy

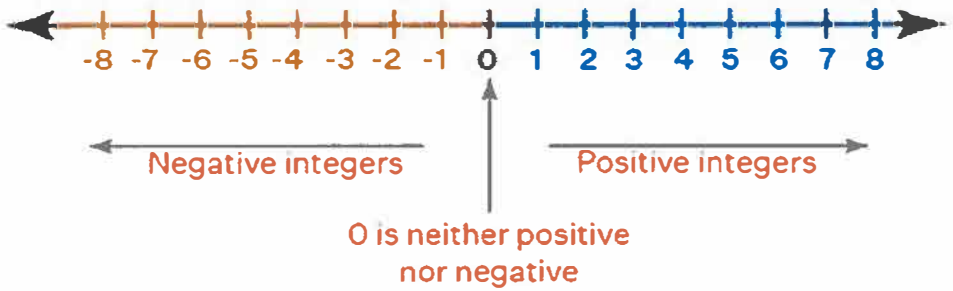
5) **Fractions, Decimals, Percents**
   - convert values between the three forms

Topic 1: Integers

Integers:

Include example with practice (eventually video links)

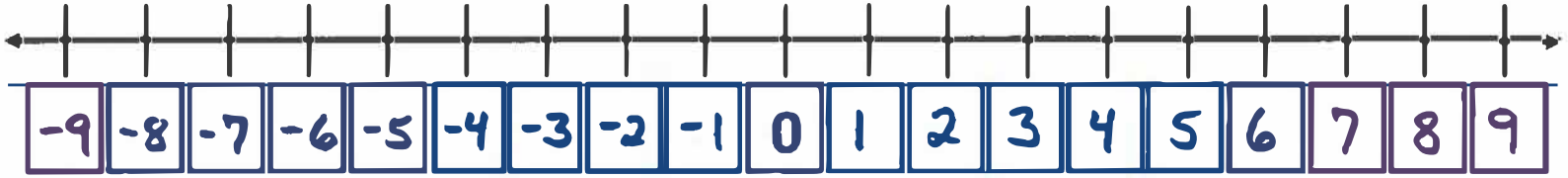
Integers – *whole numbers* (0, 1, 2, 3, 4,...) and their *opposites* (..., -4, -3, -2, -



Links to IXL Practice (grade 6):

- M.1 – Understanding Integers
- M.2 – Integers on Number Lines
- M.3 – Graph Integers on Horizontal and Vertical Number Lines
- M.4 – Understanding Opposite Integers
- M.5 – Understanding Absolute Value
- M.6 – Absolute Value
- M.7 – Compare Integers
- M.8 – Put Integers in Order
- M.9 – Integer Inequalities with Absolute Value
- M.10 – Absolute Value and Integers: Word Problems

1) Write the missing integers on the number line below.



2) Arrange the integers from least to greatest.

a) 5, -2, 0, 8, -3

-3, -2, 0, 5, 8

b) 4, -5, 0, -7, 2

-7, -5, 0, 2, 4

c) 7, -8, -3, 4, -1

-8, -3, -1, 4, 7

d) -9, 0, -5, 2, 6

-9, -5, 0, 2, 6

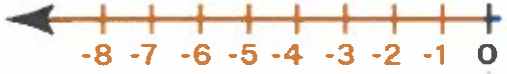
Inequality – a mathematical sentence that compares expressions.

Symbol	<	>
Key Phrases	• is less than • is fewer than	• is greater than • is more than

When comparing two expressions, the inequality symbol opens to the greater value.

The farther a **negative** number is from zero, the smaller its value.

The closer a **negative** number is to zero, the greater its value.



Ex 1: 14 > 10

Ex 2: -8 < 2

Ex 3: -3 > -9

3) Compare using > or < .

a) 12 < 18

b) 4 > -6

c) -9 < 1

d) -6 < 0

e) -8 < -3

f) -7 > -11

Absolute Value – a number's distance from zero (the number of spaces away from zero on a number line).
Distance is always positive

Symbol: | |

Ex 1: $|5| = 5$

The absolute value of 5 is 5 because it is 5 spaces from zero

Ex 2: $|-8| = 8$

The absolute value of -8 is 8 because it is 8 spaces from zero

Ex 3: $|326| = 326$

The absolute value of 326 is 326 because it is 326 spaces from zero

4) Evaluate.

a) $|18| = 18$

b) $|-24| = 24$

c) $|0| = 0$

d) $|1{,}293| = 1{,}293$

e) $|-348| = 348$

Topic 2: Expressions

Links to IXL Practice (grade 6):

P.17 – Evaluate Numerical Expressions Involving Integers

Numerical Expression: an expression that contains numbers and operations.

To **evaluate**, or find the value of, a numerical expression, use a set of rules called the **order of operations**.

Example: $4^2 \div 8 \bullet (9 + 3) - |-10|$

$4^2 \div 8 \bullet 12 - |-10|$

$4^2 \div 8 \bullet 12 - 10$

$16 \div 8 \bullet 12 - 10$

$2 \bullet 12 - 10$

$24 - 10$

14

Order of Operations: **P E M/D A/S**

1) **P**arentheses (also including the following grouping symbols: absolute value, fraction bar)
   - *If more than one exists in the expression, **break the tie** by focusing on the grouping symbol that appears **first** from left to right
2) **E**xponents
3) **M**ultiplication and **D**ivision (break tie from left to right)
4) **A**ddition and **S**ubtraction (break tie from left to right)

Remember: There are several ways to represent multiplication:

3×2 $\quad 3 \bullet 2$ $\quad 3(2)$

5) Evaluate the following expressions:

a) $3^2 + 12 \div (6 - 3) \times 8$

$3^2 + 12 \div 3 \times 8$

$9 + 12 \div 3 \times 8$

$9 + 4 \times 8$

$9 + 32$

41

b) $4(3 - 1)^3 + 7 \bullet 6 - 5^2$

$4(2)^3 + 7 \cdot 6 - 5^2$

$4(8) + 7 \cdot 6 - 5^2$

$4(8) + 7 \cdot 6 - 25$

$32 + 7 \cdot 6 - 25$

$32 + 42 - 25$

$74 - 25$

49

c) $\dfrac{7^2 - 3(6) + 9}{2^3}$

$\dfrac{49 - 3(6) + 9}{2^3}$

$\dfrac{49 - 3(6) + 9}{8}$

$\dfrac{49 - 18 + 9}{8}$

$\dfrac{31 + 9}{8}$

$\dfrac{40}{8} = 5$

When dealing with a fraction bar, perform all of the operations on the top first (numerator), then all of the operations on the bottom (denominator), before finally dividing.

(get 1 value on the top and 1 value on the bottom before you divide)

$\dfrac{\square}{\square}$

d) $3 \times 7 - 2(3)$

$21 - 2(3)$

$21 - 6$

15

e) $9(3+2) - 3(3-2)$

$9(5) - 3(3-2)$

$9(5) - 3(1)$

$45 - 3(1)$

$45 - 3$

42

f) $\dfrac{54 \div 6 + 31}{4^2 + 4}$

$\dfrac{9 + 31}{4^2 + 4}$

$\dfrac{40}{4^2+4}$

$\dfrac{40}{16+4} \rightarrow \dfrac{40}{20} \rightarrow 2$

g) $33 \div 11 \bullet 12 \div 2$

$3 \bullet 12 \div 2$

$36 \div 2$

18

h) $4 \times (10.1 + 1.9) \div 2$

$4 \times 12 \div 2$

$48 \div 2$

24

i) $\dfrac{2^4 \times 5 + 8}{7-3}$

$\dfrac{16 \times 5 + 8}{7-3}$

$\dfrac{80 + 8}{7-3}$

$\dfrac{88}{7-3} \rightarrow \dfrac{88}{4} \rightarrow 22$

j) $\left(\frac{1}{3} + 2\frac{2}{3}\right) \times 13$

3×13

39

k) $60 \div \left(6\frac{1}{7} - \frac{1}{7}\right) \times 4$

$60 \div 6 \times 4$

10×4

40

l) $\dfrac{8^2 - 4 + 4(7)}{(11)(2)}$

$\dfrac{64 - 4 + 4(7)}{(11)(2)}$

$\dfrac{64 - 4 + 28}{(11)(2)}$

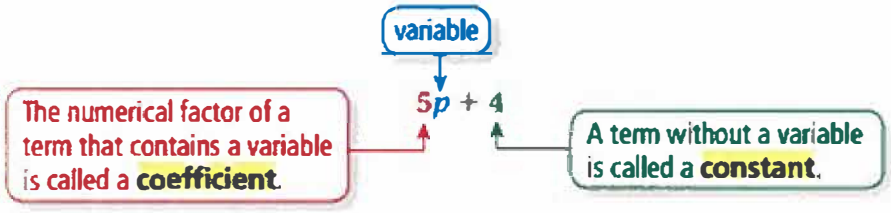
$\dfrac{60 + 28}{(11)(2)}$

$\dfrac{88}{(11)(2)}$

$\dfrac{88}{22}$

4

Algebraic Expression – an expression that may contain numbers, operations, and one or more variables.



Links to IXL Practice (grade 6):

Y.8 – Identify Terms and Coefficients

Term – a number or variable by itself, or the product of numbers and variables.

In an expression, terms are separated by addition and subtraction signs

ex 1: 5 (term) — one term

ex 2: y (term) — one term

ex 3: 5 + y (term, term) — two terms

ex 4: 5x + y - 8 (term, term, term) — three terms

Evaluating Algebraic Expressions.

Example: Evaluate the expression for when $m = 7$ and $n = 4$.

$3m - n^2$

$3(7) - 4^2$

$3(7) - 16$

$21 - 16$

5

Steps:

1. Substitute the numbers in for their assigned variables (letters)
2. Simplify the expression by following the order of operations (P E M/D A/S)

Links to IXL Practice (grade 6):

Y.4 – Evaluate Variable Expressions with Whole Numbers

Y.5 – Evaluate Multi-Variable Expressions

Y.6 – Evaluate Variable Expressions with Decimals, Fractions, and Mixed Numbers

Y.7 – Evaluate Variable Expression Word Problems

6) Evaluate the expressions for when $a = 4$ and $b = 3$ and $c = 10$

a) $5 + a$

$5 + 4$

9

b) $c - 2.5$

$10 - 2.5$

7.5

c) $a \bullet b \bullet c$

$4 \cdot 3 \cdot 10$

$12 \cdot 10$

120

d) $\frac{21}{b} + c$

$\frac{21}{3} + 10$

$7 + 10$

17

e) $c^2 - ab$

$(10)^2 - (4)(3)$

$100 - (4)(3)$

$100 - 12$

88

Combining Like Terms

Links to IXL Practice (grade 6):

Z.11 – Add and Subtract Like Terms

Term – a number or variable by itself, or the product of numbers and variables.

In an expression, terms are separated by addition and subtraction signs

ex 1: 5 (term) — one term

ex 2: y (term) — one term

ex 3: 5 + y (term, term) — two terms

ex 4: 5x + y - 8 (term, term, term) — three terms

Like Terms – terms that have the same variables raised to the same exponents. Constant terms (a number by itself) are also like terms.

You can use simple shapes to help you identify like terms

Example A: $9x + 7 - 2 - x$

Terms: 9x, + 7, – 2, – x

Like terms: 9x and – x, + 7 and – 2

Combine like terms to simplify the expression → $8x + 5$

Example B: $7z^2 + 5z - 3z^2 + z$

Terms: $7z^2$, + 5z, $-3z^2$, +z

Like terms: $7z^2$ and $-3z^2$, +5z and +z

Combine like terms to simplify the expression → $4z^2 + 6z$

7) Simplify the following expressions by combining like terms.

Use shapes to help you identify the like terms (include the + or – sign located at the beginning of each term).

a) $4d + 9 - 1d - 8$ → $3d + 1$

Helpful Tip: When a term only has a variable (letter without a number) **place a 1 in front** of the variable

Ex: y is the same as 1y
Because 1 • y = y

b) $7y + 6 - 1 + 12y$ → $19y + 5$

c) $2v + 8v - 5v + 4v$ → $9v$

d) $3.2x + 20.8 + 1x - 1.7$ → $4.2x + 19.1$

$$\begin{array}{r} {}^{1}{}^{10} \\ 20.8 \\ -\ 1.7 \\ \hline 19.1 \end{array}$$

e) $8b + 9b^2 - 3b^2 - 6b$ → $6b^2 + 2b$

f) $\frac{2}{3}a + \frac{1}{6}a - 3$ → $\frac{5}{6}a - 3$

$\frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$

$+\ \frac{1}{6} = +\ \frac{1}{6}$

$\frac{5}{6}$

Distributive Property – allows you to multiply each term inside a set of parentheses by a term outside the parentheses.

8) Use the distributive property to simplify the following *numerical* expressions.

Examples:

$3(7 + 2)$	$3(7 - 2)$
$3 \cdot 7 + 3 \cdot 2$	$3 \cdot 7 - 3 \cdot 2$
$21 + 6$	$21 - 6$
27	15

a) $4(6 + 1)$

$4 \cdot 6 + 4 \cdot 1$

$24 + 4$

28

b) $7(5 - 3)$

$7 \cdot 5 - 7 \cdot 3$

$35 - 21$

14

c) $9(1 + 10)$

$9 \cdot 1 + 9 \cdot 10$

$9 + 90$

99

d) $8(6 - 2)$

$8 \cdot 6 - 8 \cdot 2$

$48 - 16$

32

e) $5(4 + 3)$

$5 \cdot 4 + 5 \cdot 3$

$20 + 15$

35

f) $4(8 - 5)$

$4 \cdot 8 - 4 \cdot 5$

$32 - 20$

12

9) Use the distributive property to simplify the following *algebraic* expressions.

Examples:

$3(y + 2)$	$3(4y - 2)$
$3 \cdot y + 3 \cdot 2$	$3 \cdot 4y - 3 \cdot 2$
$3y + 6$	$12y - 6$

* Can't be simplified any further because they are unlike terms

a) $2(x + 6)$

$2 \cdot x + 2 \cdot 6$

$2x + 12$

b) $3(y - 6)$

$3 \cdot y - 3 \cdot 6$

$3y - 18$

c) $4(2a + 1)$

$4 \cdot 2a + 4 \cdot 1$

$8a + 4$

d) $8(b - 3)$

$8 \cdot b - 8 \cdot 3$

$8b - 24$

e) $5(7 + 9c)$

$5 \cdot 7 + 5 \cdot 9c$

$35 + 45c$

f) $6(2w - 4)$

$6 \cdot 2w - 6 \cdot 4$

$12w - 24$

Topic 3: Solving Equations

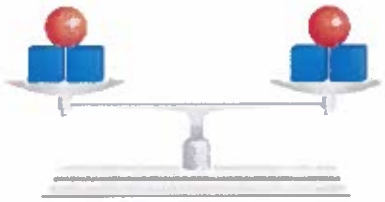
Solving One-Step Addition Equations (Formally)

How can you use addition or subtraction to solve an equation?

*Think of an equation as a balance scale.

The left side of the balance scale equals the right side, just as in an equation the left side the equal sign equals the right side*

When two sides of a scale weigh the same, the scale will balance.



When you add or subtract the same amount on each side of the scale, it will still balance.

Subtraction Property of Equality

Words When you subtract the same number from each side of an equation, the two sides remain equal.

Numbers			**Algebra**		
$8 =$	8		$x + 4 =$	5	
-5	-5		-4	-4	
$3 =$	3		$x =$	1	

Example:

Solving one-step equations by undoing addition

$x + 5 = 8$

$-5 \quad -5$

$x = 3$

Steps:

1) Undo addition by subtracting
2) Write Solution

You can use *inverse operations* to solve equations. Inverse operations "undo" each other. Addition and subtraction are inverse operations.

Check:

$x + 5 = 8$

$3 + 5 = 8$

$8 = 8$

1) Rewrite equation
2) Substitute solution for variable
3) Simplify

10) Solve and check each of the following equations. Be sure to show all of your work.

a) $y + 18 = 32$

$-18 \quad -18$

$y = 14$

check:

$y + 18 = 32$

$14 + 18 = 32$

$32 = 32$ ✓

b) $54 = x + 23$

$-23 \quad -23$

$31 = x$

check:

$54 = x + 23$

$54 = 31 + 23$

$54 = 54$ ✓

c) $r + 113 = 402$

$-113 \quad -113$

$r = 289$

check:

$r + 113 = 402$

$289 + 113 = 402$

$402 = 402$ ✓

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 289 \\ +113 \\ \hline 402 \end{array}$$

d) $63 = n + 20$

$-20 \quad -20$

$43 = n$

check:

$63 = n + 20$

$63 = 43 + 20$

$63 = 63$ ✓

e) $y + 23.4 = 54.1$

$-23.4 \quad -23.4$

$y = 30.7$

check:

$y + 23.4 = 54.1$

$30.7 + 23.4 = 54.1$

$54.1 = 54.1$ ✓

$$\begin{array}{r} {\scriptstyle 1} \\ 30.7 \\ +23.4 \\ \hline 54.1 \end{array}$$

Similar Problem:

$12.1 = y + 4.8$

$-4.8 \quad -4.8$

$7.3 = y$

Steps:

1) Undo addition by subtracting
2) Write Solution

Check:

$12.1 = y + 4.8$

$12.1 = 7.3 + 4.8$

$12.1 = 12.1$

1) Rewrite equation
2) Substitute solution for variable
3) Simplify

f) $m + \frac{2}{3} = \frac{5}{6} \rightarrow \frac{5}{6}$

$-\frac{2}{3} \quad -\frac{2\times2}{3\times2} \quad -\frac{4}{6}$

$m = \frac{1}{6}$

no check required

Similar Problem:

$m + \frac{3}{4} = \frac{7}{8} \rightarrow \frac{7}{8}$

$-\frac{3}{4} \quad -\frac{3\times2}{4\times2} \quad -\frac{6}{8}$

$m = \frac{1}{8}$

Steps:

1) Undo addition by subtracting
2) Write Solution

* When adding or subtracting fractions, they must have common denominators (same number on the bottom).

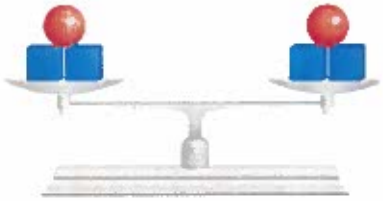
Solving One-Step Subtraction Equations (Formally)

How can you use addition or subtraction to solve an equation?

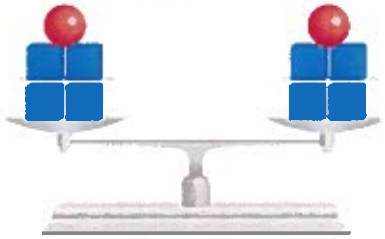
*Think of an equation as a balance scale.

The left side of the balance scale equals the right side, just as in an equation the left side the equal sign equals the right side*

When two sides of a scale weigh the same, the scale will balance.



When you add or subtract the same amount on each side of the scale, it will still balance.



Addition Property of Equality

Words When you add the same number to each side of an equation, the two sides remain equal.

Numbers			**Algebra**		
$8 =$	8		$x - 4 =$	5	
$+5$	$+5$		$+4$	$+4$	
$13 =$	13		$x =$	9	

Solving one-step equations by undoing subtraction

$X - 3 = 7$

$+3 \quad +3$

$X = 10$

Steps:

1) Undo subtraction by adding
2) Write solution

Check:

$X - 3 = 7$

$10 - 3 = 7$

$7 = 7$

1) Rewrite equation
2) Substitute solution in for variable
3) Simplify

You can use *inverse operations* to solve equations. Inverse operations "undo" each other. Addition and subtraction are inverse operations.

II) Solve and check each of the following equations. Be sure to show all of your work.

a) $a - 29 = 14$

$+29 \quad +29$

$a = 43$

$43 - 29 = 14$

$14 = 14$ ✓

check:

b) $47 = b - 33$

$+33 \quad +33$

$80 = b$

check:

$47 = 80 - 33$

$47 = 47$ ✓

c) $b - 251 = 463$

$+251 \quad +251$

$b = 714$

check:

$b - 251 = 463$

$714 - 251 = 463$

$463 = 463$ ✓

$$\begin{array}{r} {\scriptstyle 6\ 11} \\ 714 \\ -251 \\ \hline 463 \end{array}$$

d) $88 = y - 4$

$+4 \quad +4$

$92 = y$

check:

$88 = y - 4$

$88 = 92 - 4$

$88 = 88$ ✓

e) $y - 3.7 = 54.6$

$+3.7 \quad +3.7$

$y = 58.3$

check:

$y - 3.7 = 54.6$

$58.3 - 3.7 = 54.6$

$54.6 = 54.6$ ✓

$$\begin{array}{r} {\scriptstyle 7\ 13} \\ 58.3 \\ -\ 3.7 \\ \hline 54.6 \end{array}$$

Similar Problem:

$16.4 = y - 9.3$

$+9.3 \quad +9.3$

$25.7 = y$

Steps:

1) Undo subtraction by adding
2) Write Solution

Check:

$16.4 = y - 9.3$

$16.4 = 25.7 - 9.3$

$16.4 = 16.4$

1) Rewrite equation
2) Substitute solution in for variable
3) Simplify

f) $m - \frac{1}{4} = \frac{5}{8}$

$+\frac{1}{4} \quad +\frac{1\times2}{4\times2} = +\frac{2}{8}$

$m = \frac{7}{8}$

no check required

Similar Problem:

$m - \frac{2}{3} = \frac{1}{6} \rightarrow \frac{1}{6}$

$+\frac{2}{3} \quad +\frac{2}{3}\ {\scriptstyle \times 2} \quad +\frac{4}{6}$

$m = \frac{5}{6}$

Steps:

1) Undo subtraction by adding
2) Write Solution

* When adding or subtracting fractions, they must have common denominators (same number on the bottom).

Solving One-Step Multiplication Equations (Formally)

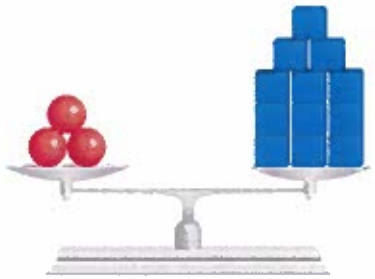
How can you use division to solve an equation?

*Think of an equation as a balance scale.

The left side of the balance scale equals the right side, just as in an equation the left side the equal sign equals the right side*

Use a model to solve the problem.

Three people go out to lunch. They decide to share the $12 bill evenly. How much does each person pay?



- How much does one ● weigh? How do you know?

Division Property of Equality

Words When you divide each side of an equation by the same nonzero number, the two sides remain equal.

Numbers		**Algebra**	
$8 \cdot 4 = 32$		$4x = 32$	
$8 \cdot 4 \div 4 = 32 \div 4$		$\frac{4x}{4} = \frac{32}{4}$	
$8 = 8$		$x = 8$	

Solving one-step equations by undoing multiplication

$\frac{5y}{5} = \frac{10}{5}$

$y = 2$

Steps:

1) Undo multiplication by dividing
2) Write solution

Check:

$5y = 10$

$5(2) = 10$

$10 = 10$

1) Rewrite equation
2) Substitute solution in for variable
3) simplify

You can use *inverse operations* to solve equations. Inverse operations "undo" each other. Multiplication and division are inverse operations.

12) Solve and check each of the following equations. Be sure to show all of your work.

a) $\frac{10x}{10} = \frac{60}{10}$

$x = 6$

check:

$10x = 60$

$10(6) = 60$

$60 = 60$ ✓

b) $\frac{56}{8} = \frac{8j}{8}$

$7 = j$

check:

$56 = 8j$

$56 = 8(7)$

$56 = 56$ ✓

c) $\frac{7x}{7} = \frac{35}{7}$

$x = 5$

check:

$7x = 35$

$7(5) = 35$

$35 = 35$ ✓

d) $\frac{88}{11} = \frac{11z}{11}$

$8 = z$

check:

$88 = 11z$

$88 = 11(8)$

$88 = 88$ ✓

e) $\frac{1.2a}{1.2} = \frac{8.76}{1.2}$

$a = 7.3$

$1.2 \overline{)8.76}$ → 7.3

-84

36

-36

0

check:

$1.2a = 8.76$

$1.2(7.3) = 8.76$

$8.76 = 8.76$ ✓

7.3 ①

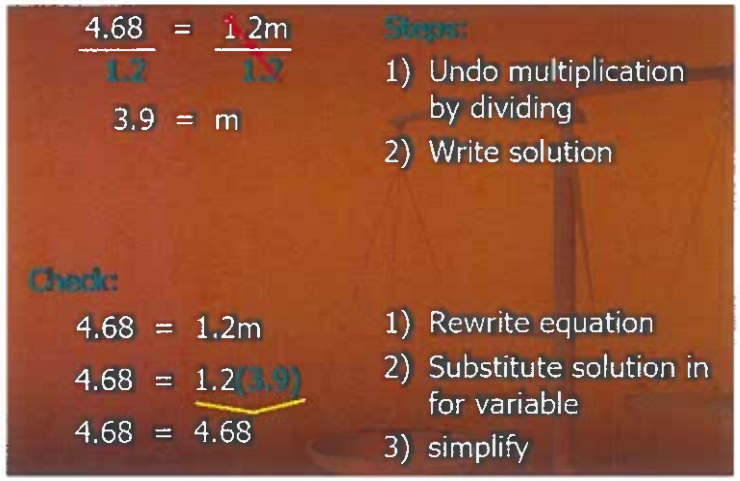
$\times 1.2$ ①

$14\,6$

$+\ 7\,3\,0$

$8.7\,6$ ②

Similar Problem:



f) $\frac{4}{3} \cdot \frac{3}{4} b = 9 \cdot \frac{4}{1}$ $\quad \frac{3}{1}$

$\frac{12}{12} b = \frac{36}{3}$

$b = 12$

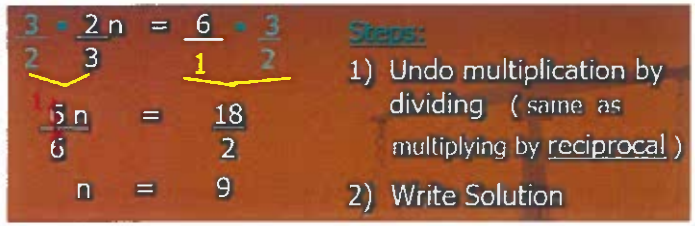
no check required

OR $\frac{3}{4} b = 9 \cdot 4$

$\frac{3b}{3} = \frac{36}{3}$

$b = 12$

Similar Problem:



Solving One-Step Division Equations (Formally)

How can you use multiplication to solve an equation?

*Think of an equation as a balance scale.

The left side of the balance scale equals the right side, just as in an equation the left side the equal sign equals the right side*

Multiplication Property of Equality

Words When you multiply each side of an equation by the same nonzero number, the two sides remain equal.

Numbers $\frac{8}{4} = 2$

$\frac{8}{4} \cdot 4 = 2 \cdot 4$

$8 = 8$

Algebra $\frac{x}{4} = 2$

$\frac{x}{4} \cdot 4 = 2 \cdot 4$

$x = 8$

Multiplicative Inverse Property

Words The product of a nonzero number n and its reciprocal, $\frac{1}{n}$, is 1.

Numbers $5 \cdot \frac{1}{5} = 1$ **Algebra** $n \cdot \frac{1}{n} = \frac{1}{n} \cdot n = 1, n \neq 0$

Solving one-step equations by undoing division

$\frac{2}{1} \cdot \frac{X}{2} = 8 \cdot 2$

$X = 16$

$X = 16$

Steps:

1) Undo division by multiplying
2) Write solution

Check:

$\frac{X}{2} = 8$

$\frac{16}{2} = 8$

$8 = 8$

1) Rewrite equation
2) Substitute solution in for variable
3) Simplify

You can use *inverse operations* to solve equations. Inverse operations "undo" each other. Multiplication and division are inverse operations.

13) Solve and check each of the following equations. Be sure to show all of your work.

a) $\frac{8}{1} \cdot \frac{x}{8} = 60 \cdot 8$

$\frac{8x}{8} = 480$

$x = 480$

check:

$\frac{x}{8} = 60$

$\frac{480}{8} = 60$

$60 = 60$

b) $2 \cdot 21 = \frac{h}{2} \cdot \frac{2}{1}$

$42 = \frac{2h}{2}$

$42 = h$

check:

$21 = \frac{h}{2}$

$21 = \frac{42}{2}$

$21 = 21$

c) $\frac{12}{1} \cdot \frac{g}{12} = 3 \cdot 12$

$\frac{12g}{12} = 36$

$g = 36$

check:

$\frac{9}{12} = 3$

$\frac{36}{12} = 3$

$3 = 3$ ✓

d) $5 \cdot 8 = \frac{y}{5} \cdot \frac{5}{1}$

$40 = \frac{5y}{5}$

$40 = y$

check:

$8 = \frac{y}{5}$

$8 = \frac{40}{5}$

$8 = 8$ ✓

e) $\frac{0.3}{1} \cdot \frac{a}{0.3} = 8.6 \ (0.3)$

$\frac{0.3a}{0.3} = 2.58$

$a = 2.58$

no check required

8.6 ①
$\times 0.3$ ①
258
$+ 000$
2.58 ②

Similar Problem:

$\frac{4.3}{1} \cdot \frac{w}{4.3} = 2.6 \cdot 4.3$

$\frac{4.3w}{4.3} = 11.18$

$w = 11.18$

Steps:
1) Undo division by multiplying
2) Write solution

Check:

$\frac{w}{4.3} = 2.6$

$\frac{11.18}{4.3} = 2.6$

$2.6 = 2.6$

1) Rewrite equation
2) Substitute solution in for variable
3) Simplify

f) $(.72)\ 4.1 = \frac{b}{.72} \cdot (.72)$

$2.952 = \frac{.72b}{.72}$

$2.952 = b$

$.72$ ②
$\times 4.1$ ①
72
$+2880$
2.952 ③

check:

$4.1 = \frac{b}{.72}$

$4.1 = \frac{2.952}{.72}$

$4.1 = 4.1$ ✓

$72 \overline{)2952}$ → 004.1

-288

72

-72

00

Topic 4: Ratios and Rates

Links to IXL Practice (grade 6):

S.1 – Write a Ratio

S.2 - Write a Ratio Using a Fraction

S.3 - Write a Ratio: Word Problems

S.4 - Which Model Represents the Ratio?

Ratio – a comparison of two quantities.

* can be part-to-part, part-to-whole, or whole-to-part comparisons.
* terms used to describe a ratio relationship
  - to
  - for every
  - out of every
  - out of
  - per (for one)
  - for each (for one)

Examples

1 cat *to* 3 dogs
1 cat *for every* 3 dogs
3 dogs *out of every* 4 pets
1 cat *out of* 4 pets
3 dogs *per* cat
3 dogs *for each* cat

A ratio can be written in 3 different ways.

Using: - a **colon:** ☐ : ☐

- the word "**to**": ☐ to ☐

- a **fraction bar:** $\frac{\square}{\square}$

Examples

The ratio of cats to dogs

1 : 3 1 to 3 $\frac{1}{3}$

The ratio of dogs to total pets

3 : 4 3 to 4 $\frac{3}{4}$

#4) Write each of the following ratios 3 ways.

* The order matters*

(write the order of the numbers in the same order as the words)

a) The number of cats to the number of birds.

2 : 1

2 to 1

$\frac{2}{1}$

b) The number of dogs to the total pets.

3 : 6

3 to 6

$\frac{3}{6}$

c) the number of total pets to the number of cats.

6 : 2

6 to 2

$\frac{6}{2}$

Equivalent Ratios – two ratios that describe the same relationship.

* Equivalent ratios have the same base ratio.
* Ratios are equivalent when they represent one or more groups of the same base ratio.

base ratio – simplified ratio (lowest terms)

$\frac{6}{8} = \frac{3}{4}$ ← base ratio ($\div 2$ on both)

Ratio Table (horizontal)

base ratio: x 2, x 3, x 4

Dogs	3	6	9	12
Cats	2	4	6	8

x 2, x 3, x 4

base ratio: $\frac{3}{2} = \frac{6}{4}$ (x 2 groups)

2 groups of base ratio

base ratio: $\frac{3}{2} = \frac{9}{6}$ (x 3 groups)

3 groups of base ratio

Model

base ratio

All of the ratios in these two tables:
1) reduce to the same **base ratio.**
2) represent 1 or more groups of the base ratio

Ratio Table (Vertical)

Dogs	Cats
3	2
6	4
9	6
12	8

base ratio: x 2, x 3, x 4

base ratio: $\frac{3}{2} = \frac{12}{8}$ (x 4 groups)

4 groups of base ratio

---

15) Fill out the given ratio tables with equivalent ratios. Also, draw a model (using letters) to represent the equivalent ratios.

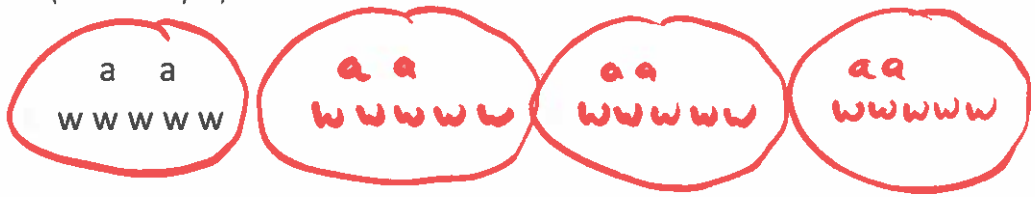
a)

base ratio: x2, x3, x4

apples	2	4	6	8
watermelon	5	10	15	20

x2, x3, x4

Model (started for you)



b)

Sharks	Dolphins
4	3
8	6
12	9
16	12

base ratio

Model
(started for you)

s s s s S S S S S S S S S S S S
d d d d d d d d d d d d

c)

pants	3	6	9	12
shirts	7	14	21	28
total	10	20	30	40

base ratio

Model
(started for you)

p p p P P P P P P P P P
s s s s s s s S S S S S S S S S S S S S S S S S S S S S

---

16) Find the missing value of the equivalent ratio in each of the ratio tables below.

Links to IXL Practice (grade 6):
S.7 – Ratio Tables

Example:

Yellow	4		16	
Blue	5	10		35

base ratio

Yellow	4	8	16	28
Blue	5	10	20	35

b)

Red	7	14	35	42	70
Blue	3	6	15	18	30

base ratio

a)

oranges	kiwi
2	3
6	9
8	12
12	18

base ratio

c)

Helpful hint: reduce to the base ratio first

pens	3	6	15	21	24
pencils	4	8	20	28	32

base ratio

Links to IXL Practice (grade 6):

S.15 – Ratios and Rates: Word Problems

Ratio Table Word Problems

Given Part-to-Part Ratio.

Example: Mark ran 8 miles in 60 minutes. If Mark continues to run at that same rate, how many minutes will it take him to run 12 miles?

Option 1: Reduce to the base ratio to help you complete the ratio table.

÷ 4 → base ratio ← x 6

miles	8	**2**	12	
minutes	60	**15**	**90**	

÷ 4 ← → x 6

Option 2: Divide the product (the number you want to get to) by given the factor (the number you are starting with) to find the other factor.

X 1.5 →

miles	8	12
minutes	60	**90**

← X 1.5

8 x _?_ = 12

12 ÷ 8 = **1.5**

Answer: It will take Mark **90** minutes to run 12 miles.

Example: Tiana has a bowl of fruit which has only apples and bananas. The ratio is 2 apples to every 3 bananas. If there are a total of 15 fruits in the bowl, how many apples are there?

apples	2	
bananas	3	
total	5	15

↑ 2 apples + 3 bananas = 5 total

x 3 →

apples	2	**6**
bananas	3	9
total	5	15

← x 3

Answer: There are **6** apples.

17) **a)** There are 230 calories in 4 ounces of a type of ice cream. How many calories are in 6 ounces of that ice cream?

÷2 BASE x3

calories	230	115	345
ounces	4 ÷2	2	6

x3

Answer: 345 calories

b) A bag contains 60 marbles, some blue and some green. The ratio of blue marbles to green marbles is 1:5. How many blue marbles are there?

BASE x10

blue	1	10	
green	5 x10	50	
total	6	60	

x10

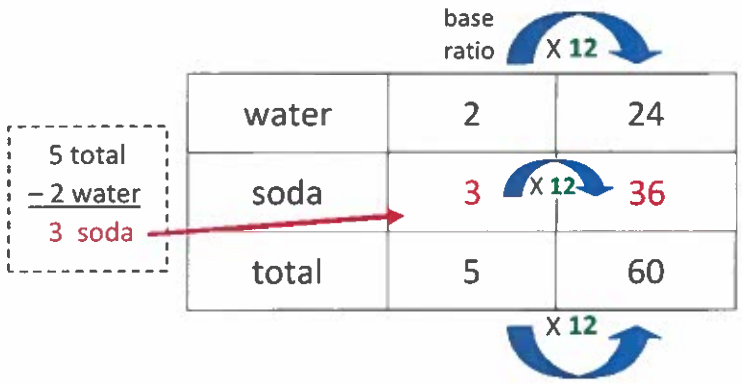
Answer: 10 blue marbles

Ratio Table Word Problems (continued)

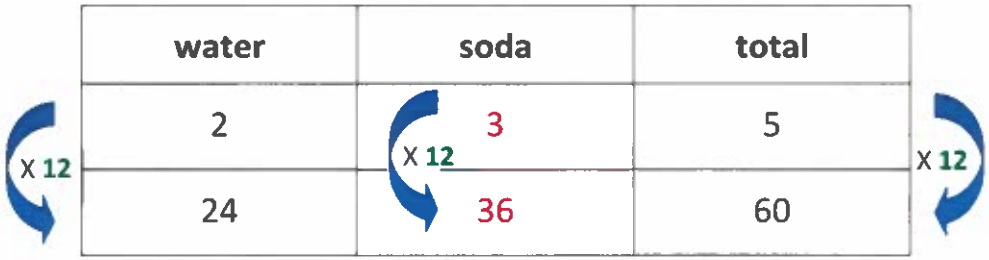
Given Part-to-Whole Ratio.

Example: A particular refrigerator contains bottles of soda and bottles of water. There are 2 bottles of water for every 5 total bottles in the refrigerator. If there are 24 bottles of water in the refrigerator, how many bottles of soda are there?

horizontal table:



vertical table:



Answer: 36 bottles of soda

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18) a) To make the perfect shade of green, Giancarlo mixes blue paint with yellow paint. He uses 5 teaspoons of blue paint for every 13 total teaspoons of paint. How many teaspoons of blue paint must he mix with 24 teaspoons of yellow paint to create the same shade of green?

13 total
– 5 blue
8 yellow

blue	5 x3	15
yellow	8 x3	24
total	13 x3	39

Answer: 15 teaspoons of blue paint

b) Colin's Donut Shop makes glazed donuts and jelly-filled donuts. The ratio of jelly-filled donuts to the total number of donuts is 4:15. If there are a total of 150 donuts, how many of them will be glazed?

15 total
– 4 jelly
11 glazed

glazed	11 x10	110
jelly-filled	4 x10	40
total	15 x10	150

Answer: 110 glazed donuts

Links to IXL Practice (grade 6):

S.9 – Unit Rates

Unit Rate – how much of the first unit corresponds to 1 of the second unit.
"How much for 1?" or "How many for 1?"

Example: Mario drives 120 miles in 3 hours. How many miles did he drive per hour (unit rate)?

miles	120	40
hour	3	1

÷3 ÷3

Answer: Mario drove 40 miles per hour (40 miles in 1 hour)

19) Set up a ratio table to help you find the unit rate for each of the scenarios below.

a) 150 calls in 6 hours. How many calls per hour? For 1

Calls	150	25
hours	6	1

÷6 ÷6

Answer: 25 calls per hour

b) 63 peaches in 7 bowls. How many peaches in each bowl? For 1

Peaches	63	9
bowls	7	1

÷7 ÷7

Answer: 9 peaches per bowl

c) 3 t.v. episodes were 135 minutes long.
How many minutes was each episode?

* order matters

minutes	135	45
episodes	3	1

÷3 ÷3

Answer: 45 minutes per episode

d) 725 buttons on 5 keyboards.
How many buttons on each keyboard?

buttons	725	145
Keyboards	5	1

÷5 ÷5

Answer: 145 buttons per keyboard

Links to IXL Practice (grade 6):

X.2 – Unit Prices

Better Buy

Unit Price – the cost (price) for 1 unit.
"How much does it cost for 1?"

Example: Caraluzzi's Supermarket sells 12 ounces of fruit salad for $8.40. What is the unit price of the fruit salad?

Start with the **cost** as the first unit (money on top) and the **size/amount** of the item as the second unit (2nd unit on the bottom)

÷ 12

Cost ($)	8.40	0.70
ounces	12	1

÷ 12

Answer: The fruit salad costs $0.70 per ounce ($0.70 for 1 ounce)

Better Buy – finding the best value (better deal) between two or more products.

Question: How do we determine the better deal when the items being compared are different sizes?

Answer: We can use unit rates to help determine which item is cheaper by finding out the cost of 1 for each item.
- Allows us to compare the price for the same amount (1) for each item.

In other words, the better buy is the item with the lowest unit price when comparing the unit prices of two or more items.

Example: Determine which of the two options below is the "better buy." Round your unit price to the nearest cent.

Option 1: Velveeta Mac N Cheese
8 boxes for $11.19

÷ 8

Cost ($)	11.19	1.39875
# of boxes	8	1

÷ 8

Unit Price for Option 1: ≈ $1.40

Option 2: Annie's Mac N Cheese
$7.95 for 4 boxes

÷ 4

Cost ($)	7.95	1.9875
# of boxes	4	1

÷ 4

Unit Price for Option 2: ≈ $1.99

Answer:

Better Buy

Option 1

20) Determine the "better buy" for each scenario. * You may use a calculator on this page.

a)

Option 1: Cape Cod potato chips
$3.50 for a 7.5 ounce (oz) bag

Option 2: Cape Cod potato chips
$6.69 for a 13 ounce (oz) bag

Answer:

Better Buy
option 1

÷7.5

$	3.50	0.46̄
oz	7.5	1

÷7.5

.4666...
≈ .47

Unit Price for Option 1: ≈ $0.47 per oz

÷13

$	6.69	0.51461538...
oz	13	

÷13

0.514...
≈ 0.51

Unit Price for Option 2: ≈ $0.51 per oz

b)

Option 1: Peelz mandarins
2 pound (lb) bag for $5.79

Option 2: Dole mandarins
5 pound (lb) bag for $9.92

Answer:

Better Buy
option 2

÷2

$	5.79	2.895
lb	2	

÷2

2.895
≈ 2.90

Unit Price for Option 1: ≈ $2.90 per lb

÷5

$	9.92	1.984
lb	5	1

÷5

1.984
≈ 1.98

Unit Price for Option 2: ≈ $1.98 per lb

Topic 5: Fractions, Decimals, and Percents

Links to IXL Practice (grade 6):

U.1 – What Percentage is Illustrated?

Converting from Fractions to Decimals and Percents.

Percent means "out of ___100___ "

For example: Look at the hundreds grid at the right → The shaded part represents:

- as a fraction: $\frac{25}{100} = \frac{1}{4}$ (÷ 25 / ÷ 25) (reduced fraction)

25 squares are shaded out of 100 total squares.

- as a decimal: 0.25

Use the place value chart to help you convert from a fraction to a decimal.

$\frac{25}{100}$ Write the top number (numerator) in the place value chart so that the last digit (5) is in the place value that matches the bottom number (denominator).

*Hundredths means out of 100

Place Value

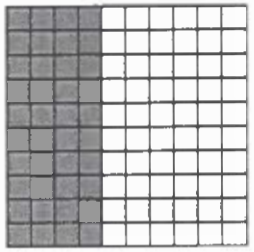
___	___	.	2	5
tens	ones		tenths	hundredths

- as a percent: 25%

Since percent means "out of 100," if your fraction is out of 100, the percent is the number on top (numerator)

21) For each grid below, write a fraction value, a decimal, and a percent to represent the shaded section.

a)

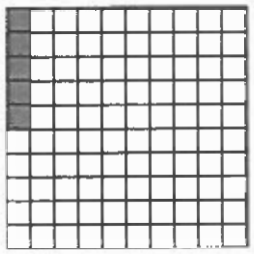


- as a fraction: $\frac{40}{100}$
- as a decimal: 0.40 or 0.4
- as a percent: 40%

b)

- as a fraction: $\frac{50}{100}$
- as a decimal: 0.50 or 0.5
- as a percent: 50%

c)



- as a fraction: $\frac{5}{100}$
- as a decimal: 0.05
- as a percent: 5%

d)

- as a fraction: $\frac{43}{100}$
- as a decimal: 0.43
- as a percent: 43%

Links to IXL Practice (grade 6):

U.3 – Convert Fractions to Percents Using Grid Models

* What if you start with a fraction that is not out of 100?

Example:

6 rectangles are shaded out of 10 total rectangles.

- as a fraction: $\frac{6}{10}$ or $\frac{60}{100}$
- as a decimal: 0.6 or 0.60
- as a percent: 60%

$\frac{6}{10}$ Write the top number (numerator) in the place value chart so that the last digit (6) is in the place value that matches the bottom number (denominator).

*Tenths means out of 10

Place Value

tens	ones	tenths	hundredths
		6	

You will need to find an **equivalent fraction** that is out of 100 in order to find the percent.

$\frac{6}{10} \xrightarrow{\times 10} \frac{60}{100}$ (x 10 above, x 10 below)

Write the top number (numerator) in the place value chart so that the last digit (0) is in the place value that matches the bottom number (denominator).

*Hundredths means out of 100

Place Value

tens	ones	tenths	hundredths
		6	0

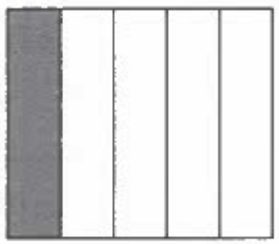
22) For each grid below, write a fraction value, a decimal, and a percent to represent the shaded section.

a)

$\frac{3}{10} \xrightarrow{\times 10} \frac{30}{100}$

- as a fraction: $\frac{3}{10}$ or $\frac{30}{100}$
- as a decimal: 0.3 or 0.30
- as a percent: 30%

b)



$\frac{1}{5} \xrightarrow{\times 20} \frac{20}{100}$

- as a fraction: $\frac{1}{5}$ or $\frac{20}{100}$
- as a decimal: 0.20 or 0.2
- as a percent: 20%

c)

$\frac{3}{4} \xrightarrow{\times 25} \frac{75}{100}$

- as a fraction: $\frac{3}{4}$ or $\frac{75}{100}$
- as a decimal: 0.75
- as a percent: 75%

Converting a Decimal to a Fraction AND Converting a Decimal to a Percent.

Decimal to Fraction

Ex: $0.26 = \frac{26}{100}$

Take the numbers in the place value chart that are to the right of the decimal and put them on the top of the fraction (numerator). The bottom number of the fraction (denominator) will be the same as the place value of the last digit in the place value chart (hundredths means out of 100).

Place Value

tens	ones	tenths	hundredths
		2	6

$= \frac{13}{50}$ (reduced fraction)

Decimal to Percent

Ex: $0.26 = \frac{26}{100} = 26\%$ % means out of 100

Convert to a fraction (out of 100), then write it as a percent.

Ex: $0.4 = \frac{4}{10}$

Take the numbers in the place value chart that are to the right of the decimal and put them on the top of the fraction (numerator). The bottom number of the fraction (denominator) will be the same as the place value of the last digit in the place value chart (tenths means out of 10).

Place Value

tens	ones	tenths	hundredths
		4	

$= \frac{2}{5}$ (reduced fraction)

Ex: $0.4 = \frac{4}{10} \overset{\times 10}{=} \frac{40}{100} = 40\%$ % means out of 100

Convert to a fraction (out of 100), then write it as a percent.

23) Complete the table below.

	Fraction	Decimal	Percent
a)	$\frac{3}{100}$	0.03	3%
b)	$\frac{88}{100}$	0.88	88%
c)	$\frac{65}{100}$	0.65	65%

	Fraction	Decimal	Percent
d)	$\frac{7}{10} \overset{\times 10}{=} \frac{70}{100}$	0.7	70%
e)	$\frac{9}{100}$	0.09	9%
f)		0.005 (challenge - think carefully)	0.5%

$\frac{5}{1000} \overset{\div 10}{\underset{\div 10}{=}} \frac{.5}{100}$

Links to IXL Practice (grade 6):

U.4 – Convert Between Percents, Fractions, and Decimals

Converting a Percent to a Fraction AND Converting a Percent to a Decimal.

Percent to Fraction

Ex: $45\% = \frac{45}{100} = \frac{9}{20}$ (reduced fraction)

% means out of 100

Place Value

___	___	. 4	5
tens	ones	tenths	hundredths

Percent to Decimal

Ex: $45\% = \frac{45}{100} = 0.45$

1) Convert to a fraction (out of 100).

2) Then, write the top number (numerator) in the place value chart so that the last digit (5) is in the place value that matches the bottom number (denominator). *Hundredths means out of 100

24) Complete the table below.

	Fraction	Decimal	Percent
a)	$\frac{18}{100}$	0.18	18%
b)	$\frac{8}{100}$	0.08	8%
c)	$\frac{90}{100}$	0.90 or 0.9	90%
d)	$\frac{39}{100}$	0.39	39%
e)	$\frac{1}{100}$	0.01	1%
f)	$\frac{100}{100}$	1	100%

work space: