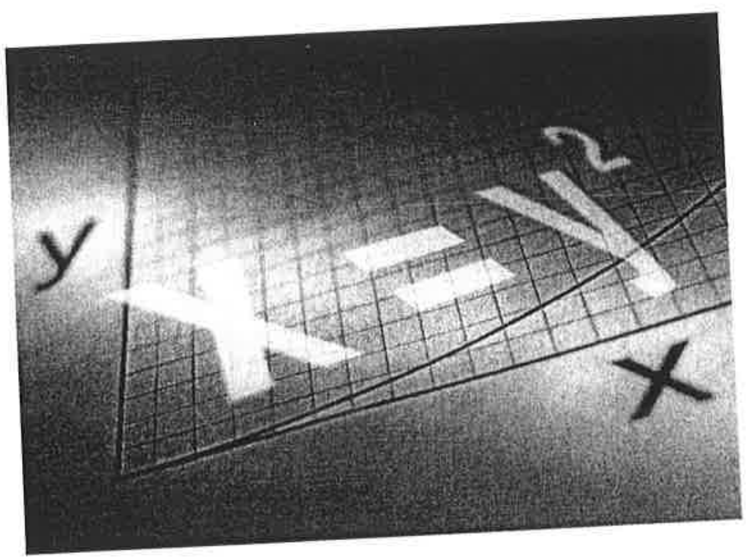
SAXON ALGEBRA 1/2

SUMMER MATH REVIEW PACKET



Dear Parents,

The Classical Math Committee would like to take this opportunity to give some reminders about and suggestions for using this math packet.

- This math packet is meant as a review of the previous year's concepts. No new material is presented here, so the problems should not require a lot of effort.

- Since this packet is meant as a review, parents and/or students may not use the packet as an indicator for math level advancement—again the problems should not require a lot of effort.

- Since the packet is a review and preparation for the next math level, students should wait until the end of July or early August to work on it. That way they will reap the benefits of having a review right before starting new curriculum in fall.

- Students should complete the math packet material over the course of several weeks. There is no benefit to rushing through the whole packet in just a few days.

- Finally, parents whose student struggles with this packet should let the administrator or the teacher who will have the student in fall know of the student's struggles.

Thank you,

Classical Math Committee

You may want to have your child visit the following sites for additional practice.

http://www.mathfactcafe.com/ For elementary math fact practice.

https://www.math-drills.com Worksheets for all levels and on-line practice.

LESSON 13 *Common Factors and the Greatest Common Factor • Multiplication Word Problems*

13.A common factors and the greatest common factor

The factors of 12 are 1, 2, 3, 4, 6, and 12. The factors of 15 are 1, 3, 5, and 15. So the shared or **common factors** of 12 and 15 are 1 and 3, and we can call 3 the **greatest common factor** (GCF) of 12 and 15. The compact notation for this is

$$\text{GCF}(12, 15) = 3$$

example 13.1 Find all the common factors and the greatest common factor of the following pairs of numbers:

(a) 24 and 30 (b) 24 and 7 (c) 24 and 6

solution

(a) The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24. The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30. So the common factors are **1, 2, 3,** and **6,** and the greatest common factor is **6.**

(b) The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24. The factors of 7 are just 1 and 7. So the only common factor is **1,** and **1** is also the greatest common factor. Note that two numbers whose GCF is 1 are said to be **relatively prime.**

(c) The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24. The factors of 6 are 1, 2, 3, and 6. So the common factors are **1, 2, 3,** and **6,** and **6** is the greatest common factor. Notice that when one number is a factor of the other, as 6 is a factor of 24, the GCF of the two must be the smaller number.

When the numbers have many factors, listing them can be time-consuming. An alternative approach is to write each number as the product of prime numbers. Then the GCF is formed by multiplying the greatest number of common (or shared) prime factors. When there are no shared prime factors, the GCF is 1, since 1 is a factor of every whole number.

13.B multiplication word problems

Some word problems require that numbers be multiplied to find the answer. These word problems sometimes contain the word *product*. Many of them contain the word *times* used in a phrase such as "5 times as many."

example 13.3 Harriet's score in the second game was the product of her first game's score and 25. If Harriet had scored 14,025 points in the first game, how many points did she score in the second game?

solution The word *product* tells us to multiply.

$$25 \times 14{,}025 \text{ points} = \mathbf{350{,}625\ points}$$

example 13.4 Roger scored 26,142 points in the first game. He scored 7 times this many points in the second game. How many points did he score in all?

solution The word *times* tells us to multiply, and the words *in all* tell us to add.

$$\begin{array}{r} 26{,}142 \\ \times \quad\quad 7 \\ \hline 182{,}994 \end{array} \longrightarrow \begin{array}{rl} 26{,}142 & \text{first game} \\ +\ 182{,}994 & \text{second game} \\ \hline 209{,}136 & \text{in all} \end{array}$$

He scored **209,136 points** in all.

1. Find all the common factors of 33 and 44 and identify the greatest common factor.

2. Find GCF $(36, 48)$.

3. Find GCF $(48, 60)$.

   [A] 80 [B] 12 [C] 4 [D] 240

4. Julie used 6308 nails to do the first roof. She used 7 times as many nails to do the second roof. How many nails did Julie use to do both roofs?

5. Ward 1 reported 4 times as many votes as Ward 2 reported. Ward 7 reported 6 times as many votes as Ward 2 reported. If Ward 2 reported 8672 votes, how many votes did the three wards report in all?

6. Jamison ordered 12,426 beads last year. This year he ordered 9 times as many beads. How many beads did Jamison order this year?

   [A] 112,934 beads

   [B] 110,774 beads

   [C] 111,834 beads

   [D] 111,854 beads

7. Find all the common factors of 30 and 15 and identify the greatest common factor.

8. Find GCF $(12, 20)$.

9. Find GCF $(24, 30)$.

   [A] 6 [B] 2 [C] 40 [D] 4

10. Renée used 4653 nails to do the first roof. She used 6 times as many nails to do the second roof. How many nails did Renée use to do both roofs?

11. Ward 3 reported 8 times as many votes as Ward 5 reported. Ward 4 reported 7 times as many votes as Ward 5 reported. If Ward 5 reported 8196 votes, how many votes did the three wards report in all?

12. Adaline ordered 15,112 buttons last year. This year she ordered 6 times as many buttons. How many buttons did Adaline order this year?

    [A] 91,722 buttons

    [B] 89,652 buttons

    [C] 90,672 buttons

    [D] 90,562 buttons

13. Find all the common factors of 70 and 18 and identify the greatest common factor.

14. Find GCF $(24, 42)$.

15. Find GCF $(18, 24)$.

    [A] 3 [B] 4 [C] 36 [D] 6

common factors: 1, 11
[1] greatest common factor: 11

[2] 12

[3] B

[4] 50,464 nails

[5] 95,392 votes

[6] C

common factors: 1, 3, 5, 15
[7] greatest common factor: 15

[8] 4

[9] A

[10] 32,571 nails

[11] 131,136 votes

[12] C

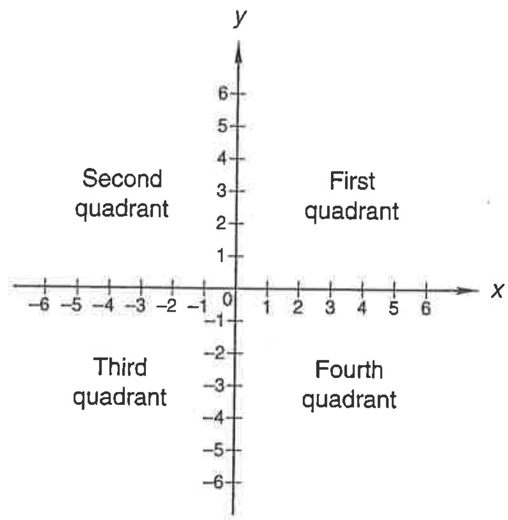
common factors: 1, 2
[13] greatest common factor: 2

[14] 6

[15] D

LESSON 38 Rectangular Coordinates

Here we show two number lines. The lines are perpendicular and intersect at the origin of both lines. We call the vertical line the **y-axis** and the horizontal line the **x-axis.** The whole figure is called a **rectangular coordinate system** or **Cartesian coordinate system.** Still other times, the figure is simply called a **coordinate plane.**

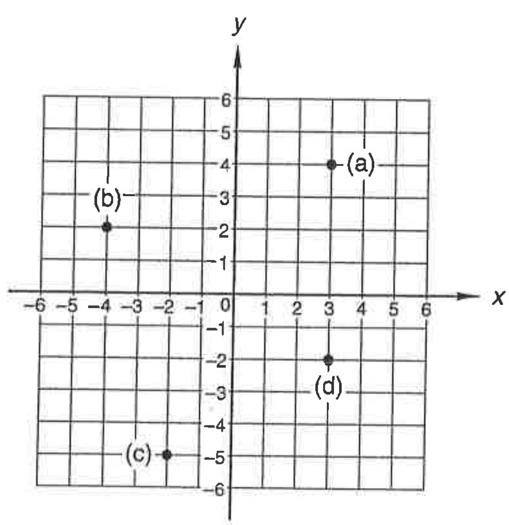


The number lines (axes) divide the plane into four quarters, or **quadrants.** The upper right quadrant is called the first quadrant. The others are numbered in a counterclockwise direction.

Any point on the plane can be designated by saying how far it is to the right or left of the y-axis and how far up or down it is from the x-axis. Two numbers are associated with every point on the plane. The first number tells how far the point is to the right (+) or to the left (–) of the y-axis. This number is called the ***x*-coordinate** of the point. The second number tells how far the point is above (+) or below (–) the x-axis. This is called the ***y*-coordinate** of the point. We name a point by referring to its coordinates. We write its coordinates as follows:

(x-coordinate, y-coordinate)

example 38.1 Four points are graphed on this rectangular coordinate system. What are the x- and y-coordinates of the points?



solution Point (a) is 3 units to the right of the y-axis and 4 units above the x-axis.

$$\text{(a)}\ x = 3,\ y = 4 \text{ or } \mathbf{(3, 4)}$$

Point (b) is 4 units to the left of the y-axis and 2 units above the x-axis.

$$\text{(b)}\ x = -4,\ y = 2 \text{ or } \mathbf{(-4, 2)}$$

Point (c) is 2 units to the left of the y-axis and 5 units below the x-axis.

$$\text{(c)}\ x = -2,\ y = -5 \text{ or } \mathbf{(-2, -5)}$$

Point (d) is 3 units to the right of the y-axis and 2 units below the x-axis.

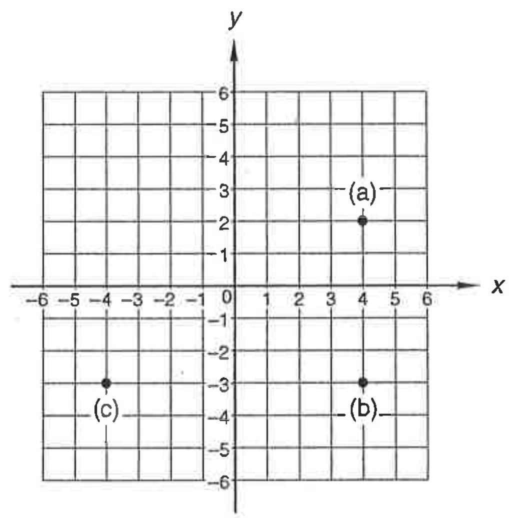
$$\text{(d)}\ x = 3,\ y = -2 \text{ or } \mathbf{(3, -2)}$$

example 38.2 Graph the following points: (a) $(4, 2)$ (b) $(4, -3)$ (c) $(-4, -3)$

solution The first number is always the x-coordinate, and the second number is always the y-coordinate. Thus,

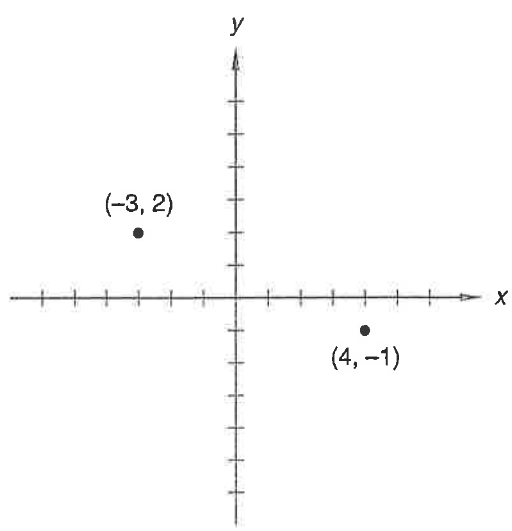
(a) is 4 units to the right of the y-axis and 2 units above the x-axis
(b) is 4 units to the right of the y-axis and 3 units below the x-axis
(c) is 4 units to the left of the y-axis and 3 units below the x-axis

Now we graph the points.

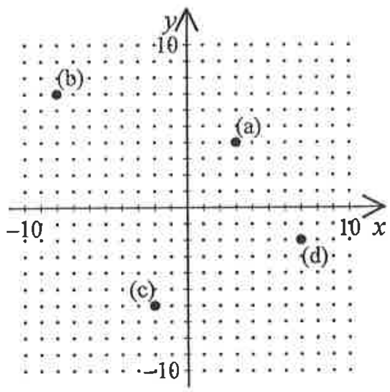


example 38.3 Make a sketch of a rectangular coordinate system, and graph the points $(-3, 2)$ and $(4, -1)$.

solution A quick sketch is all we need. We do not even need to put numbers on the axes.



1. Four points are graphed on this rectangular coordinate system. What are the x- and y-coordinates of the points?

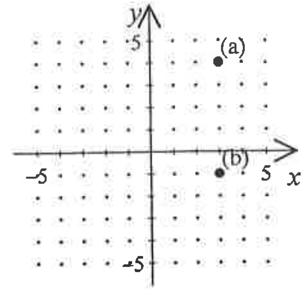


2. Graph the following points: (a) $(3, 8)$ (b) $(-9, 5)$ (c) $(-4, -2)$

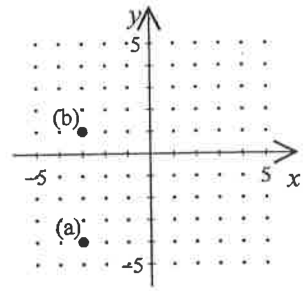
3. The vertices of a rectangle are $(-9, 8)$, $(-9, 1)$, $(-6, 1)$, and $(-6, 8)$. Draw the rectangle and find its perimeter and area.

4. Graph the following points:
(a) $(-4, -3)$ (b) $(1, -3)$

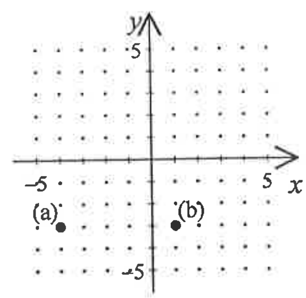
[A]



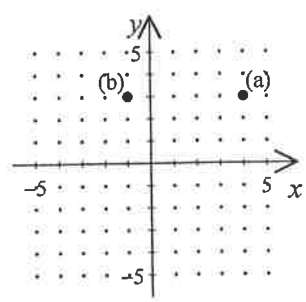
[B]



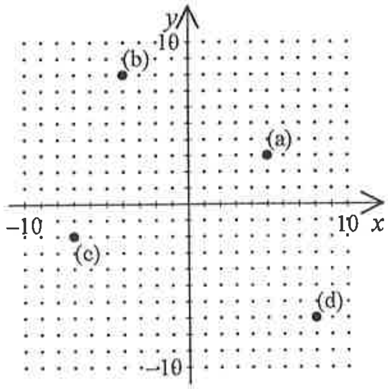
[C]



[D]



5. Four points are graphed on this rectangular coordinate system. What are the x- and y-coordinates of the points?

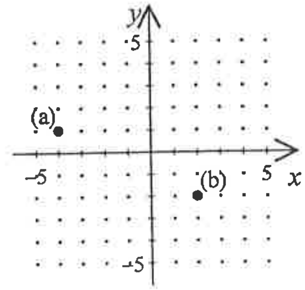


6. Graph the following points: (a) $(2, 9)$ (b) $(-7, 3)$ (c) $(-6, -8)$

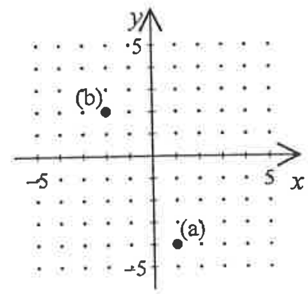
7. The vertices of a rectangle are $(7, 8)$, $(-5, 8)$, $(-5, 2)$, and $(7, 2)$. Draw the rectangle and find its perimeter and area.

8. Graph the following points:
(a) $(1, -4)$ (b) $(-2, 2)$

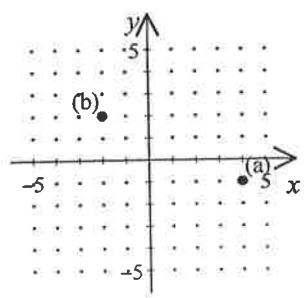
[A]



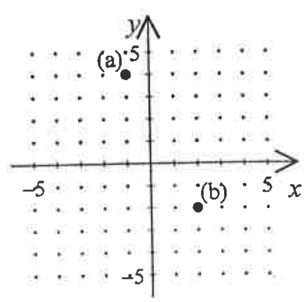
[B]



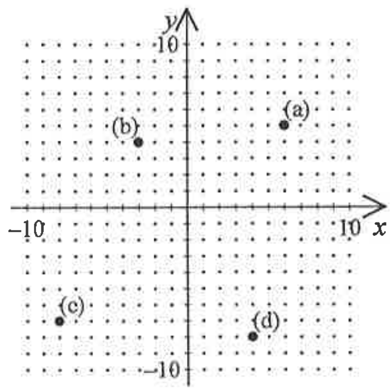
[C]



[D]



9. Four points are graphed on this rectangular coordinate system. What are the x- and y-coordinates of the points?

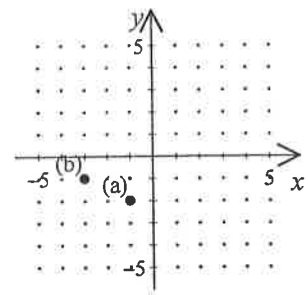


10. Graph the following points: (a) $(6,\ 4)$ (b) $(-8,\ 9)$ (c) $(-7,\ -2)$

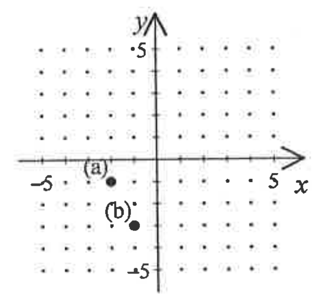
11. The vertices of a rectangle are $(-3,\ -8)$, $(-9,\ -8)$, $(-9,\ -4)$, and $(-3,\ -4)$. Draw the rectangle and find its perimeter and area.

12. Graph the following points:
(a) $(-1,\ -2)$ (b) $(-3,\ -1)$

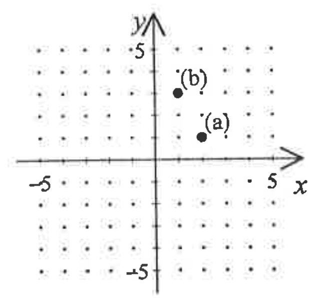
[A]



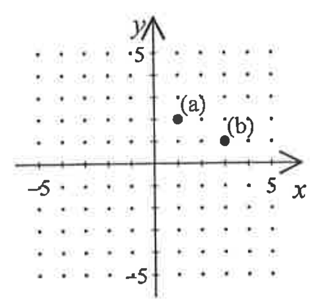
[B]



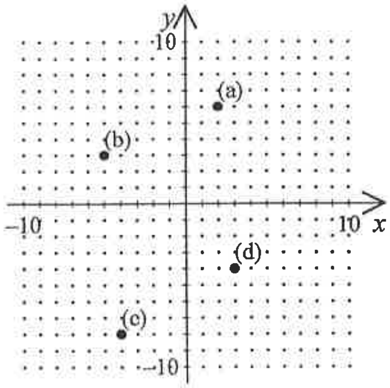
[C]



[D]



13. Four points are graphed on this rectangular coordinate system. What are the x- and y-coordinates of the points?

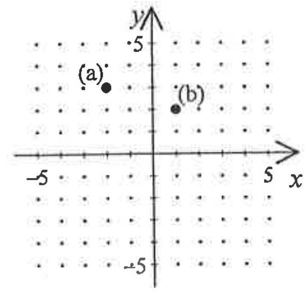


14. Graph the following points: (a) $(5, 1)$ (b) $(-3, 5)$ (c) $(-4, -6)$

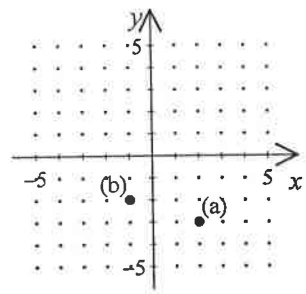
15. The vertices of a rectangle are $(-6, -7)$, $(-6, 2)$, $(5, 2)$, and $(5, -7)$. Draw the rectangle and find its perimeter and area.

16. Graph the following points:
(a) $(-2, 3)$ (b) $(1, 2)$

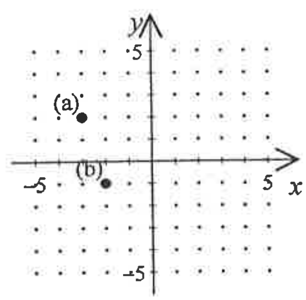
[A]



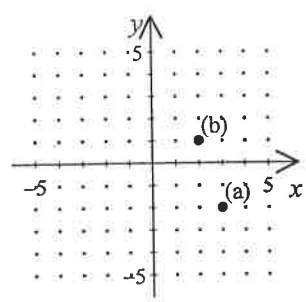
[B]



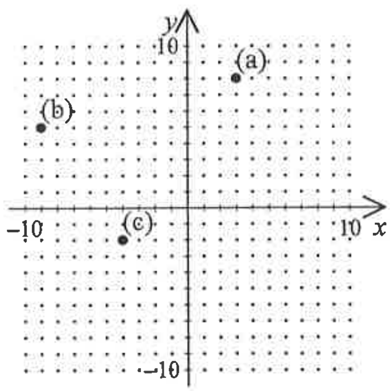
[C]



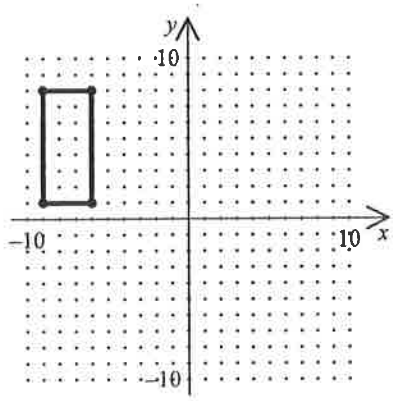
[D]



(a) $(3, 4)$
(b) $(-8, 7)$
(c) $(-2, -6)$
[1] (d) $(7, -2)$



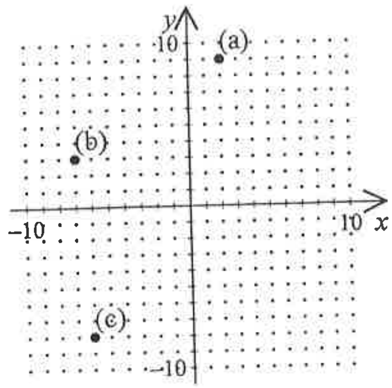
[2]



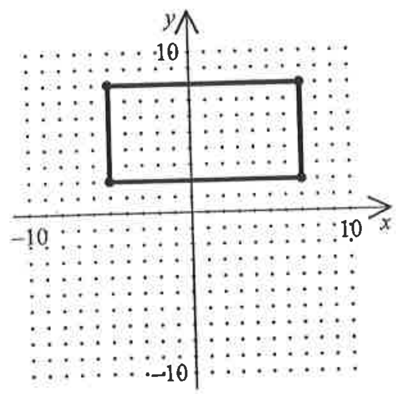
[3] Perimeter = 20 units; Area = 21 units^2

[4] C

(a) $(5, 3)$
(b) $(-4, 8)$
(c) $(-7, -2)$
[5] (d) $(8, -7)$



[6] ______

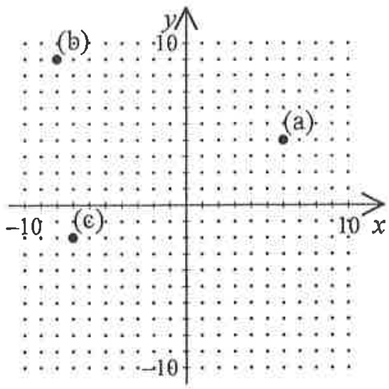


[7] Perimeter = 36 units; Area = 72 units^2

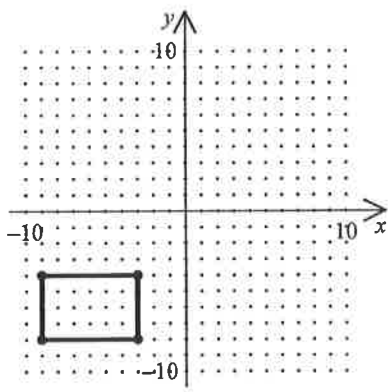
[8] B

(a) $(6, 5)$
(b) $(-3, 4)$
(c) $(-8, -7)$

[9] (d) $(4, -8)$



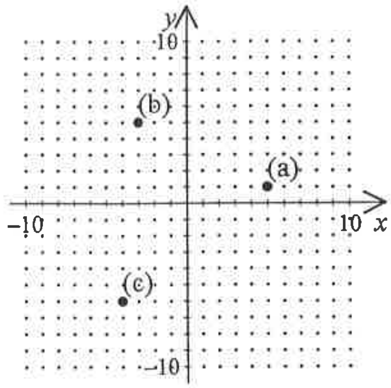
[10]



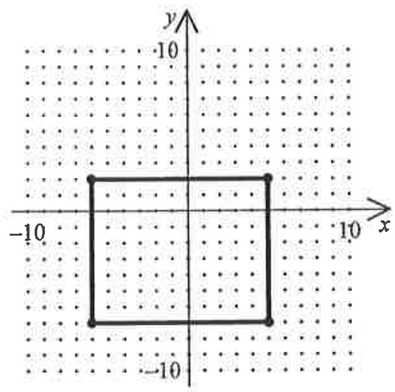
[11] Perimeter = 20 units; Area = 24 units^2

[12] A

(a) $(2, 6)$
(b) $(-5, 3)$
(c) $(-4, -8)$
[13] (d) $(3, -4)$



[14] ______



[15] Perimeter = 40 units; Area = 99 units^2

[16] A

LESSON 61 Solving Equations in Two Steps

The answer to all four of these equations is 6.

(a) $\frac{x}{3} = 2$ (b) $3x = 18$ (c) $x + 2 = 8$ (d) $x - 3 = 3$

We use the multiplication-division rule to solve (a) and (b), and we use the addition-subtraction rule to solve (c) and (d).

$$\text{(a)}\quad \cancel{3} \cdot \frac{x}{\cancel{3}} = 2 \cdot 3 \qquad\qquad \text{(b)}\quad \frac{\cancel{3}x}{\cancel{3}} = \frac{18}{3}$$

$$x = 6 \qquad\qquad x = 6$$

$$\text{(c)}\quad x + 2 - 2 = 8 - 2 \qquad\qquad \text{(d)}\quad x - 3 + 3 = 3 + 3$$

$$x = 6 \qquad\qquad x = 6$$

Often it is necessary to use both rules to solve an equation. In this lesson, we show how to solve equations in which it is easier to use the addition-subtraction rule before the multiplication-division rule.

example 61.1 Solve: $3x + 2 = 7$

solution First we subtract 2 from both sides of the equation.

$$3x + 2 - 2 = 7 - 2 \qquad \text{subtracted 2 from both sides}$$

$$3x = 5 \qquad \text{simplified}$$

Now we divide both sides of the equation by 3.

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{5}{3} \qquad \text{divided both sides by 3}$$

$$x = \mathbf{\frac{5}{3}} \qquad \text{simplified}$$

example 61.2 Solve: $\frac{2}{3}x - \frac{1}{2} = \frac{10}{3}$

solution First we add $\frac{1}{2}$ to both sides of the equation.

$$\frac{2}{3}x - \frac{1}{2} + \frac{1}{2} = \frac{10}{3} + \frac{1}{2} \qquad \text{added } \frac{1}{2} \text{ to both sides}$$

$$\frac{2}{3}x = \frac{23}{6} \qquad \text{simplified}$$

Now we multiply both sides of the equation by $\frac{3}{2}$.

$$\frac{\cancel{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{3}}x = \frac{23}{\underset{2}{\cancel{6}}} \cdot \frac{\cancel{3}}{2} \qquad \text{multiplied both sides by } \frac{3}{2}$$

$$x = \mathbf{\frac{23}{4}} \qquad \text{simplified}$$

Solve:

1. $18y+8=16$

2. $\frac{5}{6}x-\frac{1}{2}=\frac{19}{12}$

3. $6x-7=17$

4. $2\frac{1}{3}x-3\frac{1}{3}=\frac{5}{2}$

   [A] $\frac{5}{2}$ [B] $\frac{6}{3}$ [C] $\frac{6}{2}$ [D] $\frac{5}{3}$

5. $3x+5.6=23.9$

   [A] 12.2 [B] 4.4
   [C] 6.1 [D] 18.3

6. $16y-2=10$

7. $\frac{2}{3}x-\frac{1}{2}=\frac{23}{18}$

8. $23x-9=60$

9. $3\frac{1}{3}x-2\frac{1}{2}=\frac{35}{18}$

   [A] $\frac{4}{3}$ [B] $\frac{5}{4}$ [C] $\frac{4}{4}$ [D] $\frac{5}{3}$

10. $7x-1.9=16.3$

    [A] 18.2 [B] 5.2
    [C] 2.6 [D] 1.3

11. $10y-10=4$

12. $\frac{1}{2}x-\frac{1}{3}=\frac{47}{12}$

13. $17x-13=72$

14. $1\frac{1}{2}x-2\frac{4}{5}=\frac{49}{20}$

    [A] $\frac{7}{3}$ [B] $\frac{8}{3}$ [C] $\frac{8}{2}$ [D] $\frac{7}{2}$

15. $6x+4.2=18.6$

    [A] 3.5 [B] 4.8
    [C] 2.4 [D] 14.4

16. $14y-4=8$

17. $\frac{1}{2}x-\frac{1}{5}=\frac{51}{20}$

18. $11x-16=61$

19. $1\frac{1}{2}x-3\frac{1}{3}=\frac{23}{12}$

    [A] $\frac{8}{3}$ [B] $\frac{7}{2}$ [C] $\frac{8}{2}$ [D] $\frac{7}{3}$

20. $4x+3.1=21.5$

    [A] 4.6 [B] 5.8
    [C] 18.4 [D] 9.2

[1] $\frac{4}{9}$

[2] $\frac{5}{2}$

[3] 4

[4] A

[5] C

[6] $\frac{3}{4}$

[7] $\frac{8}{3}$

[8] 3

[9] A

[10] C

[11] $\frac{7}{5}$

[12] $\frac{17}{2}$

[13] 5

[14] D

[15] C

[16] $\frac{6}{7}$

[17] $\frac{11}{2}$

[18] 7

[19] B

[20] A

LESSON 82 *Evaluation with Signed Numbers*

The order of operations for simplifying expressions that contain signed numbers is the same as the order of operations for simplifying expressions that contain unsigned numbers. We always do the multiplication before we do the algebraic addition.

example 82.1 Simplify: $2(-4) - 3 - (-6)(-3)$

solution First we will do the multiplication and get

$$-8 - 3 - 18$$

We finish by adding algebraically and get -29 for our answer.

$$-8 - 3 - 18 = \mathbf{-29}$$

example 82.2 Evaluate: $-x - xy$ if $x = -2$ and $y = -4$

solution Some people find that using parentheses helps prevent making mistakes with signs. We will write parentheses for each variable.

$$-(\ ) - (\ )(\ )$$

Now we insert the proper numbers in the parentheses.

$$-(-2) - (-2)(-4)$$

We multiply before we add, so the final result is -6.

$$2 - 8 = \mathbf{-6}$$

example 82.3 Evaluate: $-a - xa$ if $x = -3$ and $a = 2$

solution We will use parentheses and write

$$-(2) - (-3)(2)$$

Now we simplify, remembering that we always multiply first.

$-2 - (-6)$	multiplied (-3) and (2)
$-2 + 6$	simplified $-\ (-6)$
$\mathbf{4}$	added

Lesson 82 - Evaluation with Signed Numbers

1. Simplify: $(-5)-(-3)(-7)-(-4)(2)$

Evaluate:

2. $pq-p$ if $p=-2$ and $q=-3$

3. $j-jk$ if $j=2$ and $k=-2$

   [A] 7 [B] 6 [C] 2 [D] –2

4. Simplify: $(-2)-(-8)(-5)-(-3)(4)$

Evaluate:

5. $-xy-x$ if $x=-5$ and $y=6$

6. $-y-yz$ if $y=-3$ and $z=5$

   [A] 12 [B] –12 [C] 18 [D] 17

7. Simplify: $(-4)-(-5)(-9)-(-6)(9)$

Evaluate:

8. $-vw-v$ if $v=6$ and $w=-5$

9. $-r-rs$ if $r=5$ and $s=-4$

   [A] 14 [B] 15 [C] –25 [D] 25

10. Simplify: $(-3)-(-2)(-2)-(-7)(8)$

Evaluate:

11. $fg-f$ if $f=3$ and $g=-5$

12. $p-pq$ if $p=4$ and $q=-6$

    [A] 20 [B] 27 [C] 28 [D] –20

13. Simplify: $(-6)-(-2)(-5)-(2)(-5)$

Evaluate:

14. $-mn-m$ if $m=6$ and $n=-3$

15. $c-cd$ if $c=6$ and $d=-3$

    [A] 25 [B] 24 [C] 12 [D] –12

16. Simplify: $(-4)-(-6)(-4)-(-3)(9)$

Evaluate:

17. $qr-q$ if $q=-2$ and $r=-4$

18. $-b-bc$ if $b=-5$ and $c=-5$

    [A] –21 [B] –20
    [C] –30 [D] 30

19. Simplify: $(-7)-(-7)(-7)-(-6)(8)$

Evaluate:

20. $st-s$ if $s=-4$ and $t=2$

21. $q-qr$ if $q=-3$ and $r=-4$

    [A] –16 [B] –15 [C] –9 [D] 9

Lesson 82 - Evaluation with Signed Numbers

[1] –18

[2] 8

[3] B

[4] –30

[5] 35

[6] C

[7] 5

[8] 24

[9] B

[10] 49

[11] –18

[12] C

[13] –6

[14] 12

[15] B

[16] –1

[17] 10

[18] B

[19] –8

[20] –4

[21] B

LESSON 85 *Equation of a Line • Graphing a Line*

85.A equation of a line

If we have an equation with one unknown, the solution to the equation is the value of the variable that will make the equation a true equation. The solution to the equation

$$x + 2 = 5$$

is 3 because if we use 3 for x we get a true equation.

$$\begin{aligned} (3) + 2 &= 5 \\ 5 &= 5 \qquad \text{true} \end{aligned}$$

If an equation has two variables, the solutions to the equation are pairs of values that make the equation a true equation. The equation

$$y = x + 2$$

is a true equation if we use 5 for y and 3 for x.

$$\begin{aligned} (5) &= (3) + 2 \\ 5 &= 5 \qquad \text{true} \end{aligned}$$

It is also a true equation if we use 9 for y and 7 for x.

$$\begin{aligned} (9) &= (7) + 2 \\ 9 &= 9 \qquad \text{true} \end{aligned}$$

There are many pairs of x and y that make this equation a true equation. If we plot these values of x and y on a rectangular coordinate plane, we find that the points they describe form a straight line. When we use points to draw a line, we say we are **graphing** the line.

85.B

graphing a line

It takes only two points to determine a line, but since this is an introduction we will use three points.

example 85.1 Graph the line $y = x - 2$.

solution To find points that lie on the line we choose values of x and use the equation to find the corresponding values of y. First we make a chart.

x	y

Then we choose values for x and put them in the first column. The numbers 0, 2, and 4 will make calculations easy using the given equation for the line.

x	y
0	
2	
4	

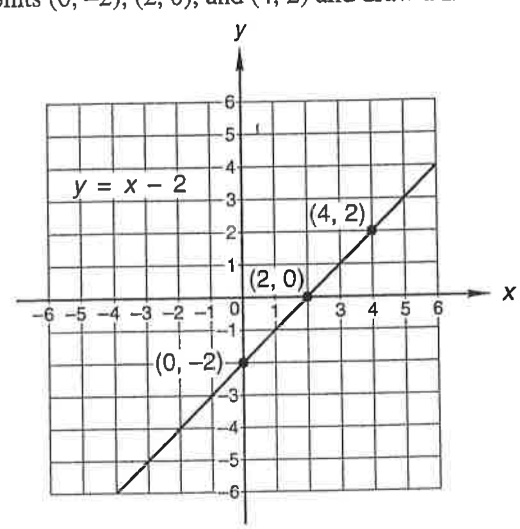
Then we use these values for x in the equation and solve for y.

$$\text{Using 0 for } x\text{: } y = (0) - 2 = -2$$
$$\text{Using 2 for } x\text{: } y = (2) - 2 = 0$$
$$\text{Using 4 for } x\text{: } y = (4) - 2 = 2$$

Now we place the values for y in the second column of the chart.

x	y
0	−2
2	0
4	2

Now we graph the points (0, −2), (2, 0), and (4, 2) and draw a line through the points.

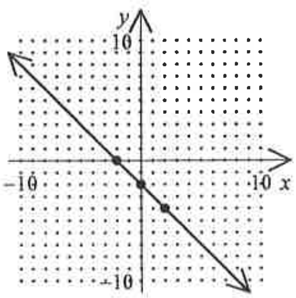


This is the graph of $y = x - 2$.

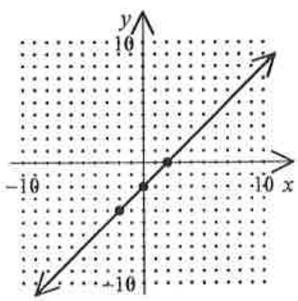
1. Graph the line $y = -x + 2$. Choose 0, 2, and –2 for x-values.

2. Which line is the graph of $y = -x + 2$?

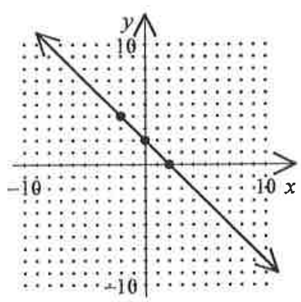
[A]



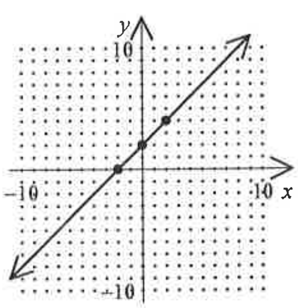
[B]



[C]



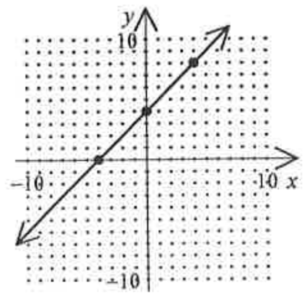
[D]



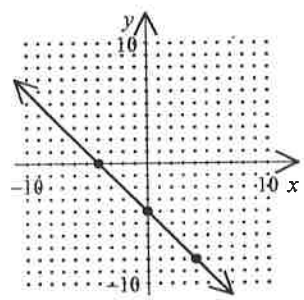
3. Graph the line $y = -x - 5$. Choose 0, –5, and 5 for x-values.

4. Which line is the graph of $y = -x - 4$?

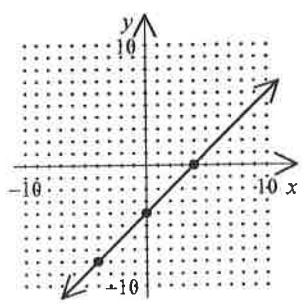
[A]



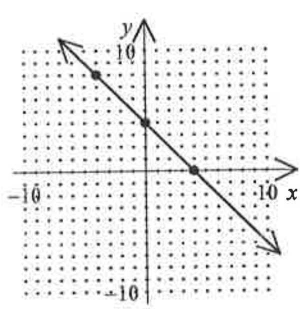
[B]



[C]



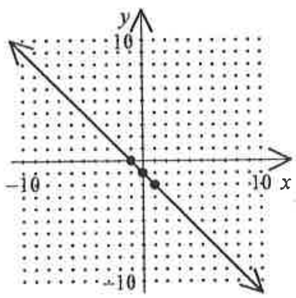
[D]



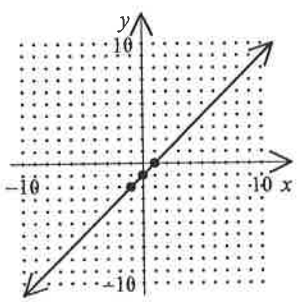
5. Graph the line $y = x + 1$. Choose 0, 1, and -1 for x-values.

6. Which line is the graph of $y = -x + 1$?

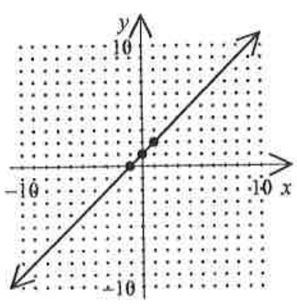
[A]



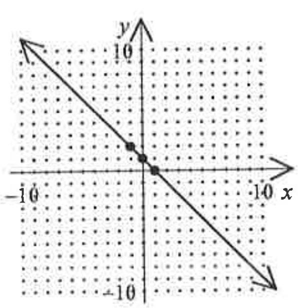
[B]



[C]



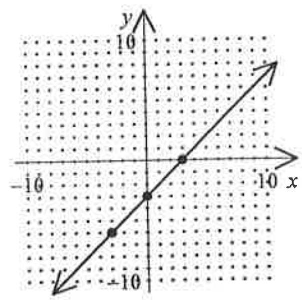
[D]



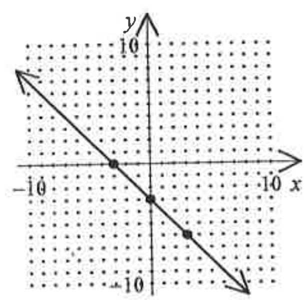
7. Graph the line $y = -x - 4$. Choose 0, -4, and 4 for x-values.

8. Which line is the graph of $y = x - 3$?

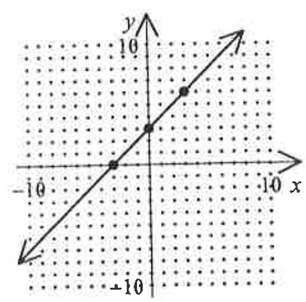
[A]



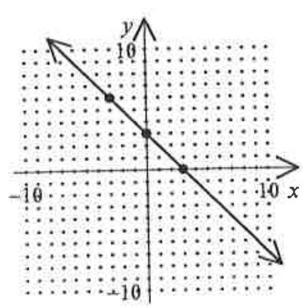
[B]



[C]



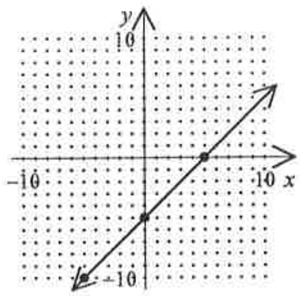
[D]



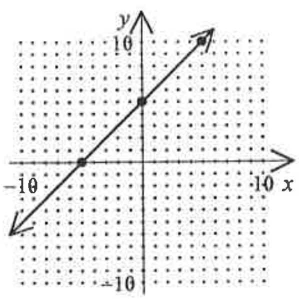
9. Graph the line $y = x + 3$. Choose 0, 3, and –3 for x-values.

10. Which line is the graph of $y = -x + 5$?

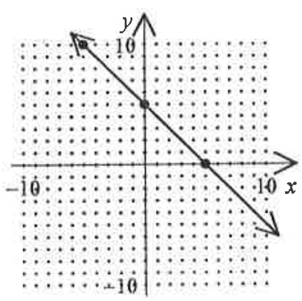
[A]



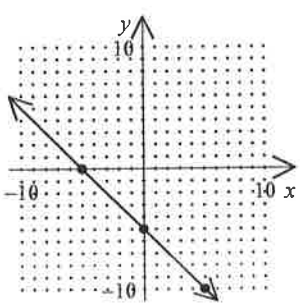
[B]



[C]



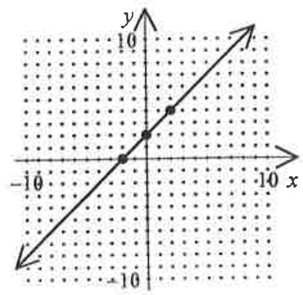
[D]



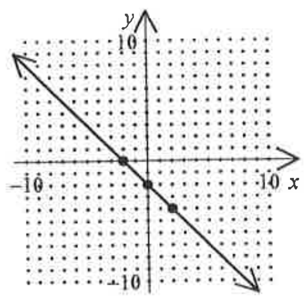
11. Graph the line $y = x - 2$. Choose 0, –2, and 2 for x-values.

12. Which line is the graph of $y = -x - 2$?

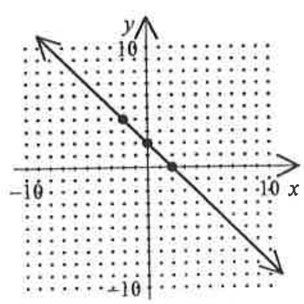
[A]



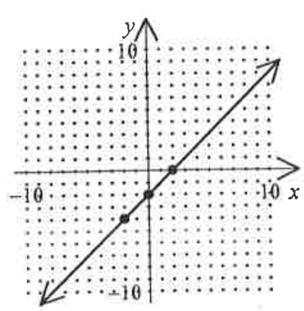
[B]



[C]



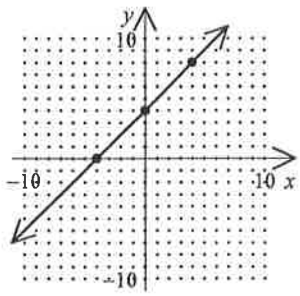
[D]



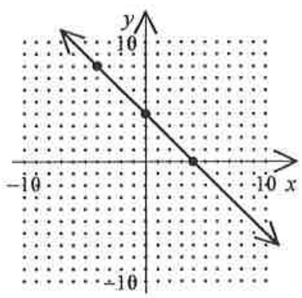
13. Graph the line $y = x + 5$. Choose 0, 5, and -5 for x-values.

14. Which line is the graph of $y = x + 4$?

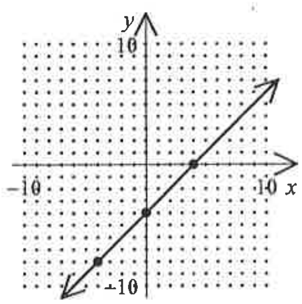
[A]



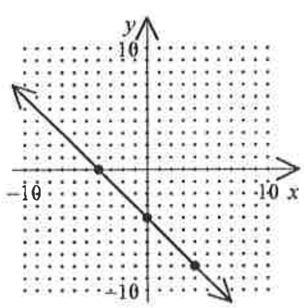
[B]



[C]



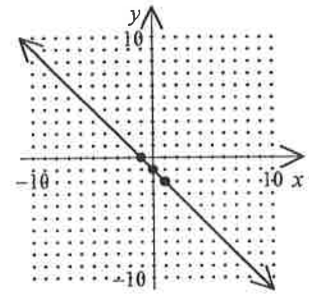
[D]



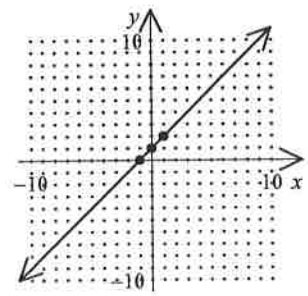
15. Graph the line $y = -x - 1$. Choose 0, -1, and 1 for x-values.

16. Which line is the graph of $y = x - 1$?

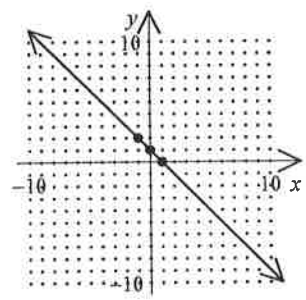
[A]



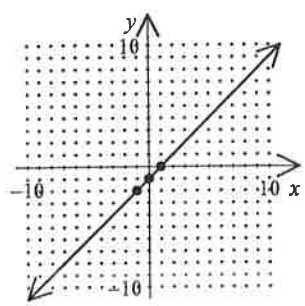
[B]



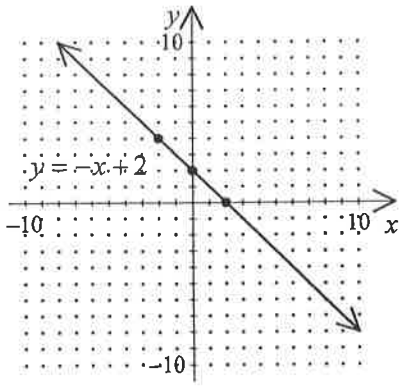
[C]



[D]

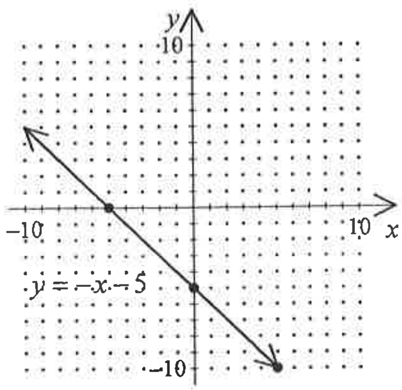


Lesson 85 - Equation of a Line; Graphing a Line



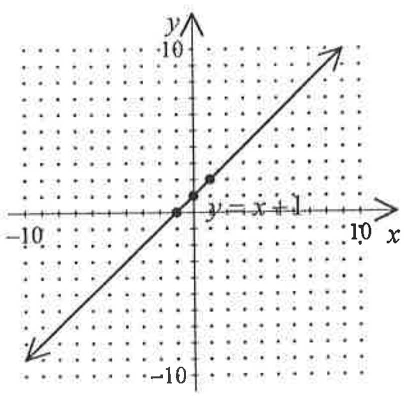
[1] ______________________________

[2] C



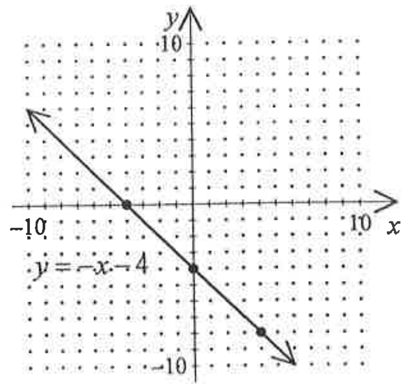
[3] ______________________________

[4] B



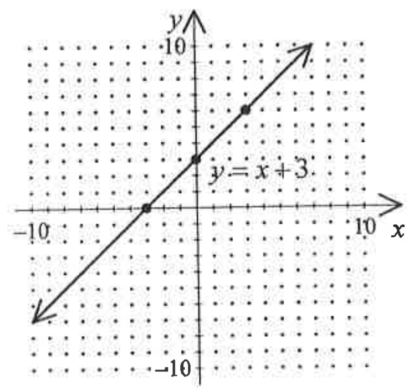
[5] ______________________________

[6] D



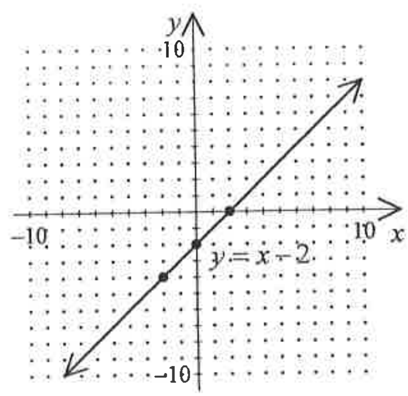
[7] ______________________________

[8] A



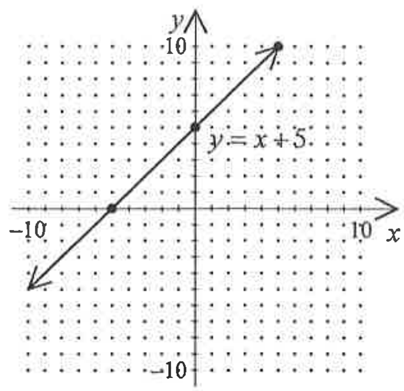
[9] ______________________________

[10] C



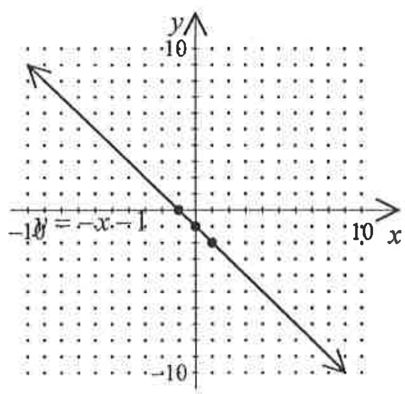
[11] ______________________________

[12] B



[13]

[14] A



[15]

[16] D

LESSON 86 *Algebraic Phrases*

This book is called *Algebra* $\frac{1}{2}$. What is algebra anyway? Here is the beginning of the answer to that question: algebra is the art of expressing problems with symbols and then solving the problems by manipulating those symbols in accordance with certain rules and techniques.

In algebra, we learn to write equations that have the same meanings as statements made with words. Then we find the solutions to the equations. These solutions give us the answers to the questions asked. When we write the equations, we use algebraic phrases. The algebraic phrases have the same meanings as the word phrases. There are several key words to look for when writing algebraic phrases. The word *sum* means things are added, as do the words *greater than* or *increased by*.

WORD PHRASE	ALGEBRAIC PHRASE
The sum of a number and 7	$N + 7$
The sum of a number and –7	$N + (-7)$
7 greater than a number	$N + 7$
A number increased by 7	$N + 7$

The words *less than* or *decreased by* mean to subtract (add the opposite of).

7 less than a number	$N - 7$
A number decreased by 7	$N - 7$

The word *product* means things are multiplied.

The product of a number and 7	$7N$

If we use N to represent an unknown number, then we will use $-N$ to represent the opposite of the unknown number. Thus, twice a number and 9 times the opposite of a number could be written as follows:

Twice a number	$2N$
9 times the opposite of a number	$9(-N)$

Sometimes two or more operations are designated by a single phrase.

The sum of twice a number and –8	$2N + (-8)$
5 times the sum of twice a number and –8	$5[2N + (-8)]$

Cover the answers in the right-hand column below and see if you can write the algebraic phrase that is indicated.

The sum of a number and 9	$N + 9$
A number decreased by 9	$N - 9$
The opposite of a number, decreased by 6	$-N - 6$
The sum of the opposite of a number and –3	$-N + (-3)$
The sum of twice a number and –3	$2N + (-3)$
The product of 5 times a number and 6	$6(5N)$
4 times the sum of twice a number and –3	$4[2N + (-3)]$
The product of –2 and the sum of a number and 5	$-2(N + 5)$

In the previous examples we used the letter N as our variable. Letters at the end of the alphabet, such as x, y, and z, are often used as variables, especially the letter x.

1. Write the algebraic phrases corresponding to the following word phrases:
   (a) The opposite of one third of a number
   (b) Seven greater than thirteen thirds of a number
   (c) The sum of six times the opposite of a number and 3
   (d) The product of –4 and the sum of 7 times a number and –2

2. Which algebraic phrase corresponds to the following word phrase:
   The product of 3 and the sum of 3 times a number and –4

   [A] $3+3N-4$ [B] $3(3N)-4$

   [C] $(N)(-N)(-N)-4$

   [D] $3(3N-4)$

3. Write the algebraic phrases corresponding to the following word phrases:
   (a) The opposite of three fourths of a number
   (b) Ten less than eleven fourths of a number
   (c) The sum of eight times the opposite of a number and 9
   (d) The product of –3 and the sum of 4 times a number and –7

4. Which algebraic phrase corresponds to the following word phrase:
   The sum of five times the opposite of a number and –3

   [A] $5(-N)-3$ [B] $N-5-3$

   [C] $(-N)(-N)(-N)(-N)(-N)-3$

   [D] $-3(-N)+5$

5. Write the algebraic phrases corresponding to the following word phrases:
   (a) The opposite of nine elevenths of a number
   (b) Eleven greater than ten sevenths of a number
   (c) The sum of three times the opposite of a number and 2
   (d) The product of 9 and the sum of 2 times a number and 3

6. Which algebraic phrase corresponds to the following word phrase:
   The sum of two times the opposite of a number and –2

   [A] $2(-N)-2$ [B] $-2(-N)+2$

   [C] $N-2-2$

   [D] $(-N)(-N)-2$

7. Write the algebraic phrases corresponding to the following word phrases:
(a) The opposite of four fifths of a number
(b) Eight less than seven sixths of a number
(c) The sum of seven times the opposite of a number and –5
(d) The product of 6 and the sum of 9 times a number and –9

8. Which algebraic phrase corresponds to the following word phrase:
The product of –5 and the sum of 4 times a number and –5

[A] $(N)(-N)(-N)(-N)-5$

[B] $-5(4N)-5$

[C] $-5(4N-5)$ [D] $-5+4N-5$

9. Write the algebraic phrases corresponding to the following word phrases:
(a) The opposite of two thirds of a number
(b) Five less than nineteen ninths of a number
(c) The sum of four times the opposite of a number and 9
(d) The product of –2 and the sum of 6 times a number and 3

10. Which algebraic phrase corresponds to the following word phrase:
The sum of six times the opposite of a number and 8

[A] $8(-N)+6$ [B] $6(-N)+8$

[C] $N-6+8$ [D] $(-N)(-N)(-N)(-N)(-N)(-N)+8$

11. Write the algebraic phrases corresponding to the following word phrases:
(a) The opposite of four elevenths of a number
(b) Two greater than fourteen elevenths of a number
(c) The sum of two times the opposite of a number and 2
(d) The product of 9 and the sum of 8 times a number and –9

12. Which algebraic phrase corresponds to the following word phrase:
The product of 6 and the sum of 3 times a number and –9

[A] $6(3N-9)$ [B] $6(3N)-9$ [C] $(N)(-N)(-N)-9$ [D] $6+3N-9$

13. Write the algebraic phrases corresponding to the following word phrases:
(a) The opposite of four sevenths of a number
(b) Four greater than eleven tenths of a number
(c) The sum of nine times the opposite of a number and 9
(d) The product of –5 and the sum of 6 times a number and 9

14. Which algebraic phrase corresponds to the following word phrase:
The product of –2 and the sum of 5 times a number and 1

[A] $(N)(-N)(-N)(-N)(-N)+1$ [B] $-2+5N+1$

[C] $-2(5N+1)$ [D] $-2(5N)+1$

[1]
(a) $-\frac{1}{3}N$
(b) $\frac{13}{3}N+7$
(c) $6(-N)+3$
(d) $-4(7N-2)$

[2] D

[3]
(a) $-\frac{3}{4}N$
(b) $\frac{11}{4}N-10$
(c) $8(-N)+9$
(d) $-3(4N-7)$

[4] A

[5]
(a) $-\frac{9}{11}N$
(b) $\frac{10}{7}N+11$
(c) $3(-N)+2$
(d) $9(2N+3)$

[6] A

[7]
(a) $-\frac{4}{5}N$
(b) $\frac{7}{6}N-8$
(c) $7(-N)-5$
(d) $6(9N-9)$

[8] C

[9]
(a) $-\frac{2}{3}N$
(b) $\frac{19}{9}N-5$
(c) $4(-N)+9$
(d) $-2(6N+3)$

[10] B

[11]
(a) $-\frac{4}{11}N$
(b) $\frac{14}{11}N+2$
(c) $2(-N)+2$
(d) $9(8N-9)$

[12] A

[13]
(a) $-\frac{4}{7}N$
(b) $\frac{11}{10}N+4$
(c) $9(-N)+9$
(d) $-5(6N+9)$

[14] C

LESSON 87 Properties of Algebra

In Lesson 84 we looked at the properties of equality. In this lesson we discuss properties of numbers. The commutative property means that with both addition and multiplication we can change the order of the numbers, and the result is the same.

$$2 + 5 = 7 \qquad \text{and} \qquad 5 + 2 = 7$$
$$2 \cdot 5 = 10 \qquad \text{and} \qquad 5 \cdot 2 = 10$$

COMMUTATIVE PROPERTIES
$a + b = b + a \qquad ab = ba$

The associative property means that when more than two numbers are added or multiplied, we can group (or associate) any two first. So

$$2 + 3 + 4 = (2 + 3) + 4 = 5 + 4 = 9$$
and
$$2 + 3 + 4 = 2 + (3 + 4) = 2 + 7 = 9$$

And

$$2 \cdot 3 \cdot 4 = (2 \cdot 3) \cdot 4 = 6 \cdot 4 = 24$$
and
$$2 \cdot 3 \cdot 4 = 2 \cdot (3 \cdot 4) = 2 \cdot 12 = 24$$

ASSOCIATIVE PROPERTIES
$(a + b) + c = a + (b + c) \qquad (ab)c = a(bc)$

The reflexive property states the obvious fact that any number is equal to itself!

REFLEXIVE PROPERTY
$a = a$

The symmetric property means that we can change the order of an equation. We usually prefer to write the variable on the left. This property justifies rewriting

$$10 = x \qquad \text{as} \qquad x = 10$$

SYMMETRIC PROPERTY
If $a = b$, then $b = a$.

The transitive property means that if two things are equal to the same thing, they are equal to each other. If Steve and Sherri are the same age, and Sherri and Brooke are the same age, it is correct to conclude that Steve and Brooke are the same age.

TRANSITIVE PROPERTY
If $a = b$ and $b = c$, then $a = c$.

Use the commutative and associative properties to simplify:

1. $9.97+4.78+0.03+5.22$

2. $\frac{7}{4}\cdot\frac{11}{2}\cdot\frac{4}{7}\cdot2\cdot\frac{14}{11}$

3. $\frac{2}{3}+3\frac{1}{2}+\frac{1}{3}+\frac{1}{2}$

   [A] $4\frac{2}{3}$ [B] 5 [C] $4\frac{5}{6}$ [D] 6

4. $10\cdot4\cdot30\cdot25$

   [A] 30,000 [B] 4000
   [C] 15,000 [D] 75,000

5. $49+76+51+24$

6. $\frac{5}{2}\cdot\frac{12}{11}\cdot4\cdot\frac{2}{5}\cdot\frac{11}{4}$

7. $\frac{2}{9}+\frac{3}{4}+\frac{7}{9}+2\frac{1}{4}$

   [A] 5 [B] $3\frac{35}{36}$ [C] $3\frac{4}{9}$ [D] 4

8. $10\cdot15\cdot20\cdot20$

   [A] 80,000 [B] 40,000
   [C] 45,000 [D] 60,000

9. $92+25+8+75$

10. $\frac{3}{10}\cdot8\cdot\frac{4}{13}\cdot\frac{13}{8}\cdot\frac{10}{3}$

11. $\frac{9}{11}+\frac{3}{5}+1\frac{2}{11}+\frac{2}{5}$

    [A] $2\frac{54}{55}$ [B] $2\frac{5}{11}$
    [C] 3 [D] 2

12. $25\cdot50\cdot4\cdot4$

    [A] 1600 [B] 20,000
    [C] 200,000 [D] 10,000

13. $4.22+3.58+5.78+6.42$

14. $\frac{11}{10}\cdot\frac{3}{14}\cdot\frac{10}{11}\cdot14\cdot\frac{4}{3}$

15. $\frac{1}{6}+\frac{1}{7}+\frac{5}{6}+1\frac{6}{7}$

    [A] $2\frac{41}{42}$ [B] 3
    [C] 4 [D] $2\frac{6}{7}$

16. $4\cdot20\cdot50\cdot5$

    [A] 8000 [B] 50,000
    [C] 4000 [D] 20,000

17. $2.46+6.18+7.54+3.82$

18. $\frac{3}{8}\cdot\frac{2}{15}\cdot4\cdot\frac{8}{3}\cdot\frac{15}{4}$

19. Use the commutative and associative properties to simplify: $\frac{1}{8}+\frac{1}{2}+2\frac{7}{8}+\frac{1}{2}$

[A] $3\frac{15}{16}$ [B] 4 [C] 3 [D] $3\frac{1}{4}$

[1] 20

[2] 14

[3] B

[4] A

[5] 200

[6] 12

[7] D

[8] D

[9] 200

[10] 4

[11] C

[12] B

[13] 20

[14] 4

[15] B

[16] D

[17] 20

[18] 2

[19] B

LESSON 90 Algebraic Sentences

An **algebraic sentence** is a group of mathematical symbols arranged to form an algebraic relationship. The symbol = functions much like the English verb *is*. Here are some examples of word sentences, and their algebraic translations.

WORD SENTENCE	ALGEBRAIC SENTENCE
A number is three.	$N = 3$
Five more than a number is nine.	$N + 5 = 9$
Twice a number is eleven.	$2N = 11$
One less than thrice a number is fourteen.	$3N - 1 = 14$
Six times a number, increased by five is seventeen.	$6N + 5 = 17$

We can solve algebraic sentences using the Properties of Equality taught in Lesson 84.

example 90.1 The sum of twice a number and 42 is 128. Find the number.

solution The word *is* means *equals*. Thus, the sum of twice a number and 42 equals 128. We can rewrite this as $2N + 42 = 128$. Now we can solve for N.

$$\begin{array}{rcll} 2N + 42 &=& 128 & \text{equation} \\ -42 && -42 & \text{added } -42 \text{ to both sides} \\ \hline 2N &=& 86 & \text{simplified} \\ \dfrac{2N}{2} &=& \dfrac{86}{2} & \text{divided both sides by 2} \\ N &=& 43 & \text{simplified} \end{array}$$

We will use 43 for N in the original equation to check.

$$\begin{aligned} 2N + 42 &= 128 \\ 2(43) + 42 &= 128 \\ 86 + 42 &= 128 \\ 128 &= 128 \quad \text{check} \end{aligned}$$

So the number is **43.**

example 90.2 Twenty-seven less than the product of 3 and a number is 144. What is the number?

solution If we use N for the number, then the product of 3 and the number is $3N$. We can translate the statement as $3N - 27 = 144$. Now we can solve for N.

$$\begin{array}{rcll} 3N - 27 &=& 144 & \text{equation} \\ +27 && +27 & \text{added 27 to both sides} \\ \hline 3N &=& 171 & \text{simplified} \\ \dfrac{3N}{3} &=& \dfrac{171}{3} & \text{divided both sides by 3} \\ N &=& 57 & \text{simplified} \end{array}$$

We will use 57 for N in the original equation to check.

$$\begin{aligned} 3N - 27 &= 144 \\ 3(57) - 27 &= 144 \\ 171 - 27 &= 144 \\ 144 &= 144 \quad \text{check} \end{aligned}$$

So the number is **57.**

1. The sum of four times a number and 16 is 16. Find the number.

2. Twenty-two less than the product of 2 and a number is –64. What is the number?

3. Seven more than the product of a number and 8 equals –97. What is the number?

   [A] –14 [B] –10
   [C] –13 [D] –15

4. The sum of three times a number and 17 is 14. Find the number.

5. Twenty-six less than the product of 3 and a number is 37. What is the number?

6. Ten more than the product of a number and 26 equals –276. What is the number?

   [A] –13 [B] –12
   [C] –8 [D] –11

7. The sum of twice a number and 14 is 30. Find the number.

8. Six less than the product of 4 and a number is –30. What is the number?

9. Eleven more than the product of a number and 15 equals 56. What is the number?

   [A] 5 [B] 2 [C] 3 [D] 0

10. The sum of three times a number and 22 is 55. Find the number.

11. Nineteen less than the product of 5 and a number is –64. What is the number?

12. Nine more than the product of a number and 12 equals –75. What is the number?

    [A] –8 [B] –5 [C] –7 [D] –10

13. The sum of four times a number and 12 is 128. Find the number.

14. Sixteen less than the product of 5 and a number is –71. What is the number?

15. Five more than the product of a number and 29 equals 701. What is the number?

    [A] 26 [B] 23 [C] 24 [D] 27

16. The sum of twice a number and 19 is –19. Find the number.

17. Seventeen less than the product of 2 and a number is –47. What is the number?

18. Twelve more than the product of a number and 6 equals –138. What is the number?

    [A] –27 [B] –25
    [C] –28 [D] –26

19. The sum of three times a number and 21 is –9. Find the number.

[1] 0

[2] –21

[3] C

[4] –1

[5] 21

[6] D

[7] 8

[8] –6

[9] C

[10] 11

[11] –9

[12] C

[13] 29

[14] –11

[15] C

[16] –19

[17] –15

[18] B

[19] –10

LESSON 91 Order of Operations with Signed Numbers and Symbols of Inclusion

If an expression involving signed numbers contains symbols of inclusion, we simplify within these symbols first, always beginning with the innermost symbols of inclusion. Within each pair of inclusion symbols, we always multiply before we add algebraically.

example 91.1 Simplify: $-2(-2 - 3 + 1) - [(-3 - 2) \cdot 4]$

solution We first simplify within the innermost parentheses.

$$-2(-4) - [(-5) \cdot 4]$$

Then we simplify within the brackets.

$$-2(-4) - [-20]$$

Now we multiply.

$$8 - [-20]$$

Then we add.

$$\mathbf{28}$$

example 91.2 Simplify: $-2(-2 - 3 \cdot 5) - 2[(3 - 5)2 + 2]$

solution First we simplify within the innermost parentheses.

$$-2(-17) - 2[(-2)2 + 2]$$

Now we simplify within the brackets.

$$-2(-17) - 2[-2]$$

Now we multiply.

$$34 - [-4]$$

Then we add.

Simplify:

1. $-5(-1-5+1)-[(-3-2)\cdot 4]$

2. $-5(3+2\cdot 3)-4[(3+2)5+3]$

   [A] –53 [B] –157
   [C] –72 [D] –77

3. $-4(-4-4+3)-[(-2-4)\cdot(-3)]$

4. $-4(-5-3\cdot 4)+3[(5+3)2-1]$

   [A] 25 [B] 113 [C] 83 [D] 74

5. $-3(-3-3+2)-[(1-5)\cdot 5]$

6. $-3(1+4\cdot 2)-5[(-1-1)4+5]$

   [A] –46 [B] –27
   [C] –12 [D] –2

7. $-2(-2-1+4)-[(-5-3)\cdot(-2)]$

8. $-5(2-4\cdot 2)+4[(-2+5)2+4]$

   [A] 70 [B] –42
   [C] –40 [D] –138

9. $-3(-4-5+3)-[(-3-3)\cdot(-2)]$

10. $-2(-3-3+5)-[(4-5)\cdot(-3)]$

11. $-4(-3+5\cdot 4)-5[(5+2)5-4]$

    [A] –83 [B] –163
    [C] –79 [D] –223

12. $-2(5-2\cdot 5)+2[(-1+4)4+5]$

    [A] 44 [B] –46
    [C] –80 [D] –40

13. $-5(-2-2+4)-[(-1-2)\cdot 4]$

14. $-3(4+3\cdot 3)-3[(3+3)3+1]$

    [A] –36 [B] –96
    [C] –39 [D] –33

15. $-4(-5-1+1)-[(-5-3)\cdot 2]$

16. $-3(-1-1+4)-[(2-4)\cdot 3]$

17. $-5(-1-5\cdot 3)+3[(4+1)5-2]$

    [A] 131 [B] 149
    [C] 54 [D] 107

18. $-4(2+3\cdot 2)-2[(-2+5)4-3]$

    [A] –18 [B] 34
    [C] –50 [D] 18

19. $-2(-4-3+3)-[(-4-5)\cdot(-5)]$

[1] 45

[2] B

[3] 2

[4] B

[5] 32

[6] C

[7] –18

[8] A

[9] 6

[10] –1

[11] D

[12] A

[13] 12

[14] B

[15] 36

[16] 0

[17] B

[18] C

[19] –37

LESSON 95 Variables on Both Sides

Some equations have a variable on both sides of the equation. To solve these equations, we first add as necessary to eliminate the variable on one side. Then we add as necessary to place all of the constants on the other side. It may seem strange to add variables to an equation, but this is allowed by the addition property of equality, as long as we add the same variable to both sides.

example 95.1 Solve: $3x + 3 = x - 5$

solution We can eliminate either the $3x$ or the x. We decide to eliminate the x, so we add $-x$ to both sides.

$$\begin{array}{rcrl} 3x + 3 &=& x - 5 & \text{equation} \\ \underline{-x } & & \underline{-x } & \text{added } -x \text{ to both sides} \\ 2x + 3 &=& -5 & \end{array}$$

Now we eliminate the 3 by adding -3 to both sides. Then we divide both sides by 2.

$$\begin{array}{rcrl} 2x + 3 &=& -5 & \\ -3 & & -3 & \text{added } -3 \text{ to both sides} \\ \hline 2x &=& -8 & \\ x &=& -4 & \text{divided both sides by 2} \end{array}$$

Check:

$$\begin{aligned} 3x + 3 &= x - 5 \\ 3(-4) + 3 &= (-4) - 5 \\ -12 + 3 &= -9 \\ -9 &= -9 \quad \text{check} \end{aligned}$$

Thus, $\mathbf{x = -4}$.

example 95.2 Solve: $4x - 2 = -x$

solution We will eliminate the $-x$ on the right side of the equation by adding $+x$ to both sides.

$$\begin{array}{rcrl} 4x - 2 &=& -x & \text{equation} \\ \underline{+x } & & \underline{+x} & \text{added } +x \text{ to both sides} \\ 5x - 2 &=& 0 & \end{array}$$

Now we add +2 to both sides and then divide by 5.

$$\begin{array}{rcrl} 5x - 2 &=& 0 & \\ +2 & & +2 & \text{added } +2 \text{ to both sides} \\ \hline 5x &=& 2 & \\ x &=& \dfrac{2}{5} & \text{divided both sides by 5} \end{array}$$

Check:

$$\begin{aligned} 4x - 2 &= -x \\ 4\left(\frac{2}{5}\right) - 2 &= -\left(\frac{2}{5}\right) \\ \frac{8}{5} - \frac{10}{5} &= -\frac{2}{5} \\ -\frac{2}{5} &= -\frac{2}{5} \quad \text{check} \end{aligned}$$

So, $\mathbf{x = \frac{2}{5}}$.

Lesson 95 - Variables on Both Sides

Solve:

1. $4x-2=5x+1$

2. $3x-4=-x-3$

   [A] $\frac{1}{4}$ [B] $\frac{1}{2}$ [C] 4 [D] $-\frac{7}{2}$

3. $2x+4=x+5$

4. $2x+3=4x-2$

   [A] 3 [B] $\frac{5}{2}$ [C] $\frac{1}{6}$ [D] $\frac{2}{5}$

5. $-6x-1=-4x+1$

6. $4x+5=-3x-1$

   [A] 4 [B] $-\frac{7}{6}$

   [C] $-\frac{6}{7}$ [D] -1

7. $6x+4=-2x-4$

8. $6x+2=2x-4$

   [A] $-\frac{2}{3}$ [B] $-\frac{3}{2}$

   [C] $-\frac{1}{4}$ [D] $-\frac{7}{4}$

9. $-5x-5=-x+3$

10. $x+3=-x-5$

    [A] $-\frac{1}{4}$ [B] -5

    [C] $-\frac{9}{2}$ [D] -4

11. $x-3=-4x+2$

12. $x-4=4x-3$

    [A] $-\frac{2}{3}$ [B] $-\frac{7}{5}$

    [C] $-\frac{1}{3}$ [D] -3

13. $-4x-2=-5x+5$

14. $-6x+2=x+4$

    [A] $-\frac{3}{7}$ [B] $-\frac{2}{7}$

    [C] $-\frac{7}{2}$ [D] $-\frac{6}{5}$

15. $x+4=5x-4$

16. $-4x+4=2x+2$

    [A] -3 [B] 3 [C] $\frac{1}{2}$ [D] $\frac{1}{3}$

17. $6x-5=5x-2$

[1] -3

[2] A

[3] 1

[4] B

[5] -1

[6] C

[7] -1

[8] B

[9] -2

[10] D

[11] 1

[12] C

[13] 7

[14] B

[15] 2

[16] D

[17] 3

LESSON 96 Multiple-Term Equations

When equations have like terms on either side of the equals sign, the first step in the solution is to combine the like terms.

example 96.1 Solve: $4x + 2 + 3x - 2x = 12$

solution The first step is to combine the like terms.

$$5x + 2 = 12 \qquad \text{combined like terms}$$

Now we finish by adding –2 to both sides and then dividing both sides by 5.

$$\begin{array}{rcll} 5x + 2 &=& 12 & \\ -2 && -2 & \text{added } -2 \text{ to both sides} \\ \hline 5x &=& 10 & \\ x &=& 2 & \text{divided both sides by 5} \end{array}$$

Check:

$$\begin{aligned} 4x + 2 + 3x - 2x &= 12 \\ 4(2) + 2 + 3(2) - 2(2) &= 12 \\ 8 + 2 + 6 - 4 &= 12 \\ 12 &= 12 \quad \text{check} \end{aligned}$$

So $x = 2$.

example 96.2 Solve: $3x - 3 - x - 4x = 5x + 12 - 2x$

solution First we simplify both the left and right sides of the equation by adding like terms.

$$-2x - 3 = 3x + 12 \qquad \text{added like terms}$$

Next we decide to eliminate the variable on the left side, so we add $+2x$ to both sides.

$$\begin{array}{rcll} -2x - 3 &=& 3x + 12 & \\ +2x && +2x & \text{added } +2x \text{ to both sides} \\ \hline -3 &=& 5x + 12 & \end{array}$$

Now we finish by adding –12 to both sides and then dividing both sides by 5.

$$\begin{array}{rcll} -3 &=& 5x + 12 & \\ -12 && -12 & \text{added } -12 \text{ to both sides} \\ \hline -15 &=& 5x & \\ -3 &=& x & \text{divided both sides by 5} \end{array}$$

Check:

$$\begin{aligned} 3x - 3 - x - 4x &= 5x + 12 - 2x \\ 3(-3) - 3 - (-3) - 4(-3) &= 5(-3) + 12 - 2(-3) \\ -9 - 3 + 3 + 12 &= -15 + 12 + 6 \\ 3 &= 3 \quad \text{check} \end{aligned}$$

So $x = -3$.

Lesson 96 - Multiple Term Equations

Solve:

1. $7x-2+6x+14=-12+x$

2. $6x+9+4x=-x-14+3x+12$

3. $-4x+50-3x-2x=-3x+5+3x$

   [A] 2 [B] 5 [C] 4 [D] 9

4. $6x-23+2x-2=17-6x$

5. $3x+12-14x=5x+7+11x+9$

6. $-5x+8+5x+4x=2x+4+4x$

   [A] –3 [B] 2 [C] 6 [D] 3

7. $-4x+11+x+5=40-7x$

8. $8x+8-10x=-4x+10-9x+2$

9. $-3x-16+x+3x=-4x-28+2x$

   [A] –7 [B] –6 [C] –4 [D] –3

10. $-2x-4+4x-2=-36-4x$

11. $2x-11-6x=14x+1+7x+3$

12. $-x-6+2x+x=-5x+6-5x$

    [A] 2 [B] 1 [C] –4 [D] –1

13. $-2x-5+3x+2=5+3x$

14. $-5x+2-6x-10=10+7x$

15. $-5x+1+8x=11x-15+15x+11$

16. $4x-42+5x-2x=4x-36+2x$

    [A] 1 [B] 6 [C] 5 [D] 2

17. $3x+2-2x+3=11+2x$

18. $4x+14+13x=12x-9-4x-7$

19. $x+4x-5x=5x+50+5x$

    [A] –3 [B] –6 [C] –5 [D] –9

20. $4x-19-7x+7=-40+4x$

21. $14x+5+12x=-13x-6+10x-1$

22. $3x-16-2x+x=x-3x$

    [A] 9 [B] 3 [C] 2 [D] 4

23. $10x-10-x=-8x+12+14x-4$

24. $-5x+25-4x+13=78+x$

25. $-6x+4+9x=-3x-4-9x-2$

26. $3x+12+5x-4x=-x+9+4x$

    [A] –3 [B] –8 [C] –1 [D] –2

27. $5x-2-3x-4x=-3x+1+4x$

    [A] 1 [B] 4 [C] 3 [D] –1

[1] -2

[2] $-\frac{11}{8}$

[3] B

[4] 3

[5] $-\frac{4}{27}$

[6] B

[7] 6

[8] $\frac{4}{11}$

[9] C

[10] -5

[11] $-\frac{3}{5}$

[12] B

[13] -4

[14] -1

[15] $\frac{5}{23}$

[16] B

[17] -6

[18] $-\frac{10}{3}$

[19] C

[20] 4

[21] $-\frac{12}{29}$

[22] D

[23] 6

[24] -4

[25] $-\frac{2}{3}$

[26] A

[27] D

LESSON 97 Two-Step Problems

Some of the problems that have appeared in the problem sets thus far have required two steps for their solutions. In this lesson we will look at two-step problems that require the solution of an equation as the first step.

example 97.1 If $2x + 4 = 6$, what is the value of $3x - 7$?

solution We begin by solving the equation to find the value of x that makes it true.

$$\begin{array}{rcll} 2x + 4 & = & 6 & \text{equation} \\ -4 & & -4 & \text{added } -4 \text{ to both sides} \\ \hline 2x & = & 2 & \\ x & = & 1 & \text{divided both sides by 2} \end{array}$$

Now we use 1 for x to find the value of $3x - 7$.

$$\begin{array}{rl} 3(1) - 7 & \text{substituted} \\ 3 - 7 & \text{multiplied} \\ \mathbf{-4} & \text{added} \end{array}$$

example 97.2 If $4x - 2 = 3$, what is the value of $\frac{2}{5}x - \frac{1}{4}$?

solution As the first step, we solve the equation to find the value of x that makes it true.

$$\begin{array}{rcll} 4x - 2 & = & 3 & \text{equation} \\ +2 & & +2 & \text{added } +2 \text{ to both sides} \\ \hline 4x & = & 5 & \\ x & = & \dfrac{5}{4} & \text{divided both sides by 4} \end{array}$$

Now we use $\frac{5}{4}$ for x to find the value of $\frac{2}{5}x - \frac{1}{4}$.

$$\begin{array}{rl} \dfrac{2}{5}\left(\dfrac{5}{4}\right) - \dfrac{1}{4} & \text{substituted} \\ \dfrac{1}{2} - \dfrac{1}{4} & \text{multiplied} \\ \mathbf{\dfrac{1}{4}} & \text{added} \end{array}$$

example 97.3 If $\frac{4}{3}x - 2 = 4$, what is the value of $6x - \frac{2}{5}$?

solution We will solve the equation for x.

$$\begin{array}{rcll} \dfrac{4}{3}x - 2 & = & 4 & \text{equation} \\ +2 & & +2 & \text{added } +2 \text{ to both sides} \\ \hline \dfrac{4}{3}x & = & 6 & \\ \dfrac{3}{4} \cdot \dfrac{4}{3}x & = & 6 \cdot \dfrac{3}{4} & \text{multiplied both sides by } \dfrac{3}{4} \\ x & = & \dfrac{9}{2} & \text{simplified} \end{array}$$

Now we use $\frac{9}{2}$ for x to find the value of $6x - \frac{2}{5}$.

$$\begin{array}{rl} 6\left(\dfrac{9}{2}\right) - \dfrac{2}{5} & \text{substituted} \\ 27 - \dfrac{2}{5} & \text{multiplied} \\ \mathbf{26\dfrac{3}{5}} & \text{added} \end{array}$$

1. If $x-2=-6$, what is the value of $7x-9$?

2. If $3x+4=3$, what is the value of $\frac{3}{2}x-\frac{9}{10}$?

3. If $\frac{8}{5}x-5=5$, what is the value of $4x-\frac{7}{8}$?

   [A] $\frac{505}{8}$ [B] $\frac{207}{8}$

   [C] $\frac{519}{8}$ [D] $\frac{193}{8}$

4. If $3x+7=7$, what is the value of $4x-5$?

5. If $2x+3=2$, what is the value of $\frac{2}{3}x-\frac{1}{6}$?

6. If $\frac{5}{4}x-4=-2$, what is the value of $5x-\frac{1}{2}$?

   [A] 12 [B] $\frac{17}{2}$

   [C] $\frac{15}{2}$ [D] 13

7. If $6x+1=-29$, what is the value of $5x-6$?

8. If $7x-2=-7$, what is the value of $\frac{3}{5}x-\frac{6}{7}$?

9. If $\frac{3}{2}x+3=-3$, what is the value of $2x-\frac{7}{10}$?

   [A] $-\frac{73}{10}$ [B] $-\frac{187}{10}$

   [C] $-\frac{173}{10}$ [D] $-\frac{87}{10}$

10. If $8x+3=43$, what is the value of $9x+4$?

11. If $4x-6=3$, what is the value of $\frac{2}{5}x-\frac{3}{10}$?

12. If $\frac{2}{3}x+2=4$, what is the value of $3x-\frac{5}{6}$?

    [A] $\frac{59}{6}$ [B] $\frac{19}{6}$

    [C] $\frac{49}{6}$ [D] $\frac{29}{6}$

13. If $2x+8=4$, what is the value of $x+8$?

14. If $4x+3=5$, what is the value of $\frac{1}{2}x-\frac{3}{8}$?

15. If $\frac{1}{5}x+1=3$, what is the value of $4x-\frac{1}{7}$?

[A] $\frac{61}{35}$ [B] $\frac{51}{35}$

[C] $\frac{279}{7}$ [D] $\frac{281}{7}$

16. If $3x+3=12$, what is the value of $2x-4$?

17. If $2x+5=2$, what is the value of $\frac{1}{2}x-\frac{1}{2}$?

18. If $\frac{5}{2}x-2=2$, what is the value of $5x-\frac{4}{5}$?

[A] $\frac{44}{5}$ [B] $\frac{36}{5}$

[C] $\frac{246}{5}$ [D] $\frac{254}{5}$

19. If $7x+1=29$, what is the value of $6x+2$?

20. If $5x+4=6$, what is the value of $\frac{5}{2}x-\frac{9}{10}$?

21. If $4x+6=14$, what is the value of $5x-9$?

22. If $3x-6=-7$, what is the value of $\frac{7}{3}x-\frac{8}{9}$?

23. If $\frac{1}{3}x+5=4$, what is the value of $2x-\frac{2}{3}$?

[A] $-\frac{20}{3}$ [B] $-\frac{16}{3}$

[C] $-\frac{4}{3}$ [D] 0

24. If $9x-5=4$, what is the value of $8x+7$?

25. If $5x-6=-2$, what is the value of $\frac{5}{3}x-\frac{2}{3}$?

26. If $\frac{4}{3}x+3=-3$, what is the value of $3x-\frac{3}{4}$?

[A] $-\frac{57}{4}$ [B] $-\frac{99}{4}$

[C] $-\frac{51}{4}$ [D] $-\frac{93}{4}$

[1] -37

[2] $-\frac{7}{5}$

[3] D

[4] -5

[5] $-\frac{1}{2}$

[6] C

[7] -31

[8] $-\frac{9}{7}$

[9] D

[10] 49

[11] $\frac{3}{5}$

[12] C

[13] 6

[14] $-\frac{1}{8}$

[15] C

[16] 2

[17] $-\frac{5}{4}$

[18] B

[19] 26

[20] $\frac{1}{10}$

[21] 1

[22] $-\frac{5}{3}$

[23] A

[24] 15

[25] $\frac{2}{3}$

[26] A

LESSON 100 Advanced Ratio Problems

In this lesson we will consider advanced ratio problems. We begin with an example.

example 100.1 The ratio of nuts to bolts was 11 to 2. If there were 260 nuts and bolts in the pile, how many were bolts?

solution We begin confidently by writing

$$\frac{N}{B} = \frac{11}{2}$$

Now where does the 260 go? What do we do now? The answer is that this problem is more difficult than the ratio problems we have been working, and we have begun the wrong way. This problem has nuts and bolts but also has the **total.** There is a trick to working these problems. **The trick is to begin by listing the given parts of the ratio, including the ratio total.**

$$\begin{aligned} \text{Nuts} &= 11 \\ \text{Bolts} &= 2 \\ \text{Ratio Total} &= 13 \end{aligned}$$

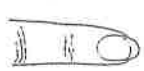
There are three ratios hidden here. We can see the ratios if we cover up the lines one at a time with a finger. First we cover the top line. We see that what remains concerns bolts and ratio total. We will use the subscript R to represent ratio values and the subscript A to represent actual values.

$$\text{Bolts} = 2 \longrightarrow \frac{B_R}{T_R} = \frac{2}{13} \qquad \text{(a)}$$

$$\text{Ratio Total} = 13$$

Now we cover the middle line and see that what remains concerns nuts and ratio total.

$$\text{Nuts} = 11$$



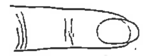
$$\longrightarrow \frac{N_R}{T_R} = \frac{11}{13} \qquad \text{(b)}$$

$$\text{Ratio Total} = 13$$

Now we cover the bottom line and see that what remains concerns nuts and bolts.

$$\text{Nuts} = 11$$

$$\text{Bolts} = 2 \longrightarrow \frac{N_R}{B_R} = \frac{11}{2} \qquad \text{(c)}$$



We see that the given information was enough to write three ratios. Ratio (a) is between bolts and ratio total. Ratio (b) is between nuts and ratio total. Ratio (c) is between nuts and bolts. We were told that we had a total of 260 and were asked for the number of bolts. So we want to use Ratio (a).

$$\frac{B_R}{T_R} = \frac{2}{13} \qquad \text{ratio (a)}$$

Ratio (a) is the reduced form of the actual number of bolts to the actual total. Thus we can write a proportion showing that ratio (a) is equal to the ratio of the actual number of bolts to the actual total.

$$\frac{B_A}{T_A} = \frac{B_R}{T_R}$$

Now we can substitute and solve.

$$\frac{B_A}{260} = \frac{2}{13} \qquad \text{replaced } T_A \text{ with 260}$$

$$B_A = \frac{2 \cdot 260}{13} \qquad \text{multiplied both sides by 260}$$

$$B_A = \textbf{40 bolts} \qquad \text{simplified}$$

In ratio problems that discuss the total, it is helpful to begin by writing all three ratios. Then reread the problem to see which ratio should be used.

example 100.2 Farmer Dunncowski wanted to plant his farm with wheat and corn in the ratio of 7 acres to 9 acres. If he had 640 acres, how many should be planted in wheat?

solution First we write down the figures given and the ratio total.

$$W_R = 7$$

$$C_R = 9$$

$$T_R = 16$$

With this information we can write three ratios.

$$\text{(a)} \ \frac{W_R}{C_R} = \frac{7}{9} \qquad \text{(b)} \ \frac{W_R}{T_R} = \frac{7}{16} \qquad \text{(c)} \ \frac{C_R}{T_R} = \frac{9}{16}$$

We were given the total and were asked for wheat, so we will use (b).

$$\frac{W_A}{T_A} = \frac{W_R}{T_R} \qquad \text{proportion}$$

$$\frac{W_A}{640} = \frac{7}{16} \qquad \text{replaced } T_A \text{ with 640}$$

$$16W_A = 7 \cdot 640 \qquad \text{cross multiplied}$$

$$\frac{\cancel{16}W_A}{\cancel{16}} = \frac{7 \cdot 640}{16} \qquad \text{divided both sides by 16}$$

$$W_A = \textbf{280 acres} \qquad \text{simplified}$$

If he plants 280 acres with wheat, then $640 - 280 = 360$ acres will be planted in corn.

1. If $x-5=-6$, what is the value of $6x+3$?

2. If $3x-6=-2$, what is the value of $\frac{9}{4}x-\frac{1}{2}$?

3. If $\frac{3}{5}x+2=-2$, what is the value of $4x-\frac{1}{4}$?

   [A] $-\frac{317}{12}$ [B] $-\frac{323}{12}$

   [C] $-\frac{197}{20}$ [D] $-\frac{187}{20}$

4. If $5x+6=6$, what is the value of $2x-1$?

5. If $6x+3=4$, what is the value of $\frac{7}{3}x-\frac{1}{6}$?

6. If $\frac{2}{3}x+4=-3$, what is the value of $5x-\frac{1}{3}$?

   [A] $-\frac{317}{6}$ [B] $-\frac{313}{6}$

   [C] $-\frac{71}{3}$ [D] -23

7. If $8x+9=41$, what is the value of $4x-2$?

8. If $3x+7=5$, what is the value of $\frac{3}{2}x-\frac{7}{9}$?

9. If $\frac{1}{2}x-1=-5$, what is the value of $3x-\frac{1}{2}$?

   [A] $-\frac{49}{2}$ [B] $-\frac{13}{2}$

   [C] $-\frac{47}{2}$ [D] $-\frac{11}{2}$

10. If $7x-4=-18$, what is the value of $9x+7$?

11. If $5x-5=-3$, what is the value of $\frac{5}{2}x-\frac{5}{8}$?

12. If $\frac{5}{2}x-5=4$, what is the value of $4x-\frac{1}{10}$?

    [A] $\frac{29}{2}$ [B] $\frac{143}{10}$

    [C] $\frac{899}{10}$ [D] $\frac{901}{10}$

13. If $3x-8=-20$, what is the value of $3x+9$?

14. If $5x+3=6$, what is the value of $\frac{5}{3}x-\frac{1}{2}$?

15. If $x-5=-8$, what is the value of $4x-4$?

16. If $3x-2=3$, what is the value of $\frac{3}{2}x-\frac{3}{8}$?

17. If $5x-2=13$, what is the value of $9x-3$?

18. If $6x+6=2$, what is the value of $\frac{3}{4}x-\frac{7}{10}$?

19. If $\frac{5}{3}x-1=-5$, what is the value of $5x-\frac{1}{5}$?

[A] $-\frac{497}{15}$ [B] $-\frac{503}{15}$

[C] $-\frac{61}{5}$ [D] $-\frac{59}{5}$

20. If $\frac{2}{5}x+4=-2$, what is the value of $3x-\frac{3}{7}$?

[A] $-\frac{312}{7}$ [B] $-\frac{237}{35}$

[C] $-\frac{318}{7}$ [D] $-\frac{267}{35}$

21. If $8x+7=23$, what is the value of $2x-1$?

22. If $3x-3=4$, what is the value of $\frac{5}{4}x-\frac{1}{4}$?

23. If $\frac{1}{4}x-5=3$, what is the value of $3x-\frac{5}{6}$?

[A] $\frac{571}{6}$ [B] $\frac{31}{6}$

[C] $\frac{581}{6}$ [D] $\frac{41}{6}$

24. If $\frac{3}{5}x-1=5$, what is the value of $3x-\frac{2}{3}$?

[A] $\frac{172}{15}$ [B] $\frac{152}{15}$

[C] $\frac{88}{3}$ [D] $\frac{92}{3}$

[1] -3

[2] $\frac{5}{2}$

[3] B

[4] -1

[5] $\frac{2}{9}$

[6] A

[7] 14

[8] $-\frac{16}{9}$

[9] A

[10] -11

[11] $\frac{3}{8}$

[12] B

[13] -3

[14] $\frac{1}{2}$

[15] -16

[16] $\frac{17}{8}$

[17] 24

[18] $-\frac{6}{5}$

[19] C

[20] C

[21] 3

[22] $\frac{8}{3}$

[23] A

[24] C

Lesson 100 - Advanced Ratio Problems

1. The ratio of frogs to toads was 2 to 15. If there were 2108 frogs and toads in all, how many were frogs?

2. Farmer Ivanov wanted to plant his orchards with plums and apricots in the ratio of 12 acres to 13 acres. If he had 1575 acres, how many should be planted with apricots?

3. The ratio of advanced skiers to novice skiers on the ski slope was 7 to 17. If there were 1416 skiers on the ski slope, how many were advanced skiers?

   [A] 413 skiers [B] 24 skiers

   [C] 59 skiers [D] 1003 skiers

4. The ratio of frogs to toads was 11 to 7. If there were 2106 frogs and toads in all, how many were toads?

5. Farmer Nguyen wanted to plant her orchards with almonds and walnuts in the ratio of 5 hectares to 7 hectares. If she had 732 hectares, how many should be planted with almonds?

6. The ratio of advanced skiers to novice skiers on the ski slope was 8 to 11. If there were 1406 skiers on the ski slope, how many were advanced skiers?

   [A] 592 skiers [B] 74 skiers

   [C] 19 skiers [D] 814 skiers

7. The ratio of frogs to toads was 6 to 5. If there were 1463 frogs and toads in all, how many were toads?

8. Farmer Sanders wanted to plant his orchards with peaches and plums in the ratio of 5 hectares to 3 hectares. If he had 536 hectares, how many should be planted with peaches?

9. The ratio of advanced skiers to novice skiers on the ski slope was 15 to 2. If there were 748 skiers on the ski slope, how many were novice skiers?

   [A] 17 skiers [B] 660 skiers

   [C] 88 skiers [D] 44 skiers

10. The ratio of frogs to toads was 3 to 8. If there were 1749 frogs and toads in all, how many were frogs?

11. Farmer Mendoza wanted to plant her orchards with walnuts and nectarines in the ratio of 2 acres to 3 acres. If she had 440 acres, how many should be planted with nectarines?

12. The ratio of advanced skiers to novice skiers on the ski slope was 5 to 13. If there were 846 skiers on the ski slope, how many were novice skiers?

    [A] 611 skiers [B] 235 skiers

    [C] 18 skiers [D] 47 skiers

13. The ratio of frogs to toads was 17 to 18. If there were 8785 frogs and toads in all, how many were frogs?

14. Farmer Mendoza wanted to plant his orchards with nectarines and peaches in the ratio of 11 hectares to 2 hectares. If he had 1235 hectares, how many should be planted with nectarines?

15. The ratio of advanced skiers to novice skiers on the ski slope was 7 to 8. If there were 1125 skiers on the ski slope, how many were advanced skiers?

    [A] 15 skiers   [B] 75 skiers

    [C] 525 skiers   [D] 600 skiers

16. The ratio of frogs to toads was 1 to 3. If there were 964 frogs and toads in all, how many were toads?

17. Farmer Sanders wanted to plant her orchards with apricots and almonds in the ratio of 3 acres to 2 acres. If she had 340 acres, how many should be planted with almonds?

18. The ratio of advanced skiers to novice skiers on the ski slope was 5 to 3. If there were 440 skiers on the ski slope, how many were advanced skiers?

    [A] 275 skiers   [B] 8 skiers

    [C] 165 skiers   [D] 55 skiers

19. The ratio of frogs to toads was 13 to 5. If there were 2736 frogs and toads in all, how many were toads?

20. Farmer Ivanov wanted to plant his orchards with plums and apricots in the ratio of 13 acres to 15 acres. If he had 2296 acres, how many should be planted with plums?

21. The ratio of advanced skiers to novice skiers on the ski slope was 9 to 4. If there were 598 skiers on the ski slope, how many were novice skiers?

    [A] 184 skiers   [B] 46 skiers

    [C] 13 skiers   [D] 414 skiers

22. The ratio of frogs to toads was 2 to 7. If there were 1800 frogs and toads in all, how many were frogs?

23. Farmer Nguyen wanted to plant his orchards with walnuts and nectarines in the ratio of 7 hectares to 12 hectares. If he had 1349 hectares, how many should be planted with nectarines?

24. The ratio of advanced skiers to novice skiers on the ski slope was 18 to 7. If there were 925 skiers on the ski slope, how many were novice skiers?

    [A] 37 skiers   [B] 666 skiers

    [C] 259 skiers   [D] 25 skiers

[1] 248 frogs

[2] 819 acres

[3] A

[4] 819 toads

[5] 305 hectares

[6] A

[7] 665 toads

[8] 335 hectares

[9] C

[10] 477 frogs

[11] 264 acres

[12] A

[13] 4267 frogs

[14] 1045 hectares

[15] C

[16] 723 toads

[17] 136 acres

[18] A

[19] 760 toads

[20] 1066 acres

[21] A

[22] 400 frogs

[23] 852 hectares

[24] C

LESSON 102 Adding Like Terms, Part 2

Terms can have more than one variable and the variables can be written as indicated multiplications or divisions. The expression

$$4xy + 6yx$$

has two terms. The coefficients are 4 and 6 respectively. To see if

$$4xy \qquad \text{and} \qquad 6yx$$

are like terms, we consider only the variables, so we discard the coefficients and get

$$xy \qquad \text{and} \qquad yx$$

If these both represent the same number regardless of the numbers that are used for x and y, we say that the expressions are like terms. To investigate, we let $x = -2$ and $y = 3$. Then

$$xy = (-2)(3) = -6 \qquad \text{and} \qquad yx = (3)(-2) = -6$$

Since the commutative property states that the order of factors does not affect a product, we see that this will hold true for any numbers that we use for x and y. So we say that $4xy$ and $6yx$ are like terms because xy and yx will have the same value no matter what numbers are used as replacements for the variables. If we have four (-6)'s plus six (-6)'s, we have ten (-6)'s.

$$4(-6) + 6(-6) = 10(-6)$$

This shows why we can add $4xy$ and $6yx$ and get $10xy$.

$$4xy + 6yx = 10xy$$

To add like terms, we add the coefficients of the terms.

example 102.1 Are $2xy^2$ and $3y^2x$ and $7yxy$ like terms?

solution We first discard the coefficients and get

$$xy^2 \qquad \text{and} \qquad y^2x \qquad \text{and} \qquad yxy$$

If we expand these expressions, we get

$$xyy \qquad \text{and} \qquad yyx \qquad \text{and} \qquad yxy$$

Each of these has as factors two y's and one x. Since the order of multiplication does not affect the product, these expressions will always represent the same number. To demonstrate we will use -2 for x and 3 for y.

xyy	yyx	yxy
$(-2)(3)(3)$	$(3)(3)(-2)$	$(3)(-2)(3)$
-18	-18	-18

Thus, the terms $2xy^2$ and $3y^2x$ and $7yxy$ are like terms. Since xy^2, y^2x, and yxy are like terms, we can add them by adding the coefficients.

$$2xy^2 + 3y^2x + 7yxy = 12y^2x \qquad \text{or} \qquad 12xy^2$$

example 102.2 Simplify by adding like terms: $4x + 3 + 6x + 4 + 2xy + 3yx + 2$

solution We add the constants for a sum of 9.

$$3 + 4 + 2 = 9$$

We add the x terms and get $10x$.

$$4x + 6x = 10x$$

And we add the xy terms and get $5xy$.

$$2xy + 3yx = 5xy$$

So the sum is

$$\mathbf{9 + 10x + 5xy}$$

We could write $5yx$ instead, but usually we write the variables in alphabetical order.

1. Are $5g^2f$, $3fg^2$, and $4gfg$ like terms?

Simplify by adding like terms:

2. $4b+bc+9+2bc+5b+2+5b$

3. $3+3q^2+8qq+2q+5+5q$

   [A] $3q^2+8qq+7q+8$

   [B] $18q^6+8$

   [C] $24q^2+10q+15$

   [D] $11q^2+7q+8$

4. Are dc^2, $4c^2d$, and $4cd$ like terms?

Simplify by adding like terms:

5. $6x+9xy+7+4xy+7x+3+2x$

6. $4+8a^2+4aa+3a+3+4a$

   [A] $32a^2+12a+12$

   [B] $8a^2+4aa+7a+7$

   [C] $12a^2+7a+7$ [D] $19a^6+7$

7. Are $2gh$, $5h^2g$, and $4hgh$ like terms?

Simplify by adding like terms:

8. $8a+8ab+4+6ab+3a+1+a$

9. $6+7d^2+2dd+5d+6+3d$

   [A] $7d^2+2dd+8d+12$

   [B] $17d^6+12$

   [C] $14d^2+15d+36$

   [D] $9d^2+8d+12$

10. Are u^2v, $2uvu$, and $3vu^2$ like terms?

Simplify by adding like terms:

11. $7f+7fg+8+8fg+6f+9+7f$

12. $2+5y^2+6yy+6y+4+2y$

   [A] $19y^6+6$

   [B] $5y^2+6yy+8y+6$

   [C] $11y^2+8y+6$

   [D] $30y^2+12y+8$

13. Are rqr, $5qr^2$, and $5q^2r$ like terms?

Simplify by adding like terms:

14. $9m+5mn+6+mn+2m+7+8m$

15. $1+2k^2+7kk+k+7+6k$

   [A] $16k^6+8$

   [B] $2k^2+7kk+7k+8$

   [C] $9k^2+7k+8$

   [D] $14k^2+6k+7$

16. Are $4yx^2$, $2xyx$, and $3x^2y^2$ like terms?

[1] yes

[2] $3bc+14b+11$

[3] D

[4] no

[5] $13xy+15x+10$

[6] C

[7] no

[8] $14ab+12a+5$

[9] D

[10] yes

[11] $15fg+20f+17$

[12] C

[13] no

[14] $6mn+19m+13$

[15] C

[16] no

LESSON 103 Distributive Property

The notation

$$4(2 + 3)$$

tells us to multiply 4 by what is inside the parentheses. There are two ways we can do this.

$4(2 + 3)$	$4(2 + 3)$
$4(5)$	$4 \cdot 2 + 4 \cdot 3$
20	$8 + 12$
	20

On the left we added 2 and 3 to get 5. Then we multiplied 4 by 5 to get 20. On the right we multiplied 4 by 2 to get 8 and multiplied 4 by 3 to get 12. Then we added 8 and 12 to get

20. We got an answer of 20 for both methods. We call this property of numbers the **distributive property.**

> DISTRIBUTIVE PROPERTY
>
> $a(b + c) = ab + ac$ or $(b + c)a = ba + ca$

This property is especially helpful when we simplify expressions that have variables. If we look at

$$4(x + y)$$

we cannot add x and y and then multiply, because x and y are not like terms. However, we can multiply x by 4 and then multiply y by 4 and then add the two products.

$$4(x + y) = 4x + 4y$$

example 103.1 Use the distributive property to multiply: $4(x + 2y - 8)$

solution The distributive property also applies to the algebraic sum of three or more terms. We multiply each of the terms in the parentheses by 4.

$$4(x + 2y - 8) = \mathbf{4x + 8y - 32}$$

example 103.2 Use the distributive property to multiply: $3x(y - 2 + 2x)$

solution We multiply each of the terms in the parentheses by $3x$.

$$3x(y - 2 + 2x) = \mathbf{3xy - 6x + 6x^2}$$

Use the distributive property to multiply:

1. $2(7c+6d-5)$

2. $2q(3q+3r+4)$

   [A] $5q^2+6qr+5q$

   [B] $6q^2+6qr+8q$

   [C] $6q^2+8qr+6q$

   [D] $6q^2+5qr+6q$

3. $3(8f-2g-4)$

4. $6d(-8d+4e-8)$

   [A] $-48d^2-48de+24d$

   [B] $-48d^2+24de-48d$

   [C] $10d^2-2de-2d$

   [D] $-48d^2+10de-2d$

5. $8(6j+9k+2)$

6. $8r(-7r+8s+6)$

   [A] $-56r^2+64rs+48r$

   [B] $-56r^2+48rs+64r$

   [C] $-56r^2+16rs+14r$

   [D] $16r^2+14rs+r$

7. $6(2g+4h+3)$

8. $7b(-4b+7c-3)$

   [A] $-28b^2+14bc+4b$

   [B] $14b^2+4bc+3b$

   [C] $-28b^2+49bc-21b$

   [D] $-28b^2-21bc+49b$

9. $7(3x+8y+4)$

10. $3q(-6q+9r-7)$

    [A] $-18q^2-21qr+27q$

    [B] $12q^2-4qr-3q$

    [C] $-18q^2+27qr-21q$

    [D] $-18q^2+12qr-4q$

11. $5(4r-5s-2)$

12. $2b(-9b+4c+5)$

    [A] $-18b^2+10bc+8b$

    [B] $6b^2+7bc-7b$

    [C] $-18b^2+8bc+10b$

    [D] $-18b^2+6bc+7b$

13. $9(5a+7b-5)$

14. $6c(-3c+8d-3)$

    [A] $-18c^2+14cd+3c$

    [B] $14c^2+3cd+3c$

    [C] $-18c^2+48cd-18c$

    [D] $-18c^2-18cd+48c$

[1] $14c+12d-10$

[2] B

[3] $24f-6g-12$

[4] B

[5] $48j+72k+16$

[6] A

[7] $12g+24h+18$

[8] C

[9] $21x+56y+28$

[10] C

[11] $20r-25s-10$

[12] C

[13] $45a+63b-45$

[14] C

LESSON 117 *Equations with x^2 • Pythagorean Theorem • Demonstration of the Pythagorean Theorem*

117.A equations with x^2

If we consider the equation

$$x^2 = 4$$

we see that there are two solutions that will satisfy the equation. These are +2 and −2.

$$(2)^2 = 4 \qquad \text{check}$$
$$(-2)^2 = 4 \qquad \text{check}$$

From this we conclude that we can solve an equation such as $x^2 = 4$ by taking the square root of both sides. We can use the symbol ± 2 to represent the two solutions to this equation, +2 and −2. It is important to note that the radical symbol, $\sqrt{}$, indicates only the positive or **principal square root.** So $\sqrt{4}$ is +2 only, and does not include −2.

example 117.1 Solve: $p^2 = 16$

solution We take the square root of p^2 and we get p. Then we take the square root of 16.

$$p^2 = 16 \qquad \text{equation}$$
$$p = \mathbf{\pm 4} \qquad \text{square root of both sides}$$

Since $(+4)^2 = 16$ and $(-4)^2 = 16$, the problem has two solutions.

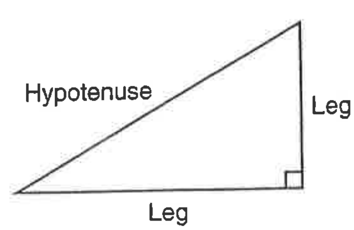
example 117.2 Solve: $p^2 = 42$

solution We take the square root of both sides.

$$p^2 = 42 \qquad \text{equation}$$
$$p = \mathbf{\pm\sqrt{42}} \qquad \text{square root of both sides}$$

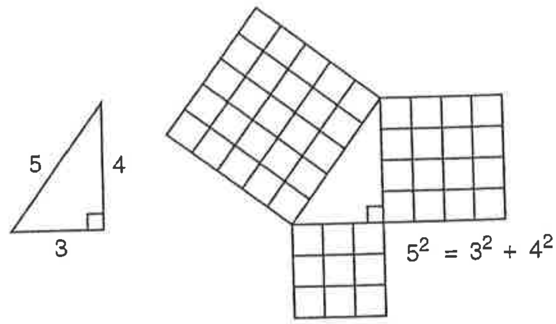
117.B Pythagorean theorem

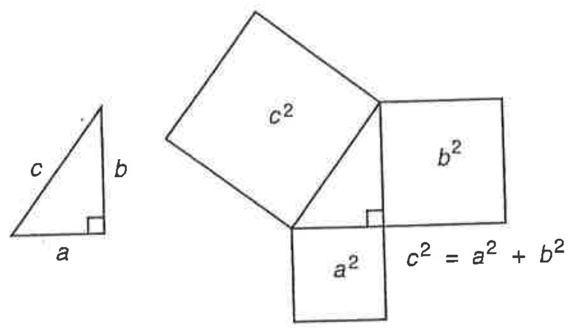
A right triangle is a triangle that has a right angle. The side opposite the right angle is called the **hypotenuse.** The other two sides are called **legs.**



A right triangle has a property that is especially useful. The property is named after a Greek mathematician who was born on the Greek island of Samos in the sixth century B.C. Later he lived in southern Italy in a town called Crotona. His name was Pythagoras. The theorem that bears his name is so simple that it is difficult to appreciate how important it is.

> **PYTHAGOREAN THEOREM**
>
> The area of a square drawn on the hypotenuse of a right triangle equals the sum of the areas of the squares drawn on the other two sides.



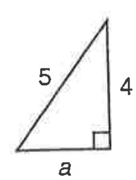


In the triangle on the upper left, the area of the square on the hypotenuse is 25. The areas of the squares on the legs are 9 and 16, which add to 25. The area of the square on the hypotenuse in the other triangle is c^2. The areas of the squares on the legs are a^2 and b^2. An algebraic statement of the theorem is

> **PYTHAGOREAN THEOREM**
>
> $$c^2 = a^2 + b^2$$

where c is the length of the hypotenuse, and a and b are the lengths of the legs.

example 117.3 Given the triangle with the lengths of the sides as shown, use the Pythagorean theorem to find a.



solution The square of the hypotenuse equals the sum of the squares of the other two sides. Thus,

$$
\begin{aligned}
5^2 &= 4^2 + a^2 && \text{Pythagorean theorem} \\
25 &= 16 + a^2 && \text{simplified} \\
9 &= a^2 && \text{added } -16 \text{ to both sides}
\end{aligned}
$$

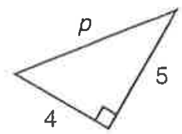
We take the square root of each side to finish the solution.

$$
\begin{aligned}
a^2 &= 9 && \text{equation} \\
a &= \pm 3 && \text{square root of both sides}
\end{aligned}
$$

While -3 is a solution to the equation $a^2 = 9$, it is not a solution to the problem at hand, because physical lengths are designated by positive numbers. Thus, we reject the negative solution and say that

$$a = 3$$

example 117.4 Find p in the triangle shown.



solution The square of the hypotenuse equals the sum of the squares of the other two sides.

$$\begin{aligned} p^2 &= 5^2 + 4^2 && \text{Pythagorean theorem} \\ p^2 &= 25 + 16 && \text{simplified} \\ p^2 &= 41 && \text{added} \\ p &= \pm\sqrt{41} && \text{square root of both sides} \end{aligned}$$

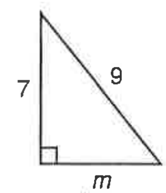
This equation has two answers. They are $+\sqrt{41}$ and $-\sqrt{41}$. But only one of these answers is the length of the hypotenuse because every length is positive.

$$p = \sqrt{\mathbf{41}}$$

Since $6 = \sqrt{36}$ and $7 = \sqrt{49}$, we can conclude $6 < \sqrt{41} < 7$. We can get a numerical approximation with a calculator.

$$p = \sqrt{41} \approx 6.40$$

example 117.5 Find m.

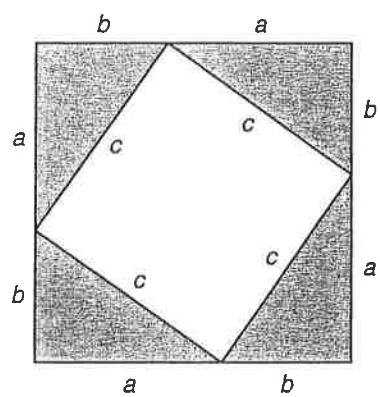


solution The hypotenuse squared equals the sum of the squares of the other two sides.

$$\begin{aligned} 9^2 &= 7^2 + m^2 && \text{Pythagorean theorem} \\ 81 &= 49 + m^2 && \text{simplified} \\ 32 &= m^2 && \text{added } -49 \text{ to both sides} \\ \sqrt{\mathbf{32}} &= m && \text{square root of both sides} \end{aligned}$$

117.C demonstration of the Pythagorean theorem

One way to see why the Pythagorean theorem is true is to consider a large square made up of four shaded right triangles and an inner square.



Area of the large square = area of the inner square + area of the 4 right triangles

$$\begin{aligned} &= c^2 + 4\left(\frac{1}{2}ab\right) \\ &= c^2 + 2ab \end{aligned}$$

Solve:

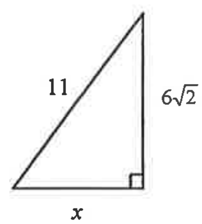
1. $g^2 = 25$

2. $p^2 = 109$

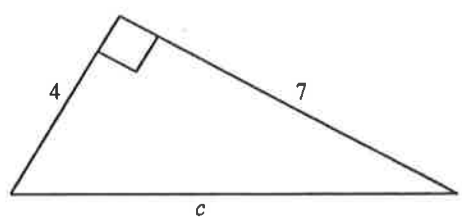
[A] $\dfrac{109}{2}$ [B] $\pm\dfrac{109}{2}$

[C] $\pm\sqrt{109}$ [D] $\sqrt{109}$

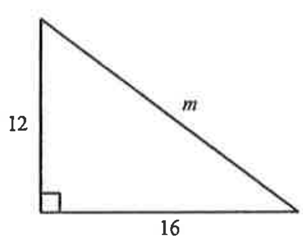
3. Use the Pythagorean theorem to find x.



4. Find c.



5. Find m.



[A] 20 [B] 22 [C] 19 [D] 21

Solve:

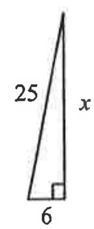
6. $j^2 = 16$

7. $p^2 = 45$

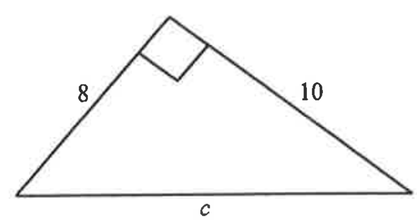
[A] $\dfrac{45}{2}$ [B] $\pm\sqrt{45}$

[C] $\pm\dfrac{45}{2}$ [D] $\sqrt{45}$

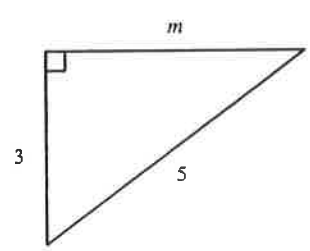
8. Use the Pythagorean theorem to find x.



9. Find c.



10. Find m.



[A] 5 [B] 6 [C] 2 [D] 4

Solve:

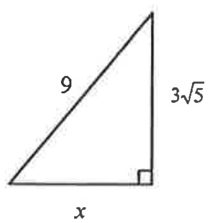
11. $r^2 = 4$

12. $p^2 = 94$

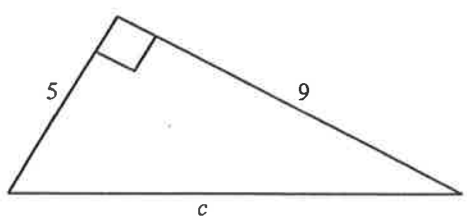
[A] $\pm\sqrt{94}$ [B] ± 47

[C] 47 [D] $\sqrt{94}$

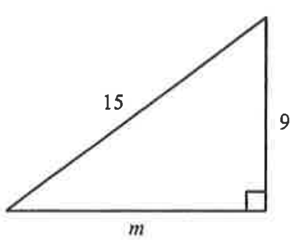
13. Use the Pythagorean theorem to find x.



14. Find c.



15. Find m.



[A] 13 [B] 12 [C] 15 [D] 10

Solve:

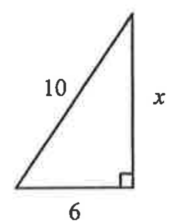
16. $p^2 = 100$

17. $p^2 = 141$

[A] $\sqrt{141}$ [B] $\pm\dfrac{141}{2}$

[C] $\pm\sqrt{141}$ [D] $\dfrac{141}{2}$

18. Use the Pythagorean theorem to find x.



[1] ± 5

[2] C

[3] 7

[4] $\sqrt{65}$

[5] A

[6] ± 4

[7] B

[8] $\sqrt{589}$

[9] $2\sqrt{41}$

[10] D

[11] ± 2

[12] A

[13] 6

[14] $\sqrt{106}$

[15] B

[16] ± 10

[17] C

[18] 8