Module 4: Maximizing and Minimizing

TOPIC 2: SOLVING QUADRATIC EQUATIONS

Students review polynomials. They use different methods to add, subtract, multiply, and divide polynomials. Students use what they know about square roots and graphs of quadratic equations to solve equations of the forms $x^2 = n$ and $ax^2 - c = n$. Students see in the graphs that the solutions are both equidistant from the axis of symmetry. Students then learn to factor or complete the square to solve quadratic equations and real-world problems. Next, students derive the Quadratic Formula. They see the structure of solutions to quadratic equations in the Quadratic Formula: the axis of symmetry plus or minus the distance to the parabola. Finally, students are presented with a real-world situation and use familiar strategies to complete a quadratic regression to determine the curve of best fit.

Where have we been?

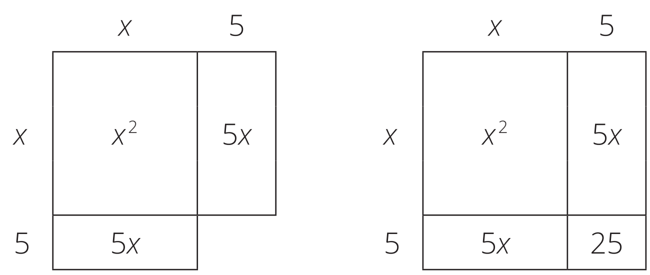
Students know the characteristics that define a quadratic function. They have explored zeros of functions and have interpreted their meaning in contexts. Students know that the factored form of a quadratic equation gives the zeros of the function. They can sketch quadratic equations using key characteristics from an equation written in general form, factored form, or vertex form. Students have extensive experience with locating solutions to equations using a graphical representation.

Where are we going?

The techniques for solving quadratics will be applicable as students solve higher-order polynomials in Algebra 2 and beyond. Understanding the structure and symmetry of a quadratic equation allows students to solve quadratics with complex roots as well as higher-order polynomials.

Completing the Square

The quadratic expression $x^2 + 10x$ can be represented in a square shape as $x^2 + 5x + 5x$. To complete the square, add $5 \cdot 5$, or 25. The expression $x^2 + 10x + 25$ can then be written in factored form as $(x + 5)(x + 5)$, or $(x + 5)^2$.



Math Legends

One of the most brilliant of ancient Greek mathematicians was a man named Pythagoras. He believed that every number could be expressed as a ratio of two integers.

Yet, legend tells us that one day at sea, one of Pythagoras's students pointed out to him that the diagonal of a square which measures 1 unit by 1 unit would be $\sqrt{2}$, a number that could not possibly be represented as a ratio of two integers.

This student was allegedly thrown overboard, and the rest of the group was sworn to secrecy!

Talking Points

Equivalent forms of quadratic equations is an important topic to know about for college admissions tests.

Here is a sample question:

The graph of $y = (x - 8)(x + 2)$ is a parabola in the xy-plane. Rewrite the equation in an equivalent form so that the x- and y-coordinates of the vertex of the parabola appear as constants.

To solve this, students might use the process of completing the square.

$y = (x - 8)(x + 2)$
$y = x^2 - 6x - 16$
$y + 16 = x^2 - 6x$
$y + 16 + 9 = x^2 - 6x + 9$
$y + 25 = (x - 3)^2$
$y = (x - 3)^2 - 25$ is the vertex form of the equation of the parabola with vertex at $(3, -25)$.

Key Terms

polynomial
A polynomial is a mathematical expression involving the sum of powers in one or more variables multiplied by coefficients.

degree of a polynomial
The greatest exponent in a polynomial determines the degree of the polynomial.

difference of two squares
The difference of two squares is an expression in the form $a^2 - b^2$ that has factors $(a + b)$ and $(a - b)$.

double root
The quadratic function $q(x) = x^2$ has two solutions at $y = 0$, so the function $q(x) = x^2$ is said to have a double root.