

Summer Assignment

AP Calculus AB

Directions: Complete problems to show understanding of each section (at least 50% of each section). Show all work. Each problem should be completed *without your calculator*. Be ready to hand in your work the first day of school. There may be a quiz to check your understanding.

Lines and Their Graphs

1. Write the equation of the following lines in point-slope form.

- a. The line through the points (2, 4) and (4, -5).
- b. The line with slope 3 passing through the point (4, -2).
- c. The line perpendicular to $2x - 4y = 8$ passing through the point (1, -2).
- d. The line that contains the points (1, -5) and (-2, 4).

2. Find the equation of the straight line that passes through the point (2, 4) and is parallel to the line $2x + 3y - 8 = 0$.

3. Find the equation of the line that is perpendicular to the line $2x + 3y - 8 = 0$ at the point (1,2)

4. The line with the slope 5 that passes through the point (-1,3) intersects the x axis at a point. What are the coordinates of this point?

5. What are the coordinates of the point at which the line passing through the points (1, -3) and (-2,4) intersects the y axis?

6. Let f be a linear function such that $f(2) = 5$ and $f(6) = -1$. Find an equation for $f(x)$.

Algebraic Manipulation

1. Simplify the following expressions.

a. $\frac{x^3}{x^{-5}}$

b. $\frac{2x^3}{y^{-5}} \cdot \frac{y^2}{3x^7}$

c. $\frac{x^2 - 4x - 5}{x^2 + 2x + 1}$

d. $\frac{x-4}{4-x}$

e. $(x - 1)^3$

f. $x^{\frac{1}{3}}x^{\frac{3}{5}}$

g. $\frac{3x+9}{6x}$

h. $\frac{x^2}{x^{1/2}}$

i. $\frac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4}$

j. $\frac{1}{x+1} - \frac{1}{x-1} - \frac{2}{x^2-1}$

k. $\frac{x(-2x)}{2\sqrt{1-x^2}} + \sqrt{1-x^2} + \frac{1}{\sqrt{1-x^2}}$

l. $\frac{\frac{a}{b} - \frac{b}{a}}{a-b}$

m. $\frac{2(x+h)^2+1-(2x^2+1)}{h}$

2. Solve the following for all real values of x.

a. $\frac{2}{x+1} = \frac{x-2}{2}$

b. $x^2 - 9x + 9 = 0$

c. $\frac{1}{x} + x = 4$

d. $\frac{5}{e^x+1} = 1$

e. $\sqrt{x-1} - \frac{5}{\sqrt{x-1}} = 0$

f. $2x^2 + x - 3 = 0$

g. $x^4 - 4x^2 + 2 = 0$

h. $\left(\frac{x}{2}\right)^3 = 125$

j. $2x^2 - x = 2 - \frac{1}{x}$

k. $2\sqrt{x} = x - 3$

l. $2x^2 + 2x + 1 = 0$

m. $(x-2)(x+2)(x-1)^2 = 0$

n. $\frac{(x-5)(x+3)}{(x-1)(x+1)} < 0$

3. Factor as indicated.

a. $3x^4 + 4x^3 - x^2 = x^2(\quad)$

b. $\frac{1}{2x^2+4x} = \frac{1}{2x}(\quad)$

c. $\sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}}(\quad)$

d. $(2x+1)^{3/2}x^{1/2} + (2x+1)^{5/2}x^{-1/2} = (2x+1)^{3/2}x^{-1/2}(\quad)$

4. Factor completely.

a. $2x^2 + 5x - 3$

b. $e^{2x} + 2 + e^{-2x}$

c. $x^3 + 4x^2 - 2x - 8$

d. $4x^4 + 3x^2 - 1$

e. $9x^4 - 25$

f. $2x^2 + 5x - 3$

5. Let $k(x) = 3x + 2$. Find $k(a)$, $k(2a)$, and $k(a+1)$.

6. Solve $x = y^3 - 4$ for y in terms of x.

7. Solve the system:
$$\begin{cases} y^2 = 1 - x^2 \\ y^2 = x^2 - 3x + 2 \end{cases}$$

8. Given $f(x) = |x - 3|$ find $f(1) - f(5)$.

9. Given $f(x) = x^2 - 3x + 4$ find $f(x+2) - f(2)$.

10. Give $f(x) = \frac{1}{x}$ find $\frac{f(x+h)-f(x)}{h}$

11. Given $f(x) = x - 3$ and $g(x) = \sqrt{x}$ complete the following.

a. $f(g(x)) =$

b. $g(f(x)) =$

c. $f(f(x)) =$

12. Given $f(x) = \frac{1}{x-5}$ and $g(x) = x^2 - 5$ complete the following.

a. $f(g(7)) =$

b. $g(f(v)) =$

c. $g(g(x)) =$

13. If $f(x) = x^2 - 4x + 6$, find $f(0)$, $f(2)$, $f(-2)$, $f(a)$, $f(-a)$, $f(x+1)$, $f(2x)$, and $2f(x) - 2$.

14. If $f(x) = 4 - \sqrt{3x - 6}$, find $f(5)$, $f(9)$, $f(a+2)$, $f(-x)$, $f(x^2)$, and $[f(x)]^2$.

Domain and Range

1. For what value of x is the function $g(x) = \frac{2x+1}{x+7}$ undefined?

2. Find the domain of the function.

a. $f(x) = \sqrt{9 - x^2}$

b. $g(x) = -\sqrt{x - 3}$

c. $h(x) = \frac{1}{4x^2 - 21x - 18}$

d. $k(x) = \sqrt{x^2 - 5x - 14}$

e. $f(x) = t^2 - 2t + 5$

f. $g(x) = 7x + 15$

g. $h(x) = \frac{2x+1}{2x-1}$

h. $k(x) = 3x - \frac{2}{\sqrt{x+1}}$

i. $f(x) = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x+2}$

j. $g(x) = \frac{2x^2 + 5x + 3}{2x^2 - 5x - 3}$

k. $h(x) = \sqrt{4x - 1} + \sqrt{x^2 - 1}$

Trigonometric Functions

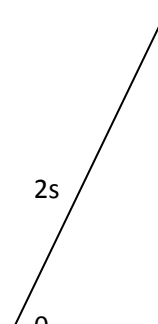
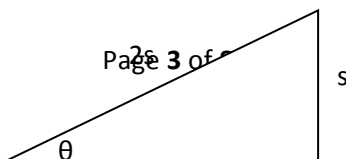
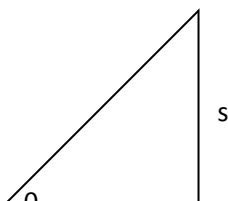
1. Solve the following for the indicated variable.

a. $3 \cos x - 1 = 2$

b. $2 \sin(2x) - \sqrt{3} = 0$

c. $\tan^2 x - 1 = 0$

2. Evaluate all six trigonometric functions for each θ .



3. Solve for θ from $0 \leq \theta \leq 2\pi$. Leave all answers in terms of radians.

a. $\sin \theta = -\frac{1}{2}$

b. $\cos \theta = \frac{\sqrt{2}}{2}$

c. $\tan \theta = -1$

d. $\csc \theta = \frac{2\sqrt{3}}{3}$

e. $\sec \theta = 1$

f. $\cot \theta = -\frac{\sqrt{3}}{3}$

g. $\sin \theta = 0$

h. $\cos \theta = \frac{\sqrt{3}}{2}$

i. $\tan \theta = \sqrt{3}$

j. $\csc \theta = 2$

k. $\sec \theta = -\frac{\sqrt{2}}{2}$

l. $\cot \theta = \text{und}$

m. $\sin \theta = -\frac{\sqrt{2}}{2}$

n. $\cos \theta = 0$

o. $\tan \theta = 0$

4. Evaluate each trigonometric function.

a. $\sin \frac{3\pi}{4}$

b. $\sec \left(-\frac{7\pi}{3}\right)$

c. $\cot \pi$

d. $\cos \left(-\frac{3\pi}{2}\right)$

e. $\tan \frac{11\pi}{6}$

f. $\csc \frac{2\pi}{3}$

5. Factor the expression.

a. $\sin x + \tan x = \sin x (\quad)$

b. $5 \cos^2 x - 5 \sin^2 x + \cos x + \sin x$

c. $1 - \sin^2 x$

d. $\cos^2 x + 4 \cos x + 4 - \tan^2 x$

6. Simplify $\frac{1 - (\sin x + \cos x)^2}{2 \sin x}$

7. Solve $\cos^2 x + 3 \cos x + 2 = 0$

8. Find the exact value of $\sin \left(\cos^{-1} \left(-\frac{1}{3} \right) \right)$

9. Find the solution of the equations for $0 \leq \theta \leq 2\pi$.

a. $2 \sin^2 \theta = 1 - \sin \theta$

e. $2 \tan \theta - \sec^2 \theta = 0$

f. $\sin 2\theta + \sin \theta = 0$

10. Which of the following expressions are identical?

a. $\cos^2 x$

b. $(\cos x)^2$

c. $\cos x^2$

11. Which of the following expressions are identical?

a. $(\sin x)^{-1}$

b. $\arcsin x$

c. $\sin x^{-1}$

Exponents and Logarithms

1. Write the equations in logarithmic form.

a. $2^6 = 64$

b. $49^{\frac{1}{2}} = \frac{1}{7}$

c. $10^x = 74$

2. Evaluate the following.

a. $\log_2 128$

b. $\log_8 1$

c. $10^{\log 45}$

d. $\log. 000001$

e. $\ln e^6$

f. $\log_4 8$

g. $\log_3 \frac{1}{27}$

h. $2^{\log_2 13}$

i. $\log_5 \sqrt{5}$

j. $e^{2 \ln 7}$

k. $\log 15 + \log 4$

l. $\log_3 \sqrt{243}$

m. $\log_2 16^{23}$

n. $\log_2 250 - \log_5 2$

o. $\log_8 6 - \log_8 3 + \log_8 2$

3. Expand the logarithmic expressions.

a. $\log_2 (x\sqrt{x^2 + 1})$

b. $\ln \sqrt{\frac{x^2 - 1}{x^2 + 1}}$

c. $\ln \left(\frac{4x^3}{y^2(x-1)^2} \right)$

4. Combine into a single logarithm.

a. $\log 6 + 4 \log 2$

b. $\log x + \log (x^2 y) + 3 \log y$

c. $\frac{3}{2} \log_2 (x - y) - 2 \log_2 (x^2 + y^2)$

5. Solve for x.

a. $2^x = 64$

b. $10^x = 1000$

c. $\log x = 0.72$

d. $4^x = 3$

e. $\ln x = 1.09$

f. $\ln e^3 = x$

g. $\ln e^x = 4$

h. $\ln x + \ln x = 0$

i. $e^{\ln 5} = x$

j. $\ln 1 - \ln e = x$

k. $\ln 6 + \ln x - \ln 2 = 3$

l. $\ln(x + 5) = \ln(x - 1) - \ln(x + 1)$

m. $\log_2 (1 - x) = 4$

n. $2^{3x-5} = 7$

o. $5^{5-3x} = 26$

p. $\ln(2x - 3) = 14$

q. $e^{\frac{3x}{4}} = 10$

r. $2^{1-x} = 3^{2x+5}$

s. $\log x + \log(x + 1) = \log 12$

t. $\log_8(x+5) - \log_8(x-2) = 1$

6. Express y in terms of x.

a. $\log y = x + 2$

b. $\ln y = 2 \ln x$

c. $\log y = 4 \log x + 3$

Graphs

1. Sketch the graphs of $y = x^2 - 4x + 3$ and $x - 2y = -6$ on the same set of axes. Find the coordinates of each intersection point.

2. Sketch the graph of each function.

$$\text{a. } f(x) = \begin{cases} 1, & x \leq 0 \\ -1, & x > 0 \end{cases} \quad \text{b. } f(x) = \begin{cases} 2x, & (-\infty, -1) \\ 2x^2, & [-1, 2) \\ -x + 3, & (2, \infty) \end{cases}$$

3. If $f(x) = x^2 - 1$, describe in words what the following would do to the graph of $f(x)$.

$$\begin{array}{lll} \text{a. } f(x) - 4 & \text{b. } f(x - 4) & \text{c. } -f(x + 2) \\ \text{d. } 5f(x) + 3 & \text{e. } f(2x) & \text{f. } |f(x)| \end{array}$$

4. Sketch the graph of the following functions

$$\begin{array}{ll} \text{a. } (x) = 1 - 2x & \\ \text{b. } f(x) = \frac{1}{3}(x - 5), 2 \leq x \leq 8 & \\ \text{c. } f(t) = 1 - \frac{1}{2}t^2 & \\ \text{d. } g(t) = t^2 - 2t & \\ \text{e. } f(x) = x^2 - 6x + 6 & \\ \text{f. } f(x) = 3 - 8x - 2x^2 & \\ \text{g. } g(x) = 1 - \sqrt{x} & \\ \text{h. } g(x) = -|x| & \\ \text{i. } f(x) = \begin{cases} x + 6, & x < -2 \\ x^2, & x \geq -2 \end{cases} & \\ \text{j. } f(x) = \begin{cases} x^2, & 0 \leq x < 2 \\ 1, & x \geq 2 \end{cases} & \end{array}$$

5. Sketch the graph of the indicated translations of $f(x) = x^2$ on the same axis. Use a different color for each equation.

$$\begin{array}{lll} \text{a. } f(x) + 2 & \text{b. } f(x+2) & \text{c. } -f(x) \\ \text{d. } f(-x) & \text{e. } 0.5f(x) & \text{f. } 2f(x) \end{array}$$

6. Sketch the graph of the indicated translations of $f(x) = \frac{1}{x}$ on the same axis. Use a different color for each equation.

- a. $f(x) + 2$ b. $f(x+2)$ c. $-f(x)$

7. Sketch the graph of the indicated translations of $f(x) = \sqrt{x}$ on the same axis. Use a different color for each equation.

- a. $f(x) + 2$ b. $f(x+2)$ c. $-f(x)$
d. $f(-x)$ e. $0.5f(x)$ f. $2f(x)$

8. Sketch the graph of the indicated translations of $f(x) = e^x$ on the same axis. Use a different color for each equation.

- a. $f(x) + 2$ b. $f(x+2)$ c. $-f(x)$

9. Sketch the graph of the indicated translations of $f(x) = \ln x$ on the same axis. Use a different color for each equation.

- a. $f(x) + 2$ b. $f(x+2)$ c. $-f(x)$

10. Graph the following translations of $f(x) = \sin x$ for at least two periods.

- a. $f(x) = -\sin x$
b. $f(x) = \sin \pi x$
c. $f(x) = \sin(x - \pi)$
d. $f(x) = \sin(x + \pi)$
e. $f(x) = \sin x + 2$
f. $f(x) = -\sin x + 2$

11. Graph the following translations of $f(x) = \cos x$ for at least two periods.

- a. $f(x) = -\cos x$
b. $f(x) = \cos \pi x$
c. $f(x) = \cos(x - \pi)$
d. $f(x) = \cos(x + \pi)$
e. $f(x) = \cos x + 2$
f. $f(x) = -\cos x + 2$

12. Graph the following translations of $f(x) = \tan x$ for at least two periods.

- a. $f(x) = -\tan x$
b. $f(x) = \tan \frac{\pi}{2} x$

c. $f(x) = \tan\left(x - \frac{\pi}{2}\right)$

d. $f(x) = \tan\left(x + \frac{\pi}{2}\right)$

e. $f(x) = \tan x + 2$

f. $f(x) = -\tan x + 2$

Geometry

1. The sides of a rectangle are x and $3 - 2x$. Express the rectangle's area as a function of x . Express the rectangle's perimeter as a function of x . Explain why x cannot equal 2.
2. The height and the diameter of a cylinder are equal. Express the volume of the cylinder as a function of its radius.
3. Give the dimensions of three different rectangles with area 6 cm^2 .
4. Each leg of an isosceles triangle is twice as long as its base. Express the perimeter of the triangle in terms of the length b of the base.
5. Sketch the graph of the circle $x^2 + (y - 2)^2 = 25$. Find the circumference and the area of the circle.
6. Find the surface area of a box of height h whose base dimensions are p and q , and that satisfies the following conditions:
 - a. The box is closed.
 - b. The box has an open top.
 - c. The box has an open top and a square base with side length p .
7. A piece of wire 5 inches long is to be cut into two pieces. One piece is x inches long and is to be bent into the shape of a square. The other piece is to be bent into the shape of a circle. Find an expression for the total area made up by the square and the circle as a function of x .
8. A car travels 360 miles in a period of 180 minutes. Find the average velocity of the car in miles per hour over this time period.
9. A 20 foot ladder rests against a building 15 feet from the floor. How far does the ladder extend from the base of the wall? What angle does the ladder make with the ground?