College Prep Algebra 2

Unit 4: Radical Expressions and Rational Exponents (Chapter 7)

Name: ______________________________

Teacher: ________________ Period: __________

Unit 4 Radical Expressions and Rational Exponents (chapter 7)

Learning Targets:

Properties of Exponents	1. I can use properties of exponents to simplify expressions.
Simplifying Radical Expressions	2. I can simplify radical algebraic expressions.
Multiplying and Dividing	3. I can multiply radical expressions. 4. I can divide radical expressions (and rationalize a denominator).
Major Operations	5. I can add and subtract radical expressions. 6. I can multiply and rationalize binomial radical expressions.
Rational Exponents	7. I can convert from rational exponents to radical expressions (and vice versa). 8. I can simplify numbers with rational exponents.
Solving Radical Equations	9. I can solve equations with roots. 10. I can solve equations with rational exponents.
Graphing Radicals	11. I can graph radical expressions & identify domain and range of radical expressions.

Corresponding Book Sections

LTs	Book Section
1	7-0
2	7-1
3,4	7-2
5,6	7-3
7,8	7-4
9,10	7-5
11	7-8

Properties of Exponents

Date: ____________

Quiz On: ____________

After this lesson and practice, I will be able to …

☐ use properties of exponents to simplify expressions.. (LT 1)

Before we learn about a new family of functions, it is important that we first pause to review the _____________________ properties you learned in Algebra 1. First, a review of the vocabulary:

A useful method of remembering the exponent properties is through ________________ ________. If the exponent of a power is a positive integer, you can write it in expanded form. For example:

Summary of exponent rules:

PROPERTIES OF EXPONENTS

Let a and b be real numbers and let m and n be integers.

Product of Powers Property	$a^m \cdot a^n = a$_______
Power of a Power Property	$(a^m)^n = a$_____
Power of a Product Property	$(ab)^m = a$____b____
Negative Exponent Property	$a^{-m} = \frac{\quad}{\_\_\_\_\_}$, $a \neq 0$
Zero Exponent Property	$a^0 =$ ___, $a \neq 0$
Quotient of Powers Property	$\frac{a^m}{a^n} = a$_______, $a \neq 0$
Power of a Quotient Property	$\left(\frac{a}{b}\right)^m = \frac{\quad}{\_\_\_\_\_}$, $b \neq 0$

Power Property of Equality:

Common Base Property of Equality:

Example 1: Simplify each expression completely.

A) $\left((-3)^2 \cdot 5\right)^3$

B) $\left(\frac{3^5}{3^3}\right)^2$

C) $\left(\frac{5^6}{5^4}\right)^{-3}$

Example 2: Simplify $\left(\frac{1^4}{3^2}\right)^{-2}$

You can use properties of exponents to simplify _______________ expressions. A simplified expression contains only __________________ exponents.

Example 3: Simplify each expression. Use only positive exponents in your solution.

A) $w^5w^{-8}w^6$ B) $\left(\dfrac{r^2}{c^{-4}}\right)^{-3}$ C) $\dfrac{16m^5n^{-4}}{2n^{-6}}$ D) $\dfrac{\left(x^4y^{-3}\right)^3}{x^3y^4}$

Example 4: Simplify each expression. Use only positive exponents in your solution.

A) $(-4x^2)(-2x^{-2})$ B) $\left(\dfrac{2w^{-3}}{m^4}\right)^{-5}$ C) $\dfrac{\left(12m^2n^6\right)^2}{8m^4n^7}$ D) $\dfrac{\left(3x^5\right)^2y^3}{2x^4y^{-7}}$

FINAL CHECK: (calc allowed)

LT 1. I can use properties of exponents to simplify expressions.

1. Simplify each expression. *Use only positive exponents in your solution.*

a. $\left(-5x^6\right)\left(2x^{-4}y\right)\left(-6x^{-3}y^4\right)$ b. $(-3x^5y)^{-2}$ c. $\dfrac{\left(x^4y^{-3}\right)^4}{x^0y^2}$

Practice Assignment (LT1)

☐ LT1. I can use properties of exponents to simplify expressions.

- BOOK 7. 0 page 368

Simplify each expression. Use only positive exponents.

1. $(3a^2)(4a^6)$

2. $(-4x^2)(-2x^{-2})$

3. $(4x^3y^5)^2$

4. $(2x^{-5}y^4)^3$

5. $\frac{8a^5}{2a^2}$

6. $\frac{6x^7y^5}{3x^{-1}}$

7. $\frac{(4x^2)^0}{2xy^5}$

8. $\left(\frac{3x^2}{2}\right)^2$

9. $(-6m^2n^2)(3mn)$

10. $(3x^4y^5)^{-3}$

11. $\frac{(2r^{-1}s^2t^0)^{-2}}{2rs}$

12. $x^5(2x)^3$

13. $\frac{x^4x^{-2}}{x^{-5}}$

14. $\frac{(12x^2y^6)^2}{8x^4y^7}$

15. $(4p^2q)(p^2q^3)$

16. $\frac{4x^3}{2x}$

17. $(p^2)^{-2}$

18. $\frac{-15x^4}{3x}$

19. $\frac{r^2s^3t^4}{r^2s^4t^{-4}}$

20. $\frac{xy^2}{2} \cdot \frac{6x}{y^2}$

21. $(s^2t)^3(st)$

22. $(3x^{-3}y^{-2})^{-2}$

23. $(h^4k^5)^0$

24. $\frac{s^2t^3}{r} \cdot \frac{sr^3}{t}$

LT 1 MORE PRACTICE #1 (Yay!!!!)

1) 9^{-2} =

2) $4x^{-3}$ =

3) $(-4x)^{-3}$ =

4) -7^{-2} =

5) 10^{-2} =

6) xy^{-5} =

7) $(xy)^{-5}$

8) $2x^{-4}$

9) $(2x)^{-4}$

10) $2^{-2} + 3^{-2}$

Answers

1) $\frac{1}{81}$ 4) $-\frac{1}{49}$ 7) $\frac{1}{x^5y^5}$ 9) $\frac{1}{16x^4}$ 2) $\frac{4}{x^3}$ 5) $\frac{1}{100}$

3) $\frac{1}{64x^3}$ 6) $\frac{x}{y^5}$ 8) $\frac{2}{x^4}$ 10) $\frac{13}{36}$

LT 1 More Practice #2

Simplify. Notice in these examples, some have negative exponents & some don't!

a) $5x^{-1}$ 1) $2x^{-5}$ b) 2^{-5} 2) 2^{-3} c) $(-2)^3$ 3) $-2x^4$

d) $-2x^3$ 4) $(-2)^4 =$ e) $(-2)^{-3}$ 5) $(-2)^{-4} =$ f) $5x^2y^{-3} =$ 6) $7a^5b^{-10} =$

g) $5y^3x^{-2}$ 7) $-10a^2b^{-3}c^{-4}$ h) 5^2ab^{-3} 8) $(a^5 b^2)^{-3}$

i) $(5b^3)^{-2}$ 9) $(4x^5)^{-2}$ j) $(\mathbf{-2a^2b^4})^3$ 10) $(-5x^4)^{-3}$

k) $(5x)^{-2} y^3$ l) $(7a)^2 b^{-3}$ m) $4x^{-2}yz^{-1}$ n) $\dfrac{8x^6y^{-10}z^{35}}{20x^{-2}y^4z^5} =$

LT 1 More Practice #3

11) $(-2)^2(-2)^3$ 12) $[\mathbf{-3}^2]^3$ 13) $(3^2x^2y)^2$

14) $5^{-5} \bullet 5^3$ 15) $(2^{-3})^2$ 16) $m^7 \bullet \dfrac{1}{m^4}$

17) $\dfrac{8^3 \bullet 8^5}{8^9}$ 18) $\left(\dfrac{5}{4}\right)^3$ 19) $-3x^2y^0z^{-4}$

20) $\dfrac{5x^4y^3}{8x^5} \bullet \dfrac{3x^3y^5}{6y^4}$

21) $\dfrac{2x^6y^4}{6x^3} \bullet \dfrac{4x^2y^3}{12y^5}$

22) $\dfrac{x^{-7}y^3z^{-2}}{xy^{-1}z^{-4}}$

23) $\dfrac{-5x^{-3}yz}{15y^{-10}z}$

24) $\left(\dfrac{2x^{-9}y^{-2}z^4a^{-7}b^3c}{5a^0bc^{-10}x^2y^{-4}z^3}\right)^0$

Simplifying Radical Expressions

Date: ____________
Quiz On: ____________

After this lesson and practice, I will be able to …

☐ simplify radical algebraic expressions. (LT 2)

Warm Up: Write each number or expression as the square of a number or expression (i.e. 16 = 4^2).

a. $\frac{4}{49}$ b. x^{10} c. $144x^6y^8$

Since 5^2 = 25, 5 is a square root of 25.   Since 5^4 = ______, 5 is a _______ root of ______.

Since 5^3 = ____, 5 is a _______ root of _____.   Since 5^5 = ______, 5 is a _______ root of ______.

Definition: **nth Root –** For real numbers *a* and *b* and any positive integer *n*, if ___________ then _____ is an nth root of ______.

Notation:

What are the real fourth root(s) of 16? _______________________________________.

What are the real fourth root(s) of -16? _______________________________________.

What are the real cube root(s) of -8? _______________________________________.

Type of Number	*Number of nth Roots when n is even*	*Example*	*Number of nth Roots when n is odd*	*Example*
Positive				
0				
Negative				

Example 1: Find all real roots of each number.

A) The cube roots of -1000 and $\frac{1}{27}$ B) The fourth roots of 1, -0.0001, and $\frac{16}{625}$

When a number has two real roots, the positive root is called the _______________ root. The ___________ sign indicates that you are to calculate the ______________ root of the radicand.

Example 2: Find each real-number root.

A) $\sqrt[3]{-8}$ B) $\sqrt{-100}$ C) $\sqrt[4]{81}$

When finding principal roots (especially when ________________ are involved), one observation is required...

Property 1: For any negative number *a*, $\sqrt[n]{a^n} =$ _____ when *n* is ______. Why?!

Example 3: Simplify each radical expression.

A) $\sqrt{4x^6}$ B) $\sqrt[3]{a^3b^6}$ C) $\sqrt[4]{x^4y^8}$

D) $\sqrt{4x^2y^4}$ E) $\sqrt[3]{-27c^6}$ F) $\sqrt[4]{x^8y^{12}}$

G) $\sqrt[3]{40x^8y^{12}}$ H) $\sqrt[4]{112ac^6}$ I) $\sqrt[4]{20{,}000a^{10}b^{15}}$

Example 5: A citrus grower wants to ship oranges that weigh between 8 and 9 ounces in gift cartons. Each carton will hold three-dozen oranges, in 3 layers of 3 oranges by 4 oranges. The weight of each orange is related to its diameter by the formula $w = \frac{d^3}{4}$, where *d* is the diameter in inches and *w* is the weight in ounces. Cartons can only be ordered in whole-number dimensions. What are the dimensions of the container the grower should order?

FINAL CHECK:

I can simplify radical algebraic expressions. (LT2).

1. Simplify each expression. Use absolute values when necessary.

a. $\sqrt{144a^6b^{20}}$ b. $\sqrt[3]{-125x^{12}y^6}$ c. $\sqrt[4]{64x^{18}y^{12}}$

LT 2 **Practice Assignment**

- ☐ I can simplify radical algebraic expressions. (LT2).
  - Worksheet (Practice 7.1) (below)

Practice 7-1 Roots and Radical Expressions

Find each real-number root.

1. $\sqrt{144}$ **2.** $-\sqrt{25}$ **3.** $\sqrt{-0.01}$ **4.** $\sqrt[3]{0.001}$

5. $\sqrt[4]{0.0081}$ **6.** $\sqrt[3]{27}$ **7.** $\sqrt[3]{-27}$ **8.** $\sqrt{0.09}$

Find all the real cube roots of each number.

9. 216 **10.** –343 **11.** –0.064 **12.** $\frac{1000}{27}$

Find all the real square roots of each number.

13. 400 **14.** –196 **15.** 10,000 **16.** 0.0625

Find all the real fourth roots of each number.

17. –81 **18.** 256 **19.** 0.0001 **20.** 625

Simplify each radical expression. Use absolute value symbols when needed.

21. $\sqrt{81x^4}$ **22.** $\sqrt{121y^{10}}$ **23.** $\sqrt[3]{8g^6}$ **24.** $\sqrt[3]{125x^9}$

25. $\sqrt[5]{243x^5y^{15}}$ **26.** $\sqrt[3]{(x-9)^3}$ **27.** $\sqrt{25(x+2)^4}$ **28.** $\sqrt[3]{\frac{64x^9}{343}}$

Find the two real-number solutions of each equation.

29. $x^2 = 4$ **30.** $x^4 = 81$ **31.** $x^2 = 0.16$ **32.** $x^2 = \frac{16}{49}$

33. A cube has volume $V = s^3$, where s is the length of a side. Find the side length for a cube with volume 8000 cm^3.

34. The velocity of a falling object can be found using the formula $v^2 = 64h$, where v is the velocity (in feet per second) and h is the distance the object has already fallen.

a. What is the velocity of the object after a 10-foot fall?

b. How much does the velocity increase if the object falls 20 feet rather than 10 feet?

Practice 7-1

1. 12 **2.** −5 **3.** not a real number **4.** 0.1 **5.** 0.3 **6.** 3 **7.** −3 **8.** 0.3 **9.** 6 **10.** −7 **11.** −0.4 **12.** $\frac{10}{3}$ **13.** −20, 20 **14.** no real square roots **15.** −100, 100 **16.** −0.25, 0.25 **17.** no real fourth roots **18.** −4, 4 **19.** −0.1, 0.1 **20.** −5, 5 **21.** $9x^2$ **22.** $11\,|y^5|$ **23.** $2g^2$ **24.** $5x^3$ **25.** $3xy^3$ **26.** $x - 9$ **27.** $5(x+2)^2$ **28.** $\frac{4x^3}{7}$ **29.** −2, 2 **30.** −3, 3 **31.** −0.4, 0.4 **32.** $-\frac{4}{7}, \frac{4}{7}$ **33.** 20 cm **34a.** about 25.30 ft/sec **34b.** about 10.48 ft/sec

LT 2 More Practice #1 : 7.1 Book Page 372

Find all the real square roots of each number.

1. 225 **2.** 0.0049 **3.** $-\frac{1}{121}$ **4.** $\frac{64}{169}$

Find all the real cube roots of each number.

5. -64 **6.** 0.125 **7.** $-\frac{27}{216}$ **8.** 0.000343

Find all the real fourth roots of each number.

9. 16 **10.** -16 **11.** 0.0081 **12.** $\frac{10{,}000}{81}$

Find each real-number root.

13. $\sqrt{36}$ **14.** $-\sqrt{36}$ **15.** $\sqrt{-36}$ **16.** $\sqrt{0.36}$

17. $-\sqrt[3]{64}$ **18.** $\sqrt[3]{-64}$ **19.** $-\sqrt[4]{81}$ **20.** $\sqrt[4]{-81}$

Simplify each radical expression. Use absolute value symbols when needed.

21. $\sqrt{16x^2}$ **22.** $\sqrt{0.25x^6}$ **23.** $\sqrt{x^8y^{18}}$ **24.** $\sqrt{64b^{48}}$

25. $\sqrt[3]{-64a^3}$ **26.** $\sqrt[3]{27y^6}$ **27.** $\sqrt[4]{x^8y^{12}}$ **28.** $\sqrt[5]{32y^{10}}$

Geometry **The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$. Find the radius to the nearest hundredth of a sphere with each volume.**

29. 10 in.^{3} **30.** 20 ft^3 **31.** 0.45 cm^3 **32.** 0.002 mm^3

Simplify each radical expression. Use absolute value symbols when needed.

39. $\sqrt[3]{0.125}$ **40.** $\sqrt[3]{\frac{8}{216}}$ **41.** $\sqrt[4]{0.0016}$ **42.** $\sqrt[4]{\frac{1}{256}}$

43. $\sqrt[4]{16c^4}$ **44.** $\sqrt[3]{81x^3y^6}$ **45.** $\sqrt{144x^3y^4z^5}$ **46.** $\sqrt[5]{y^{20}}$

47. $\sqrt[5]{-y^{20}}$ **48.** $\sqrt[5]{k^{15}}$ **49.** $\sqrt[5]{-k^{15}}$ **50.** $\sqrt{(x+3)^2}$

51. $\sqrt{(x+1)^4}$ **52.** $\sqrt[2n]{x^{2n}}$ **53.** $\sqrt[2n]{x^{4n}}$ **54.** $\sqrt[2n]{x^{6n}}$

Lesson 7-1 pp. 372–373

EXERCISES **1.** 15, -15 **3.** none **5.** -4 **7.** $-\frac{1}{2}$ **9.** 2, -2 **11.** 0.3, -0.3 **13.** 6 **15.** no real-number root **21.** $4|x|$ **23.** $x^4|y^9|$ **29.** 1.34 in. **31.** 0.48 cm **33.** 10, -10 **35.** 0.5, -0.5 **37.** $\sqrt[3]{-64}$, $\sqrt[6]{64}$, $-\sqrt[3]{-64}$, $\sqrt{64}$ **39.** 0.5 **41.** 0.2 **59.** Some; they are equal for $x \geq 0$. **61.** Some; they are equal for $x \geq 0$.

LT 2 More Practice#2

Simplify completely .Use absolute value bars when needed.

1) $\sqrt{45} = \sqrt{\quad} \bullet \sqrt{\quad}$

2) $-\sqrt{125} = -\sqrt{\quad} \bullet \sqrt{\quad}$

3) $\sqrt{20} =$

4) $\sqrt[5]{64} = \sqrt[5]{\quad} \bullet \sqrt[5]{\quad}$

5) $\sqrt[3]{-27} =$

6) $\sqrt[3]{-81} =$

7) $\pm\sqrt[4]{16} =$

8) $\sqrt[4]{-32} =$

9) $\sqrt{16x^{16}} =$

10) $\sqrt[3]{8x^{9}} =$

Note: With letters we simply ____.

If it doesn't divide evenly, we must "____________________".

11) $\sqrt{x^{5}} = \sqrt{\quad} \bullet \sqrt{\quad}$

$=$

12) $\sqrt[3]{x^{8}} = \sqrt[3]{\quad} \bullet \sqrt[3]{\quad}$

$=$

13) $\sqrt{9x^{20}} =$

14) $\sqrt[3]{-24x^{13}y^{6}} =$

15) $\sqrt[4]{a^{11}b^{3}c^{8}} =$

16) $2xy\sqrt{32x^{6}y^{2}z^{9}} =$

Multiplying and Dividing Radical Expressions

Date: ____________
Quiz On: ____________

After this lesson and practice, I will be able to …

- ☐ multiply radical expressions.(LT 3)
- ☐ divide radical expressions (and rationalize a denominator). (LT 4)

Warm Up: Simplify each expression. Assume all variables are positive (this means that you do not need to use ________________ _________ in the simplified expression).

A) $\sqrt{50x^5}$ B) $\sqrt[3]{54n^8}$

Multiplying Radical Expressions (LT 3)

In your studies this year, you have already learned several properties of multiplying and dividing ____________ roots. These same properties can be used to multiply and divide any radical expression.

Property 1: **Multiplying Radical Expressions –** If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then $\sqrt[n]{a}\cdot\sqrt[n]{b} =$ _____

Example 2: Multiply. Simplify if possible. Assume all variables are positive.

A) $\sqrt{2}\cdot\sqrt{8}$ B) $\sqrt[3]{-5}\cdot\sqrt[3]{25}$ C) $\sqrt[3]{25xy^8}\cdot\sqrt[3]{5x^4y^3}$

Example 3: Simplify each radical expression. Assume all variables are positive.

A) $\sqrt{3}\cdot\sqrt{27}$ B) $\sqrt[3]{3}\cdot\sqrt[3]{-9}$ C) $\sqrt[3]{54x^2y^3}\cdot\sqrt[3]{5x^3y^4}$

Dividing Radical Expressions (LT 4)

Property 2: **Dividing Radical Expressions –** If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers and $b \neq 0$, then

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} =$$

Example 4: Divide. Simplify if possible. Assume all variables are positive.

A) $\dfrac{\sqrt[3]{-81}}{\sqrt[3]{3}}$

B) $\dfrac{\sqrt[3]{192x^8}}{\sqrt[3]{3x}}$

C) $\dfrac{\sqrt[3]{162x^5}}{\sqrt[3]{3x^2}}$

Example 5: Divide. Simplify if possible. Assume all variables are positive.

A) $\dfrac{\sqrt{243}}{\sqrt{27}}$

B) $\dfrac{\sqrt[4]{1024x^{15}}}{\sqrt[4]{4x}}$

Rationalizing Denominators

In Unit 2, you learned about a convention used in mathematics concerning radicals in the ___________________ of fractions. For example, after completing the square, you might obtain a solution in the form $x = -3 \pm \sqrt{\frac{2}{7}}$. Let's review how to *rationalize the denominator.*

Example 6: Rationalize the denominator of each expression. Assume all variables are positive.

A) $\dfrac{\sqrt{3}}{\sqrt{5}}$

B) $\dfrac{\sqrt{x^3}}{\sqrt{5xy}}$

C) $\sqrt[3]{\dfrac{2}{3x}}$

Example 7: Rationalize the denominator of each expression. Assume all variables are positive.

A) $\dfrac{\sqrt{2x^3}}{\sqrt{10xy}}$ B) $\dfrac{\sqrt[3]{4}}{\sqrt[3]{6x}}$ C) $\dfrac{\sqrt[3]{45}}{\sqrt[3]{25x^2}}$

FINAL CHECK: LT 3-4

I can multiply radical expressions. (LT3)

I can divide radical expressions (and rationalize a denominator) (LT4)

1. Simplify each radical expression. Assume all variables are positive. Express your answer in simplified radical form. Rationalize denominators if necessary. *Show your work.*

a. $\sqrt{490a^5b}$

b. $3\sqrt{35a^5}\cdot\sqrt{14a^2}$

c. $\dfrac{\sqrt{6}}{\sqrt{9a}}$

d. $\dfrac{5}{\sqrt[3]{9a^2}}$

Practice Assignment LT 3,4

- ☐ I can multiply radical expressions. (LT3)
  - BOOK 7.2 page 377-378 #2-8 even, 18-22 even
- ☐ I can divide radical expressions (and rationalize a denominator) (LT4)
  - BOOK 7.2 page 377-378 #24-34 even

Practice both: BOOK 7.2page 377 #38-54 even

LT 3,4 Book Page 377

Multiply, if possible. Then simplify.

1. $\sqrt{8} \cdot \sqrt{32}$ **2.** $\sqrt[3]{4} \cdot \sqrt[3]{16}$ **3.** $\sqrt[3]{9} \cdot \sqrt[3]{-81}$ **4.** $\sqrt[4]{8} \cdot \sqrt[4]{32}$

5. $\sqrt{-5} \cdot \sqrt{5}$ **6.** $\sqrt[3]{-5} \cdot \sqrt[3]{-25}$ **7.** $\sqrt[3]{9} \cdot \sqrt[3]{-24}$ **8.** $\sqrt[3]{-12} \cdot \sqrt[3]{-18}$

Multiply and simplify. Assume that all variables are positive.

17. $\sqrt[3]{6} \cdot \sqrt[3]{16}$

18. $\sqrt{8y^5} \cdot \sqrt{40y^2}$

19. $\sqrt{7x^5} \cdot \sqrt{42xy^9}$

20. $4\sqrt{2x} \cdot 5\sqrt{6xy^2}$

21. $3\sqrt[3]{5y^3} \cdot 2\sqrt[3]{50y^4}$

22. $-\sqrt[3]{2x^2y^2} \cdot 2\sqrt[3]{15x^5y}$

Divide and simplify. Assume that all variables are positive.

23. $\frac{\sqrt{500}}{\sqrt{5}}$ **24.** $\frac{\sqrt{48x^3}}{\sqrt{3xy^2}}$ **25.** $\frac{\sqrt{56x^5y^5}}{\sqrt{7xy}}$ **26.** $\frac{\sqrt[3]{250x^7y^3}}{\sqrt[3]{2x^2y}}$

Simplify each expression. Rationalize all denominators. Assume that all variables are positive.

37. $\sqrt{5} \cdot \sqrt{40}$ **38.** $\sqrt[3]{4} \cdot \sqrt[3]{80}$ **39.** $\sqrt{x^5y^5} \cdot 3\sqrt{2x^7y^6}$

40. $5\sqrt{2xy^6} \cdot 2\sqrt{2x^3y}$ **41.** $\sqrt{2}(\sqrt{50} + 7)$ **42.** $3(5 + \sqrt{21})$

43. $\sqrt{5}(\sqrt{5} + \sqrt{15})$ **44.** $\sqrt[3]{2x} \cdot \sqrt[3]{4} \cdot \sqrt[3]{2x^2}$ **45.** $\sqrt[3]{3x^2} \cdot \sqrt[3]{x^2} \cdot \sqrt[3]{9x^3}$

46. $\frac{\sqrt{5x^4}}{\sqrt{2x^2y^3}}$ **47.** $\frac{5\sqrt{2}}{3\sqrt{7x}}$ **48.** $\frac{1}{\sqrt[3]{9x}}$

49. $\frac{10}{\sqrt[3]{5x^2}}$ **50.** $\frac{\sqrt[3]{14}}{\sqrt[3]{7x^2y}}$ **51.** $\frac{3\sqrt{11x^3y}}{-2\sqrt{12x^4y}}$

52. $-2\left(\sqrt[3]{32} + \sqrt[3]{54}\right)$ **53.** $\frac{3 + \sqrt{5}}{\sqrt{5}}$ **54.** $\frac{\sqrt{3} - \sqrt{2}}{\sqrt{8}}$

Lesson 7-2 **pp. 377–379**

EXERCISES **1.** 16 **3.** –9 **9.** $2x\sqrt{5x}$ **11.** $5x^2\sqrt{2x}$ **17.** $2\sqrt[3]{12}$ **19.** $7x^3y^4\sqrt{6y}$ **23.** 10 **25.** $2x^2y^2\sqrt{2}$ **27.** $\frac{\sqrt{2x}}{2}$ **29.** $\frac{\sqrt[3]{4x}}{2}$ **35.** $r = \frac{\sqrt{Gm_1m_2F}}{F}$ **37.** $10\sqrt{2}$ **39.** $3x^6y^5\sqrt{2y}$ **55.** 212 mi/h greater

LT 3,4 More Practice #1: Worksheet 7.2

Practice 7-2 — Multiplying and Dividing Radical Expressions

Multiply and simplify. Assume that all variables are positive.

1. $\sqrt{4}\cdot\sqrt{6}$ **2.** $\sqrt{9x^2}\cdot\sqrt{9y^5}$ **3.** $\sqrt[3]{50x^2z^5}\cdot\sqrt[3]{15y^3z}$

4. $4\sqrt{2x}\cdot 3\sqrt{8x}$ **5.** $\sqrt{xy}\cdot\sqrt{4xy}$ **6.** $9\sqrt{2}\cdot 3\sqrt{y}$

Rationalize the denominator of each expression. Assume that all variables are positive.

7. $\sqrt{\frac{9x}{2}}$ **8.** $\frac{\sqrt{xy}}{\sqrt{3x}}$ **9.** $\sqrt[3]{\frac{x^2}{3y}}$

10. $\frac{\sqrt[4]{2x}}{\sqrt[4]{3x^2}}$ **11.** $\sqrt{\frac{x}{8y}}$ **12.** $\sqrt[3]{\frac{3a}{4b^2c}}$

Multiply. Simplify if possible. Assume that all variables are positive.

13. $\sqrt{4}\cdot\sqrt{25}$ **14.** $\sqrt{81}\cdot\sqrt{36}$ **15.** $\sqrt{3}\cdot\sqrt{27}$

16. $\sqrt[3]{-3}\cdot\sqrt[3]{9}$ **17.** $\sqrt{3x}\cdot\sqrt{6x^3}$ **18.** $\sqrt[3]{2xy^2}\cdot\sqrt[3]{4x^2y^7}$

Simplify. Assume that all variables are positive.

19. $\sqrt{36x^3}$ **20.** $\sqrt[3]{125y^2z^4}$ **21.** $\sqrt{18k^6}$

22. $\sqrt[3]{-16a^{12}}$ **23.** $\sqrt{x^2y^{10}z}$ **24.** $\sqrt[4]{256s^7t^{12}}$

25. $\sqrt[3]{216x^4y^3}$ **26.** $\sqrt{75r^3}$ **27.** $\sqrt[4]{625u^5v^8}$

Divide and simplify. Assume all variables are positive.

28. $\frac{\sqrt{6x}}{\sqrt{3x}}$ **29.** $\frac{\sqrt[3]{4x^2}}{\sqrt[3]{x}}$ **30.** $\sqrt[4]{\frac{243k^3}{3k^7}}$

31. $\frac{\sqrt{(2x)^2}}{\sqrt{(5y)^4}}$ **32.** $\frac{\sqrt[3]{18y^2}}{\sqrt[3]{12y}}$ **33.** $\sqrt{\frac{162a}{6a^3}}$

34. The volume of a sphere of radius *r* is $V=\frac{4}{3}\pi r^3$.

a. Use the formula to find *r* in terms of *V*.

b. Find the radius of a sphere with volume 100 in³.

Practice 7-2

1. $2\sqrt{6}$ **2.** $9xy^2\sqrt{y}$ **3.** $5yz^2\sqrt[3]{6x^2}$ **4.** $48x$ **5.** $2xy$

6. $27\sqrt{2y}$ **7.** $\frac{3\sqrt{2x}}{2}$ **8.** $\frac{\sqrt{3y}}{3}$ **9.** $\frac{\sqrt[3]{9x^2y^2}}{3y}$ **10.** $\frac{\sqrt[4]{54x^3}}{3x}$

11. $\frac{\sqrt{2xy}}{4y}$ **12.** $\frac{\sqrt[3]{6abc^2}}{2bc}$ **13.** 10 **14.** 54 **15.** 9 **16.** -3

17. $3x^2\sqrt{2}$ **18.** $2xy^3$ **19.** $6x\sqrt{x}$ **20.** $5z\sqrt[3]{y^2z}$

21. $3k^3\sqrt{2}$ **22.** $-2a^4\sqrt[3]{2}$ **23.** $xy^5\sqrt{z}$ **24.** $4st^3\sqrt[4]{s^3}$

25. $6xy\sqrt[3]{x}$ **26.** $5r\sqrt{3r}$ **27.** $5uv^2\sqrt[4]{u}$ **28.** $\sqrt{2}$ **29.** $\sqrt[3]{4x}$

30. $\frac{3}{k}$ **31.** $\frac{2x}{25y^2}$ **32.** $\frac{\sqrt[3]{12y}}{2}$ **33.** $\frac{3\sqrt{3}}{a}$

34a. $r=\sqrt[3]{\frac{3V}{4\pi}}$; $r=\frac{\sqrt[3]{6\pi^2V}}{2\pi}$ **34b.** 2.88 in.

LT 3,4 More Practice #2:
Simplify. (Multiply)

1) $2\sqrt{3} \cdot 5\sqrt{7}$

2) $2\sqrt{3} \cdot 5\sqrt[4]{7}$

3) $7\sqrt{5} \cdot 10\sqrt{2}$

4) $\left(-2\sqrt{11}\right)\left(-7\sqrt{3}\right)$

5) $2\sqrt[4]{3} \cdot 5\sqrt[4]{3}$

6) $3\sqrt{2} \cdot \left(-\sqrt{5}\right)$

7) $\left(4\sqrt{5}\right)^2$

8) $5\sqrt[3]{9} \cdot 6\sqrt[3]{3}$

9) $\left(\sqrt[3]{2}\right)^3$

10) $\left(\sqrt{8x^7y}\right)\left(\sqrt{6x^5y^3z}\right)$

11) $\left(2\sqrt[3]{5}\right)^3$

Simplify (Divide and Rationalize)

12) $\sqrt{\dfrac{3}{5}}$

13) $\dfrac{\sqrt[3]{18}}{\sqrt[3]{3}}$

14) $\dfrac{8\sqrt{30}}{2\sqrt{5}} =$

15) $\dfrac{1}{\sqrt[3]{7}}$

16) $2\sqrt{6^{-1}}$

17) $\dfrac{5}{\sqrt[3]{2}}$

18) $\dfrac{1}{7\sqrt{12}}$

19) $\dfrac{6}{4\sqrt[3]{3}}$

20) $\dfrac{2}{\sqrt[3]{25}}$

21) $\dfrac{10}{\sqrt[4]{8}}$

22) $\dfrac{5m}{\sqrt[4]{10m^3}}$

23) $\dfrac{3\sqrt[4]{2a^2}}{\sqrt[4]{6a^3}}$

Major Operations with Radical Expressions

Date: ____________
Quiz On: ____________

After this lesson and practice, I will be able to …

- ☐ add and subtract radical expressions. (LT 5)
- ☐ multiply and rationalize binomial radical expressions. (LT 6)

In your studies with quadratics and polynomials, several of your solutions to quadratic equations were in the form of a _____________ radical expression. Examples of these are:

In today's lesson, you will learn how to perform the four major operations with these expressions.

Adding and Subtracting Radical Expressions (LT 5)

Just as ______________ terms can only be added or subtracted if they are ______ terms, radical expressions can be added or subtracted only if they are _______ radicals.

Definition 1: **Like Radicals –** Radical expressions that have the same _______ and the same _____________.

LIKE RADICALS OR NOT?!

A) $6\sqrt[3]{11}, 2\sqrt[3]{11}$ *B)* $6\sqrt[3]{11}, 2\sqrt{11}$ *C)* $5\sqrt{7}, 3\sqrt{7}$ *D)* $5\sqrt{7}, 3\sqrt{2}$

Example 1: Add or subtract, if possible.

A) $5\sqrt[3]{x} - 3\sqrt[3]{x}$ B) $4\sqrt{2} + 4\sqrt{3}$

When possible, you should ____________ radicals before adding or subtracting so that you can see all of the like radicals.

Example 2: Simplify $6\sqrt{18} + 4\sqrt{8} - 3\sqrt{72}$.

Example 3: Simplify each radical expression, if possible.

A) $6\sqrt{3} + 5\sqrt{3}$ B) $2\sqrt{3} - 8\sqrt{32}$ C) $\sqrt[3]{24} + 5\sqrt[3]{3}$ D) $\sqrt{50} - \sqrt{75} + \sqrt{98} + \sqrt{27}$

Multiplying Binomial Radical Expressions (LT 6)

To multiply binomial radical expressions, you can use __________ or ________________.

Example 4: Multiply and simplify.

A) $\left(3+2\sqrt{5}\right)\left(2-4\sqrt{5}\right)$

B) $\left(\sqrt{2}-\sqrt{5}\right)^2$

Last unit, you learned that ________________ differ only in the ______ of the _________ term. Let's explore what happens when you multiply binomial radical conjugates.

Example 5: Multiply and simplify $\left(3+\sqrt{7}\right)\left(3-\sqrt{7}\right)$

Since the product of radical conjugates is a ___________ number, you can use ______________ to rationalize denominators that consist of binomial radical expressions.

Rationalizing Denominators with Binomial Radical Expressions (LT 6)

Example 6: Rationalize the denominator of each expression.

A) $\dfrac{3+\sqrt{5}}{1-\sqrt{5}}$

B) $\dfrac{6+\sqrt{15}}{4-\sqrt{15}}$

Example 7: Simplify each expression. Rationalize denominators if necessary.

A) $\left(2+\sqrt{7}\right)\left(1+3\sqrt{7}\right)$

B) $\sqrt[3]{54}+\sqrt[3]{16}$

C) $\left(4+2\sqrt{3}\right)\left(4-2\sqrt{3}\right)$

D) $\dfrac{2-\sqrt{3}}{4+\sqrt{3}}$

FINAL CHECK: LT 5-6

I can add and subtract radical expressions. (LT 5)
I can multiply and rationalize binomial radical expressions. (LT 6)

1. Simplify each radical expression, if possible. Assume all variables are positive. Express your answer in simplified radical form. Rationalize denominators if necessary. *Show your work.*

a. $-3\sqrt{125}+\sqrt{40}-6\sqrt{20}$

b. $\left(1+7\sqrt{6}\right)\left(5-2\sqrt{6}\right)$

c. $\left(\sqrt{7}-\sqrt{2}\right)^2$

d. $\dfrac{4-\sqrt{5}}{3+\sqrt{5}}$

LT 5,6 Practice Assignment

- ☐ I can add and subtract radical expressions. (LT 5)
- ☐ I can multiply and rationalize binomial radical expressions. (LT 6)
  - Worksheet Practice 7.3 (below)

Practice 7-3 — Binomial Radical Expressions

Multiply each pair of conjugates.

1. $\left(3\sqrt{2}-9\right)\left(3\sqrt{2}+9\right)$ **2.** $\left(1-\sqrt{7}\right)\left(1+\sqrt{7}\right)$ **3.** $\left(5\sqrt{3}+\sqrt{2}\right)\left(5\sqrt{3}-\sqrt{2}\right)$

Add or subtract if possible.

4. $9\sqrt{3}+2\sqrt{3}$ **5.** $5\sqrt{2}+2\sqrt{3}$ **6.** $3\sqrt{7}-7\sqrt[3]{x}$ **7.** $14\sqrt[3]{xy}-3\sqrt[3]{xy}$

Rationalize each denominator. Simplify the answer.

8. $\dfrac{2}{2\sqrt{3}-4}$ **9.** $\dfrac{5}{2+\sqrt{3}}$ **10.** $\dfrac{1+\sqrt{5}}{1-\sqrt{5}}$ **11.** $\dfrac{2+\sqrt{12}}{5-\sqrt{12}}$

Simplify.

12. $3\sqrt{32}+2\sqrt{50}$ **13.** $\sqrt{200}-\sqrt{72}$ **14.** $\sqrt[3]{81}-3\sqrt[3]{3}$ **15.** $2\sqrt[4]{48}+3\sqrt[4]{243}$

Multiply.

16. $\left(1-\sqrt{5}\right)\left(2+\sqrt{5}\right)$ **17.** $\left(1+4\sqrt{10}\right)\left(2-\sqrt{10}\right)$ **18.** $\left(1-3\sqrt{7}\right)\left(4-3\sqrt{7}\right)$

19. $\left(4-2\sqrt{3}\right)^2$ **20.** $\left(\sqrt{2}+\sqrt{7}\right)^2$ **21.** $\left(2\sqrt{3}+3\sqrt{2}\right)^2$

Simplify. Rationalize all denominators. Assume that all variables are positive

22. $\sqrt{28}+4\sqrt{63}-2\sqrt{7}$ **23.** $6\sqrt{40}-2\sqrt{90}+3\sqrt{160}$

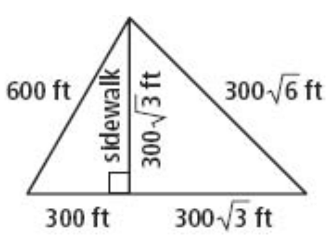
24. $3\sqrt{12}+7\sqrt{75}-\sqrt{54}$ **25.** $4\sqrt[3]{81}+2\sqrt[3]{72}-3\sqrt[3]{24}$

26. $3\sqrt{225x}+5\sqrt{144x}$ **27.** $6\sqrt{45y^2}+4\sqrt{20y^2}$

28. $\left(3\sqrt{y}-\sqrt{5}\right)\left(2\sqrt{y}+5\sqrt{5}\right)$ **29.** $\left(\sqrt{x}-\sqrt{3}\right)\left(\sqrt{x}+\sqrt{3}\right)$

30. $\dfrac{3-\sqrt{10}}{\sqrt{5}-\sqrt{2}}$ **31.** $\dfrac{2+\sqrt{14}}{\sqrt{7}+\sqrt{2}}$ **32.** $\dfrac{2+\sqrt[3]{x}}{\sqrt[3]{x}}$

33. A park in the shape of a triangle has a sidewalk dividing it into two parts.



a. If a man walks around the perimeter of the park, how far will he walk

b. What is the area of the park?

Practice 7-3

1. -63 **2.** -6 **3.** 73 **4.** $11\sqrt{3}$ **5.** $5\sqrt{2}+2\sqrt{3}$
6. $3\sqrt{7}-7\sqrt[3]{x}$ **7.** $11\sqrt[3]{xy}$ **8.** $-2-\sqrt{3}$ **9.** $10-5\sqrt{3}$
10. $-\dfrac{3+\sqrt{5}}{2}$ **11.** $\dfrac{22+14\sqrt{3}}{13}$ **12.** $22\sqrt{2}$ **13.** $4\sqrt{2}$
14. 0 **15.** $13\sqrt[4]{3}$ **16.** $-3-\sqrt{5}$ **17.** $-38+7\sqrt{10}$
18. $67-15\sqrt{7}$ **19.** $28-16\sqrt{3}$ **20.** $9+2\sqrt{14}$
21. $30+12\sqrt{6}$ **22.** $12\sqrt{7}$ **23.** $18\sqrt{10}$
24. $41\sqrt{3}-3\sqrt{6}$ **25.** $6\sqrt[3]{3}+4\sqrt[3]{9}$ **26.** $105\sqrt{x}$
27. $26y\sqrt{5}$ **28.** $6y+13\sqrt{5y}-25$ **29.** $x-3$
30. $\dfrac{\sqrt{5}-2\sqrt{2}}{3}$ **31.** $\sqrt{2}$ **32.** $\dfrac{x+2\sqrt[3]{x^2}}{x}$
33a. $(900+300\sqrt{3}+300\sqrt{6})$ ft or about 2154 ft
33b. $\dfrac{270{,}000+90{,}000\sqrt{3}}{2}$ ft^2 or about 212,942 ft^2

LT 5,6 More Practice #1: Book 7-3 Page 382

Add or subtract if possible.

1. $5\sqrt{6} + \sqrt{6}$ **2.** $6\sqrt[3]{3} - 2\sqrt[3]{3}$ **3.** $4\sqrt{3} + 4\sqrt[3]{3}$

4. $3\sqrt{x} - 5\sqrt{x}$ **5.** $14\sqrt{x} + 3\sqrt{y}$ **6.** $7\sqrt[3]{x^2} - 2\sqrt[3]{x^2}$

Simplify.

7. $6\sqrt{18} + 3\sqrt{50}$ **8.** $14\sqrt{20} - 3\sqrt{125}$ **9.** $\sqrt{18} + \sqrt{32}$

10. $\sqrt[3]{54} + \sqrt[3]{16}$ **11.** $3\sqrt[3]{81} - 2\sqrt[3]{54}$ **12.** $\sqrt[4]{32} + \sqrt[4]{48}$

Multiply.

13. $(3 + \sqrt{5})(1 + \sqrt{5})$ **14.** $(2 + \sqrt{7})(1 + 3\sqrt{7})$

15. $(3 - 4\sqrt{2})(5 - 6\sqrt{2})$ **16.** $(\sqrt{3} + \sqrt{5})^2$

17. $(\sqrt{13} + 6)^2$ **18.** $(2\sqrt{5} + 3\sqrt{2})^2$

Multiply each pair of conjugates.

19. $(5 - \sqrt{11})(5 + \sqrt{11})$ **20.** $(4 - 2\sqrt{3})(4 + 2\sqrt{3})$

21. $(2\sqrt{6} + 8)(2\sqrt{6} - 8)$ **22.** $(\sqrt{3} + \sqrt{5})(\sqrt{3} - \sqrt{5})$

Rationalize each denominator. Simplify the answer.

23. $\frac{4}{1 + \sqrt{3}}$ **24.** $\frac{4}{3\sqrt{3} - 2}$ **25.** $\frac{5 + \sqrt{3}}{2 - \sqrt{3}}$ **26.** $\frac{3 + \sqrt{8}}{2 - 2\sqrt{8}}$

Simplify. Rationalize all denominators. Assume that all the variables are positive.

27. $\sqrt{72} + \sqrt{32} + \sqrt{18}$ **28.** $\sqrt{75} + 2\sqrt{48} - 5\sqrt{3}$

29. $5\sqrt{32x} + 4\sqrt{98x}$ **30.** $\sqrt{75} - 4\sqrt{18} + 2\sqrt{32}$

31. $4\sqrt{216y^2} + 3\sqrt{54y^2}$ **32.** $3\sqrt[3]{16} - 4\sqrt[3]{54} + \sqrt[3]{128}$

33. $(\sqrt{3} - \sqrt{7})(\sqrt{3} + 2\sqrt{7})$ **34.** $(2\sqrt{5} + 3\sqrt{2})(5\sqrt{5} - 7\sqrt{2})$

35. $(1 + \sqrt{72})(5 + \sqrt{2})$ **36.** $(2 - \sqrt{98})(3 + \sqrt{18})$

37. $(\sqrt{x} + \sqrt{3})(\sqrt{x} + 2\sqrt{3})$ **38.** $(2\sqrt{y} - 3\sqrt{2})(4\sqrt{y} - 5\sqrt{2})$

39. $\frac{4 + \sqrt{27}}{2 - 3\sqrt{27}}$ **40.** $\frac{4 + \sqrt{6}}{\sqrt{2} + \sqrt{3}}$

41. $\frac{5 - \sqrt{21}}{\sqrt{3} - \sqrt{7}}$ **42.** $\frac{3 + \sqrt[3]{2}}{\sqrt[3]{2}}$

43. $\frac{5 + \sqrt[4]{x}}{\sqrt[4]{x}}$ **44.** $\frac{4 - 2\sqrt[3]{6}}{\sqrt[3]{4}}$

Lesson 7-3 pp. 382–384

EXERCISES 1. $6\sqrt{6}$ **3.** cannot combine **7.** $33\sqrt{2}$ **9.** $7\sqrt{2}$ **13.** $8 + 4\sqrt{5}$ **15.** $63 - 38\sqrt{2}$ **19.** 14 **21.** −40 **23.** $-2 + 2\sqrt{3}$ **25.** $13 + 7\sqrt{3}$ **27.** $13\sqrt{2}$ **29.** $48\sqrt{2x}$

LT 5,6 More Practice #2

Simplify (Adding and Subtracting)

1) $6\sqrt{3} + 5\sqrt{3}$

2) $5\sqrt[3]{4} - 2\sqrt[3]{4} =$

3) $2\sqrt{3} - 8\sqrt{32}$

4) $2\sqrt{40} + 5\sqrt{90}$

5) $4\sqrt[3]{5} + 13\sqrt[7]{5}$

6) $a\sqrt{ab^3} + 7ab\sqrt{ab}$

7) $\sqrt[3]{24} + 5\sqrt[3]{3}$

8) $d^2\sqrt[4]{16d^5} - 8\sqrt[4]{d^{13}}$

9) $\sqrt{a^3} + \sqrt{b^5} + a\sqrt{25a} + b^2\sqrt{49b}$

10) $\sqrt{50} - \sqrt{75} + \sqrt{98} + \sqrt{27}$

Simplify. (Multiplying)

11) $\sqrt{3}\left(2\sqrt{6} + 5\right)$

12) $3\sqrt{2}\left(\sqrt{8} + 2\sqrt{3} - 5\sqrt{12}\right)$

13) $\left(\sqrt{6} - 5\sqrt{2}\right)^2$

14) $\left(\sqrt[3]{4} - \sqrt[3]{2}\right)\left(3\sqrt[3]{4} - \sqrt[3]{2}\right)$

Simplify (Rationalize)

15) $\dfrac{3+5\sqrt{7}}{2-\sqrt{7}}$

16) $\dfrac{1-\sqrt{5}}{1+2\sqrt{5}}$

Rational Exponents

Date: ____________

Quiz On: ____________

After this lesson and practice, I will be able to …

- ☐ convert from rational exponents to radical expressions (and vice versa). (LT 7)
- ☐ simplify numbers with rational exponents. (LT 8)

There exists another way of representing a radical expression that, in some applications, is easier to use when performing _________________ with the expression. This alternate way of writing radical expressions is by using _____________ _________________.

<u>**Exponent Form**</u>	<u>**Radical Form**</u>
$B^{\frac{P}{R}}$	$= \sqrt[R]{B^P}$ OR $\left(\sqrt[R]{B}\right)^P$

P, the numerator, is ___________. R, the denominator, is ________. B stands for ________.

Note: When you use rational exponents, you are indicating the ______________ root.

<u>Simplifying Expressions with Rational Exponents – Part 1 (LT 8)</u>

<u>Example 1</u>: Simplify each expression.

A) $125^{1/3}$

B) $5^{1/2} \cdot 5^{1/2}$

C) $10^{1/3} \cdot 100^{1/3}$

The _________________ of rational exponents do not always need to be _____.

$25^{3/2}$

$25^{3/2}$

<u>Definition 1</u>: **Rational Exponents** – If the *n*th root of *a* is a real number and *m* is an integer, then:

$a^{1/n} =$

$a^{m/n}$

- The denominator becomes the __________ of the radical.
- The numerator becomes the ________ to which *either* the ___________ or ___________ is raised.

Converting Between Radical Expressions and Rational Exponents (LT 7)

Example 2: Write each of the following expressions in radical form.

a. $x^{3/4}$ b. $y^{-1.5}$ c. $a^{-1/4}$

Example 3: Write each of the following expressions in exponential form.

a. $\left(\sqrt[5]{x}\right)^9$ b. $\sqrt[4]{3r}$ c. $\left(\sqrt[3]{4m}\right)^2$

Example 4: Rewrite each exponential form expression in radical form, and vice versa.

A) $(5x)^{4/3}$ B) $w^{-0.4}$ C) $\sqrt[3]{c^4}$ D) $\left(\sqrt{8k}\right)^3$

Simplifying Expressions with Rational Exponents – Part 2 (LT 8)

The same exponent properties that are true with integer exponents are also true for ____________ exponents.

Example 5: Simplify each number completely.

A) $(-32)^{3/5}$ B) $36^{-5/2}$

An expression is considered to be in ______________ form when all exponents are _____________.

Example 6: Write $\left(-27x^6\right)^{2/3}$ in simplified exponential form.

Example 7: Simplify each number or expression completely. Write answers in simplified exponential form.

A) $\left(8x^{15}\right)^{-1/3}$

B) $\left(\dfrac{h^6p^9}{1000m^3}\right)^{-2/3}$

c) $\left(\dfrac{32a^5}{b^{10}c^{20}}\right)^{-\frac{3}{5}}$

Calculator Help: When entering rational exponents in the calculator, be sure to type the fraction inside

___________________.

Final Check:

NO Calculator:

LT 7: I can convert from rational exponents to radical expressions (and vice versa).

1. Write $x^{5/7}$ in radical form. _____

2. Write $\sqrt[10]{m^7}$ in exponential form. _______

LT 8: I can simplify numbers with rational exponents.

3. Evaluate without a calculator: $64^{-3/2}$ _____

Calculator:

LT 8: I can simplify numbers with rational exponents.

4. Write the expression in simplest form. Assume all variables are positive. Express your answer in simplified exponential form. Use only positive exponents in your answer. *Show your work.*

a. $(81a^{-80}b^{32}c^{0})^{-1/4}$

b. $\left(\dfrac{a^{12}c^{-18}}{b^{-6}}\right)^{-2/3}$

LT 7,8 Practice Assignment

- ☐ I can convert from rational exponents to radical expressions (and vice versa). (LT 7)
- ☐ I can simplify numbers with rational exponents. (LT 8)
    - Worksheet Practice 7.4 (next page)

Practice 7-4 Rational Exponents

Simplify each expression. Assume that all variables are positive.

1. $27^{\frac{1}{3}}$ **2.** $\left(81^{\frac{1}{4}}\right)^4$ **3.** $\left(32^{\frac{1}{5}}\right)^5$

4. $(256^4)^{\frac{1}{4}}$ **5.** 7^0 **6.** $8^{\frac{2}{3}}$

7. $(-1)^{\frac{1}{5}}$ **8.** $(-27)^{\frac{2}{3}}$ **9.** $16^{\frac{1}{4}}$

10. $x^{\frac{1}{2}} \cdot x^{\frac{1}{3}}$ **11.** $2y^{\frac{1}{2}} \cdot y$ **12.** $(8^2)^{\frac{1}{3}}$

13. 3.6^0 **14.** $\left(\frac{1}{16}\right)^{\frac{1}{4}}$ **15.** $\left(\frac{27}{8}\right)^{\frac{2}{3}}$

16. $\sqrt[8]{0}$ **17.** $\left(3x^{\frac{1}{2}}\right)\left(4x^{\frac{2}{3}}\right)$ **18.** $\frac{12y^{\frac{1}{3}}}{4y^{\frac{1}{2}}}$

19. $\left(3a^{\frac{1}{2}}b^{\frac{1}{3}}\right)^2$ **20.** $\left(y^{\frac{2}{3}}\right)^{-9}$ **21.** $\left(a^{\frac{2}{3}}b^{-\frac{1}{2}}\right)^{-6}$

22. $y^{\frac{2}{5}} \cdot y^{\frac{3}{8}}$ **23.** $\left(\frac{x^{\frac{4}{7}}}{x^{\frac{2}{3}}}\right)$ **24.** $\left(2a^{\frac{1}{4}}\right)^3$

25. $81^{-\frac{1}{2}}$ **26.** $\left(2x^{\frac{2}{5}}\right)\left(6x^{\frac{1}{4}}\right)$ **27.** $\left(9x^4y^{-2}\right)^{\frac{1}{2}}$

Write each expression in radical form.

29. $x^{\frac{4}{3}}$ **30.** $(2y)^{\frac{1}{3}}$ **31.** $a^{1.5}$

32. $b^{\frac{1}{5}}$ **33.** $z^{\frac{2}{3}}$ **34.** $(ab)^{\frac{1}{4}}$

35. $m^{2.4}$ **36.** $t^{-\frac{2}{7}}$ **37.** $a^{-1.6}$

Write each expression in exponential form.

38. $\sqrt{x^3}$ **39.** $\sqrt[3]{m}$ **40.** $\sqrt{5y}$

41. $\sqrt[3]{2y^2}$ **42.** $\left(\sqrt[4]{b}\right)^3$ **43.** $\sqrt{-6}$

44. $\sqrt{(6a)^4}$ **45.** $\sqrt[5]{n^4}$ **46.** $\sqrt[4]{(5ab)^3}$

Practice 7-4

1. 3 **2.** 81 **3.** 32 **4.** 256 **5.** 1 **6.** 4 **7.** -1 **8.** 9 **9.** 2
10. $x^{\frac{5}{6}}$ **11.** $2y^{\frac{3}{2}}$ **12.** 4 **13.** 1 **14.** $\frac{1}{2}$ **15.** $\frac{9}{4}$ **16.** 0

ANSWERS:

17. $12x^{\frac{7}{6}}$ **18.** $\frac{3}{y^{\frac{1}{6}}}$ **19.** $9ab^{\frac{2}{3}}$ **20.** $\frac{1}{y^6}$ **21.** $\frac{b^3}{a^4}$ **22.** $y^{\frac{31}{40}}$

23. $\frac{1}{x^{\frac{2}{21}}}$ **24.** $8a^{\frac{3}{4}}$ **25.** $\frac{1}{9}$ **26.** $12x^{\frac{13}{20}}$ **27.** $\frac{3x^2}{y}$

28. 10.1% **29.** $\sqrt[3]{x^4}$ **30.** $\sqrt[3]{2y}$ **31.** $\sqrt{a^3}$ **32.** $\sqrt[5]{b}$

33. $\sqrt[3]{z^2}$ **34.** $\sqrt[4]{ab}$ **35.** $\sqrt[5]{m^{12}}$ **36.** $\frac{1}{\sqrt[7]{t^2}}$ **37.** $\frac{1}{\sqrt[5]{a^8}}$

38. $x^{\frac{3}{2}}$ **39.** $m^{\frac{1}{3}}$ **40.** $(5y)^{\frac{1}{2}}$ **41.** $2^{\frac{1}{3}}y^{\frac{2}{3}}$ **42.** $b^{\frac{3}{4}}$ **43.** $(-6)^{\frac{1}{2}}$

44. $36a^2$ **45.** $n^{\frac{4}{5}}$ **46.** $(5ab)^{\frac{3}{4}}$

LT 7,8 More Practice #1: Book 7.4 page 388

Simplify each expression.

1. $36^{\frac{1}{2}}$ **2.** $27^{\frac{1}{3}}$ **3.** $49^{\frac{1}{2}}$

4. $10^{\frac{1}{2}} \cdot 10^{\frac{1}{2}}$ **5.** $(-3)^{\frac{1}{3}} \cdot (-3)^{\frac{1}{3}} \cdot (-3)^{\frac{1}{3}}$ **6.** $3^{\frac{1}{2}} \cdot 12^{\frac{1}{2}}$

7. $2^{\frac{1}{2}} \cdot 32^{\frac{1}{2}}$ **8.** $3^{\frac{1}{3}} \cdot 9^{\frac{1}{3}}$ **9.** $3^{\frac{1}{4}} \cdot 27^{\frac{1}{4}}$

Write each expression in radical form.

10. $x^{\frac{1}{6}}$ **11.** $x^{\frac{1}{5}}$ **12.** $x^{\frac{2}{7}}$ **13.** $y^{\frac{2}{5}}$

14. $y^{-\frac{9}{8}}$ **15.** $t^{-\frac{3}{4}}$ **16.** $x^{1.5}$ **17.** $y^{1.2}$

Write each expression in exponential form.

18. $\sqrt{-10}$ **19.** $\sqrt{7x^3}$ **20.** $\sqrt{(7x)^3}$ **21.** $(\sqrt{7x})^3$

22. $\sqrt[3]{a^2}$ **23.** $(\sqrt[3]{a})^2$ **24.** $\sqrt[4]{c^2}$ **25.** $\sqrt[3]{(5xy)^6}$

Write each expression in simplest form. Assume that all variables are positive.

38. $\left(x^{\frac{2}{3}}\right)^{-3}$ **39.** $\left(x^{-\frac{4}{7}}\right)^{7}$ **40.** $\left(3x^{\frac{2}{3}}\right)^{-1}$ **41.** $5\left(x^{\frac{2}{3}}\right)^{-1}$

42. $\left(-27x^{-9}\right)^{\frac{1}{3}}$ **43.** $\left(-32y^{15}\right)^{\frac{1}{5}}$ **44.** $\left(\frac{x^3}{x^{-1}}\right)^{-\frac{1}{4}}$ **45.** $\left(\frac{x^2}{x^{-11}}\right)^{\frac{1}{3}}$

46. $\left(x^{\frac{1}{2}}y^{-\frac{2}{3}}\right)^{-6}$ **47.** $\left(x^{\frac{2}{3}}y^{-\frac{1}{6}}\right)^{-12}$ **48.** $\left(\frac{x^{\frac{1}{4}}}{y^{-\frac{3}{4}}}\right)^{12}$ **49.** $\left(\frac{x^{-\frac{2}{3}}}{y^{-\frac{1}{3}}}\right)^{15}$

Simplify each number.

50. $(-343)^{\frac{1}{3}}$ **51.** $(-243)^{\frac{1}{5}}$ **52.** $32^{1.2}$ **53.** $243^{1.2}$

54. $64^{3.5}$ **55.** $100^{4.5}$ **56.** $32^{-0.4}$ **57.** $64^{-0.5}$

58. $(-216)^{-\frac{2}{3}}$ **59.** $2(16)^{\frac{3}{4}}$ **60.** $-(-27)^{-\frac{4}{3}}$ **61.** $\frac{1000^{\frac{4}{3}}}{100^{\frac{3}{2}}}$

Simplify each expression. Assume that all variables are positive.

65. $x^{\frac{2}{7}} \cdot x^{\frac{3}{14}}$ **66.** $y^{\frac{1}{2}} \cdot y^{\frac{3}{10}}$ **67.** $x^{\frac{3}{5}} \div x^{\frac{1}{10}}$ **68.** $y^{\frac{5}{7}} \div y^{\frac{3}{14}}$

69. $\frac{x^{\frac{2}{3}}y^{-\frac{1}{4}}}{x^{\frac{1}{2}}y^{-\frac{1}{2}}}$ **70.** $\frac{x^{\frac{1}{2}}y^{-\frac{1}{3}}}{x^{\frac{3}{4}}y^{\frac{1}{2}}}$ **71.** $\left(\frac{16x^{14}}{81y^{18}}\right)^{\frac{1}{2}}$ **72.** $\left(\frac{81y^{16}}{16x^{12}}\right)^{\frac{1}{2}}$

73. $\left(x^{\frac{1}{2}} \cdot x^{\frac{5}{12}}\right)^{\frac{1}{3}} \div x^{\frac{2}{3}}$ **74.** $\left(x^{\frac{3}{4}} \div x^{\frac{7}{8}}\right) \cdot x^{-\frac{1}{6}}$ **75.** $\left[\left(x^{-\frac{1}{2}}\right)^{2}\right]^{\frac{1}{3}}$ **76.** $\left[\left(\sqrt{x^3y^3}\right)^{\frac{1}{3}}\right]^{-1}$

Lesson 7-4 pp. 388–390

EXERCISES 1. 6 **3.** 7 **11.** $\sqrt[5]{x}$ **13.** $\sqrt[5]{y^2}$ or $(\sqrt[5]{y})^2$ **19.** $7^{\frac{1}{2}}x^{\frac{3}{2}}$ **21.** $(7x)^{\frac{3}{2}}$ **27.** ≈15.1 m **29.** ≈1.6 m **31.** 16 **33.** 64 **39.** $\frac{1}{x^4}$ **41.** $\frac{5}{x^{\frac{2}{3}}}$ **51.** −3 **53.** 729 **63.** A **65.** $x^{\frac{1}{2}}$ **67.** $x^{\frac{1}{2}}$

LT 7,8 More Practice #2 (mixed):

Write in radical form.

16) $a^{\frac{1}{n}} =$

17) $a^{\frac{1}{2}} =$

18) $a^{\frac{1}{4}} =$

19) $a^{\frac{m}{n}} =$

20) $a^{\frac{2}{3}} =$

21) $a^{-\frac{4}{9}} =$

Write in exponential form. Assume all variables are positive.

22) $\sqrt[3]{x^6} =$

23) $\sqrt[4]{x^{12}} =$

24) $\sqrt[7]{x^{21}y^{49}} =$

25) $\sqrt{x^4y^8z^{14}} =$

Simplify. Use absolute value bars when needed.

26) $(-27x^6)^{\frac{2}{3}} =$

27) $(-64y^{12})^{-\frac{2}{3}} =$

28) $\left(8x^{15}\right)^{-\frac{1}{3}}$

29) $\left(\frac{27b^9}{8a^6}\right)^{\frac{1}{3}}$

30) $\left(\frac{16c^8}{81d^{16}}\right)^{-\frac{1}{4}}$

Solving Radical Equations

Date: ____________
Quiz On: ____________

After this lesson and practice, I will be able to …

- ☐ solve equations with roots (or radicals). (LT 9)
- ☐ solve equations with rational exponents. (LT 10)

Warm Up: Solve by factoring. $x^2 = 5x + 14$

A major theme throughout any study of algebra is ______________ _____________. Having learned how to perform operations with ____________ and ______________ exponents, you are now prepared to solve ______________ equations.

Definition 1: **Radical Equation** – An equation that has a ______________ in a _____________ (or a variable with a ______________ exponent).

RADICAL EQUATION OR NOT?!

A) $\sqrt{x} + 3 = 12$ B) $\sqrt{3} + 2x = 10$ C) $x^2 + 64^{1/4} = 20$ D) $x^{0.5} + 4 = 20$

Let's explore how to solve radical equations! $\sqrt[3]{x} = 8$

- When you solve radical equations, ____________ the radical on one side of the equation and then raise both sides to the power equal to the ________ of the radical.

Solving Equations with Roots (LT 9)

Example 1: Solve $-10 + \sqrt{2x+1} = -5$

Solving Equations with Rational Exponents (LT 10)

> Once the rational exponent is isolated, raise each side to the __________ of the exponent.

Example 2: Solve $3(x+1)^{3/5}=24$

Be careful! Remember $x^2=9$ then x = 3 and -3.
Whenever you take the even power root of both sides (like a square root) you need a plus and minus.
When the equation has an even number in numerator , you need a plus and minus.

A) $x^{2/3}=81$

B) $(x+1)^{4/5}=16$

Example 3: Solve each equation.

A) $\sqrt{5x+1}-6=0$

B) $3(x+3)^{3/4}=81$

C) $(x-5)^{2/3}=100$

Unfortunately, whenever you raise both sides of an equation to (an even) power, you may introduce solutions that are not actually solutions of the _____________ equation. These are called ___________________ solutions. It is important to check that your final solutions are solutions of the original equation.

Example 4: Solve $\sqrt{x+2}-3=2x$. Check for extraneous solutions.

Check for extraneous solutions by substituting back into the original equation.

Example 5: Solve $\sqrt{5x-1}+3=x$. Check for extraneous solutions.

Solving Equations with Two Radicals or Rational Exponents (LT 9 and 10)

Some equations will have multiple radicals or terms with rational exponents. In these situations, isolate one radical or term with a rational exponent and attempt to raise both sides to a power that will eliminate all radicals or rational exponents.

Example 6: Solve each equation. Check for extraneous solutions.

A) $\left(2x+1\right)^{0.5}-\left(3x+4\right)^{0.25}=0$

B) $\sqrt{3x+2}-\sqrt{2x+7}=0$

<u>Final Check:</u>

LT 9 and 10: I can solve equations with roots and rational exponents.

10. Solve each equation. Check for extraneous solutions. *Show all work.*

a. $0=-2+\sqrt{6x-1}$

b. $\sqrt{4x-20}-\sqrt{12-2x}=0$

c. $-6+6(26-a)^{4/3}=90$

d. $\sqrt{7a+2}-2=7a$

e. $(4x-3)^{\frac{3}{2}}=8$

f. $(-2-x)^{0.5}-x=2$

LT 9,10 Practice Assignment

- ☐ I can solve equations with roots (or radicals). (LT 9)
- ☐ I can solve equations with rational exponents. (LT 10)
    - o BOOK 7.5 page 394

Solve.

1. $3\sqrt{x} + 3 = 15$

2. $4\sqrt{x} - 1 = 3$

3. $\sqrt{x + 3} = 5$

4. $\sqrt{3x + 4} = 4$

5. $\sqrt{2x + 3} - 7 = 0$

6. $\sqrt{6 - 3x} - 2 = 0$

Solve.

7. $(x + 5)^{\frac{2}{3}} = 4$

8. $(x - 2)^{\frac{2}{3}} = 9$

9. $3(x - 2)^{\frac{3}{4}} = 24$

10. $3(x + 3)^{\frac{3}{4}} = 81$

11. $(x + 1)^{\frac{3}{2}} - 2 = 25$

12. $3 + (4 - x)^{\frac{3}{2}} = 11$

13. Volume A spherical water tank holds 15,000 ft^3 of water. Find the diameter of the tank. (*Hint:* $V = \frac{\pi}{6}d^3$)

14. Hydraulics The maximum flow of water in a pipe is modeled by the formula $Q = Av$, where A is the cross-sectional area of the pipe and v is the velocity of the water. Find the diameter of a pipe that allows a maximum flow of 50 ft^3/min of water flowing at a velocity of 600 ft/min. Round your answer to the nearest inch.

Solve. Check for extraneous solutions.

15. $\sqrt{11x + 3} - 2x = 0$

16. $(5x + 4)^{\frac{1}{2}} - 3x = 0$

17. $\sqrt{3x + 13} - 5 = x$

18. $\sqrt{x + 7} + 5 = x$

19. $(x + 3)^{\frac{1}{2}} - 1 = x$

20. $(5 - x)^{\frac{1}{2}} = x + 1$

Solve. Check for extraneous solutions.

21. $\sqrt{3x} = \sqrt{x + 6}$

22. $(x + 5)^{\frac{1}{2}} - (5 - 2x)^{\frac{1}{4}} = 0$

23. $(7x + 6)^{\frac{1}{2}} = (9 + 4x)^{\frac{1}{2}}$

24. $\sqrt{3x + 7} = x - 1$

25. $\sqrt{x + 7} - x = 1$

26. $\sqrt{-3x - 5} = x + 3$

27. $(3x + 2)^{\frac{1}{2}} - (2x + 7)^{\frac{1}{2}} = 0$

28. $x + 8 = (x^2 + 16)^{\frac{1}{2}}$

29. $(2x)^{\frac{1}{2}} = (x + 5)^{\frac{1}{2}}$

30. $1 = (3 + x)^{\frac{1}{2}}$

Lesson 7-5 pp. 394–396

EXERCISES 1. 16 **3.** 22 **7.** 3, −13 **9.** 18 **13.** 30.6 ft
15. 3 **17.** −3, −4 **21.** 3 **23.** 1 **35.** 4 **37.** 23

LT 9,10 More Practice #1: Worksheet 7.5

Practice 7-5 — Solving Square Root and Other Radical Equations

Solve. Check for extraneous solutions.

1. $(x-2)^{\frac{1}{3}} = 5$

2. $3x^{\frac{4}{3}} + 5 = 53$

3. $4x^{\frac{3}{2}} - 5 = 103$

4. $\sqrt{x+1} = x - 1$

5. $\sqrt{2x+1} = -3$

6. $x^{\frac{1}{2}} - 5 = 0$

7. $\sqrt{x+7} = x - 5$

8. $(2x+1)^{\frac{1}{3}} = -3$

9. $2x^{\frac{1}{3}} - 2 = 0$

10. $\sqrt{2x-5} = 7$

11. $\sqrt{2x-4} = x - 2$

12. $\sqrt{x} + 6 = x$

13. $\sqrt{x+2} = 10 - x$

14. $\sqrt{4x+2} = \sqrt{3x+4}$

15. $(7x-3)^{\frac{1}{2}} = 5$

16. $(x-2)^{\frac{2}{3}} - 4 = 5$

17. $2\sqrt{x-1} = \sqrt{26+x}$

18. $2x^{\frac{3}{4}} = 16$

19. $\sqrt{7x-6} - \sqrt{5x+2} = 0$

20. $\sqrt{3x-3} - 6 = 0$

21. $5\sqrt{x} + 2 = 12$

22. $2x^{\frac{4}{3}} - 2 = 160$

23. $4x^{\frac{1}{2}} - 5 = 27$

24. $\sqrt{x+1} = x + 1$

25. $\sqrt{2x+1} = -5$

26. $x^{\frac{1}{6}} - 2 = 0$

27. $\sqrt{x+2} = x - 18$

28. $(2x+1)^{\frac{1}{3}} = 1$

29. $x^{\frac{1}{4}} + 3 = 0$

30. $\sqrt[3]{2x-4} = -2$

For each equation, let Y1 = left side and Y2 = right side. Find where Y1 = Y2. Use the Technology Activity Steps on page 394 of the text to check that you've found all solutions.

31. $x^{\frac{1}{4}} - 1 = 0$

32. $(x-2)^{\frac{1}{3}} = -5$

33. $x^{\frac{1}{3}} - 2 = 0$

34. $\sqrt{3x} = 6$

35. $(2x+7)^{\frac{1}{2}} - x = 2$

36. $\sqrt{4x} - 8 = 0$

37. $\sqrt{3x+1} - 5 = 0$

38. $3(2x+4)^{\frac{4}{3}} = 48$

39. $2\sqrt{x} = \sqrt{x+6}$

40. $(2x+1)^{\frac{1}{2}} = (5-2x)^{\frac{1}{2}}$

41. $(x+14)^{\frac{1}{4}} = (2x)^{\frac{1}{2}}$

42. $\sqrt[3]{x-2} = 4$

Practice 7-5

1. 127 **2.** $-8, 8$ **3.** 9 **4.** 3 **5.** no solution **6.** 25 **7.** 9
8. -14 **9.** 1 **10.** 27 **11.** 4, 2 **12.** 9 **13.** 7 **14.** 2 **15.** 4
16. $29, -25$ **17.** 10 **18.** 16 **19.** 4 **20.** 13 **21.** 4
22. $-27, 27$ **23.** 64 **24.** $0, -1$ **25.** no solution **26.** 64
27. 23 **28.** 0 **29.** no solution **30.** -2 **31.** 1 **32.** -123
33. 8 **34.** 12 **35.** 1 **36.** 16 **37.** 8 **38.** $-6, 2$ **39.** 2
40. 1 **41.** 2 **42.** 66

LT 9, 10 More Practice #2:

Solve each equation. Check for extraneous solutions. (Don't forget $\pm$ when needed)

1) $\sqrt[3]{x} - 2 = 0$

2) $x^{\frac{3}{2}} = 27$

3) $\sqrt{x} = 8$

4) $x^{\frac{1}{3}} = -2$

5) $\sqrt[3]{x+1} = 3$

6) $(2x)^{\frac{3}{4}} = 8$

7) $2x^{\frac{5}{3}} = -64$

8) $x^{\frac{1}{3}} - 6 = 8$

9) $\sqrt[3]{15 - 7x} + 15 = 17$

10) $x^{\frac{2}{3}} + 13 = 17$

11) $\sqrt{3x-8}+1=3$

12) $\sqrt[3]{2x+3}-\sqrt[3]{x-1}=0$

13) $\sqrt{2x-5}-\sqrt{x}=0$

13) $\sqrt{3X-8}+1=3$

14) $\sqrt[4]{x}-3=0$

15) $8\sqrt[3]{x}-24=0$

16) $3(x-1)^{\frac{2}{5}}-12=0$

17) $\sqrt{4X+8}-3=1$

Graphing Radical Functions

Date: ____________
Quiz On: ____________

After this lesson and practice, I will be able to …

- ☐ graph radical expressions and identify domain and range of radical expressions. (LT 11)

Group Activity: Remembering Transformations

Earlier this year, you learned how different operations performed on a function cause the graph to _____________ (to shift up/down/left/right, reflect, or dilate). Let's see what you remember!

Warm Up: Describe the transformations applied to the parent quadratic function.

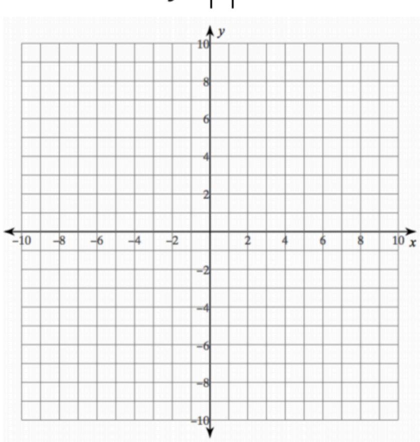
1. $y=(x-3)^2$
2. $y=x^2+6$
3. $y=3x^2$
4. $y=-x^2+4$

In this final lesson in this unit, you will learn how to apply these transformations to radical functions.

Type of Transformation	**Transformation Equation**	**Change to Parent Graph**	**Examples**
Translation (addition and subtraction)		Translates graph *c* units ____ Translates graph *c* units ____ Translates graph *c* units ____ Translates graph *c* units ____	
Reflection (negative signs)		Reflects graph across the ______________	
Dilation (multiplication)		__Vertical__: Multiplies y-coordinates by *a*.	

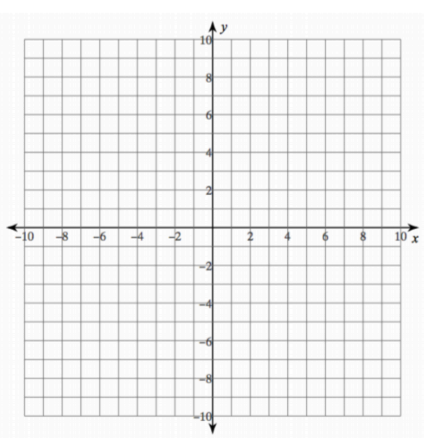
As part of this lesson, you will apply transformations to the three primary parent functions you have learned this year. ***<u>You will want to memorize these functions and their graphs</u>***.

$y=|x|$



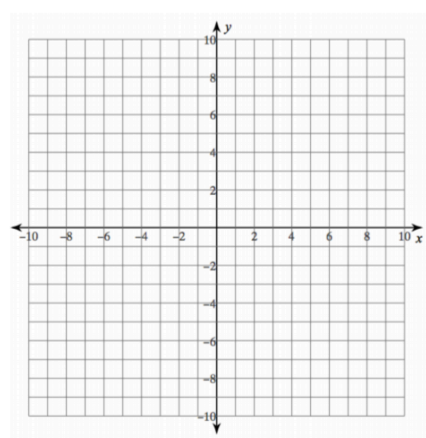
x	0	-1	1	-2	2
y					

$y=x^2$



x	0	-1	1	-2	2
y					

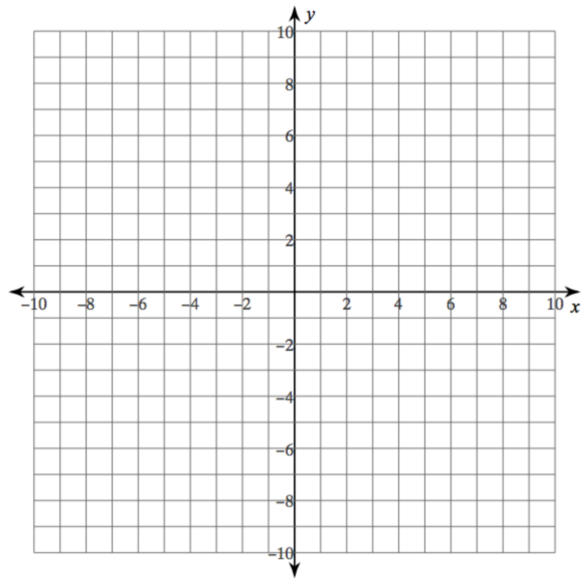
$y=\sqrt{x}$



x	0	1	4	9
y				

Graphing Transformations

<u>Example 1</u>: Graph the parent <u>quadratic</u> function with a dashed line. Then, using solid lines, graph and label each equation. Show at least the transformations of the “five key points.”



$g(x) = x^2 + 2$

$h(x) = (x-5)^2$

$k(x) = -2(x+1)^2$

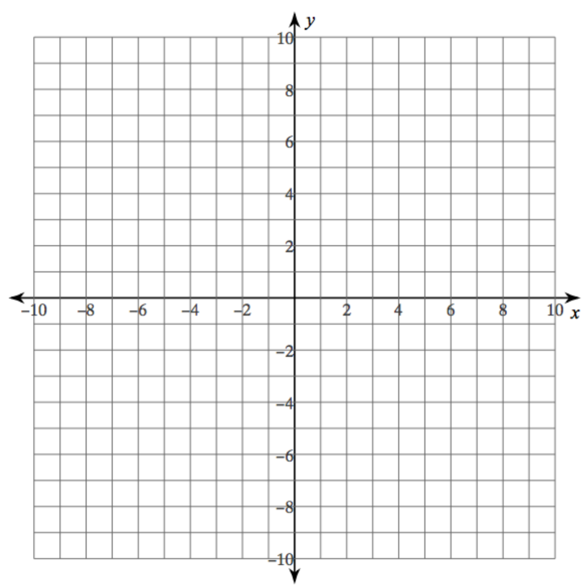
What is the domain and range of each function?

Domain of g: ___________ Range of g: __________

Domain of h: ___________ Range of h: __________

Domain of k: ___________ Range of k: __________

<u>Example 2</u>: Graph the parent <u>square root</u> function with a dashed line. Then, using solid lines, graph and label each equation. Show at least the transformations of the “four key points.”



$g(x) = \sqrt{x-3}$

$h(x) = \sqrt{x} - 5$

$k(x) = -3\sqrt{x+2}$

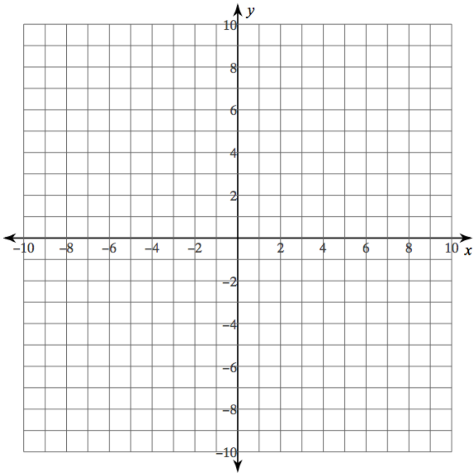
What is the domain and range of each function?

Domain of g: ___________ Range of g: ___________

Domain of h: ___________ Range of h: ___________

Domain of k: ___________ Range of k: ___________

Finally, let's explore one final parent function: the _________ ___________ function!



x	0				
y					

Domain: ____________________

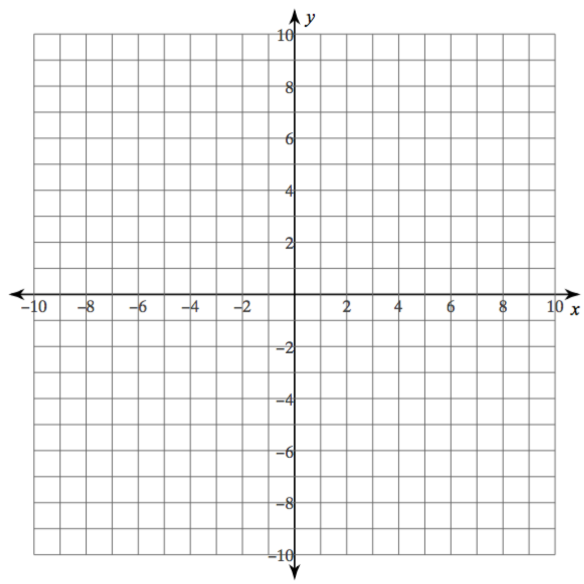
Range: ______________________

Example 3: Graph the parent cube root function with a dashed line. Then, using solid lines, graph and label each equation. Show at least the transformations of the "five key points."

$g(x)=\sqrt[3]{x}+4$

$h(x)=-\sqrt[3]{x}$

$k(x)=3\sqrt[3]{x-2}$



What is the domain and range of each function?

Domain of g: ___________ Range of g: __________

Domain of h: ___________ Range of h: __________

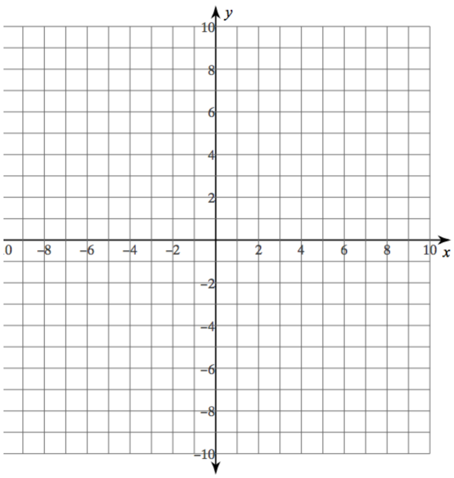
Domain of k: ___________ Range of k: __________

Final Check (NO Calculator)

LT 11: I can graph radical expressions and identify domain and range of radical expressions.

1. For each equation, sketch the graph of the equation with a solid line. The graph of the transformation equation must include visible, plotted “key points.” Then state the domain and range of each equation.

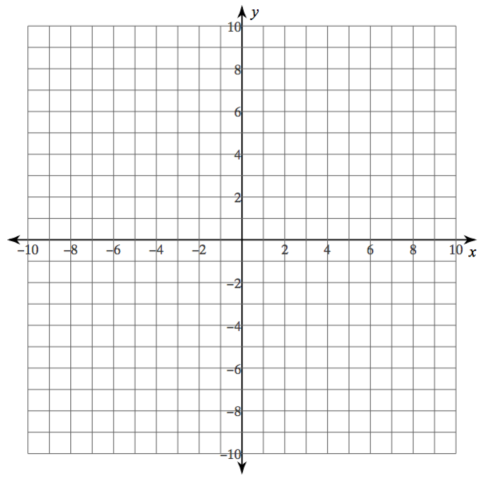
a. $y = |x+4| - 1$



Domain: ______________

Range: ______________

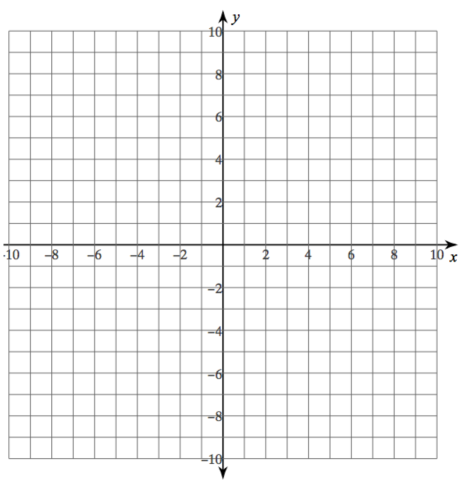
b. $y = -(x+1)^2 + 4$



Domain: ______________

Range: ______________

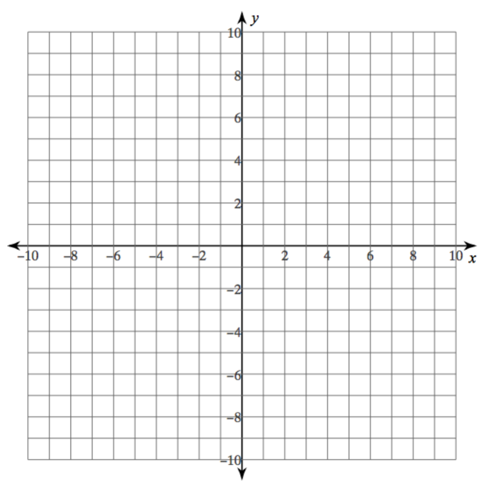
c. $y = 2\sqrt{x+5} - 1$



Domain: ______________

Range: ______________

d. $y = -\sqrt[3]{x-7}$



Domain: ______________

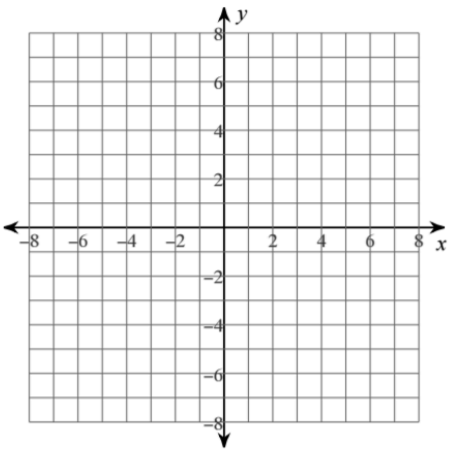
Range: ______________

LT 11 Practice Assignment

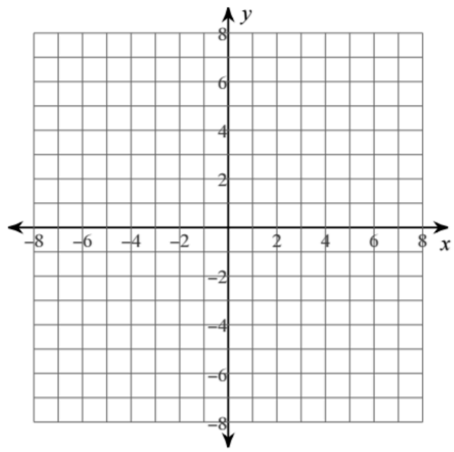
- ☐ I can graph radical expressions and identify domain and range of radical expressions. (LT11)
  - Worksheet, (next page)
    - NOTE: State the domain and range for each problem as well!

Sketch the graph of each function (sketch the transformations of at least the "key points" for each function). Then identify the domain and range of each. HINT: You may want to start by sketching the parent function with a dashed line before graphing each transformation.

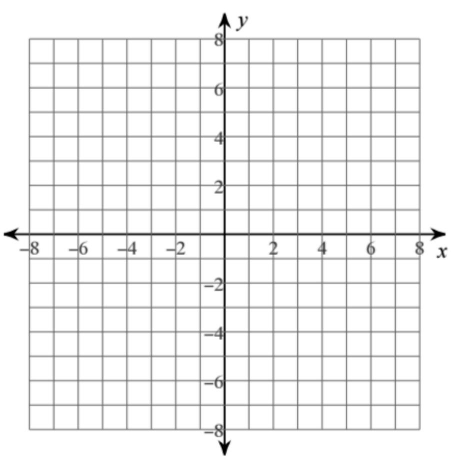
1) $y = \sqrt{x - 3}$



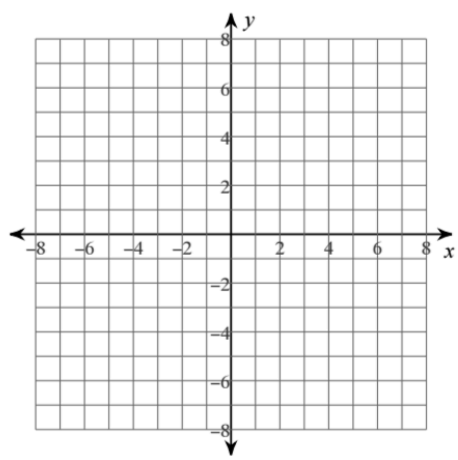
2) $y = \sqrt{x + 3} - 3$



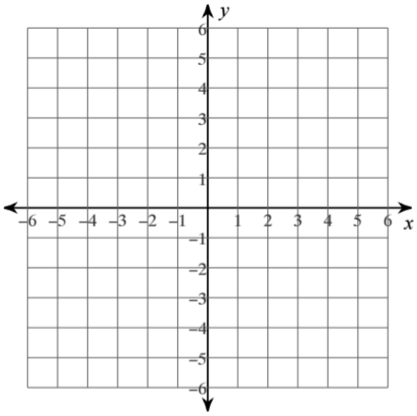
3) $y = 4\sqrt{x - 3}$



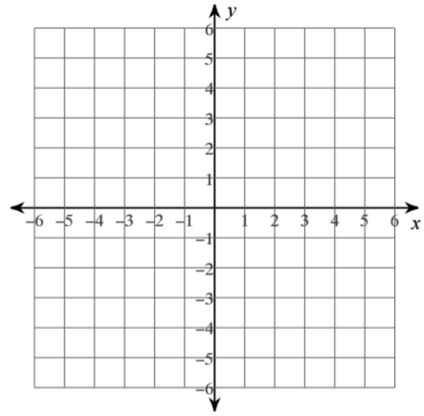
4) $y = -2\sqrt{x + 1} + 3$



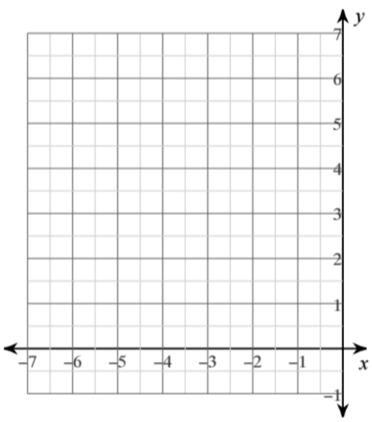
5) $y = |x + 2| - 1$



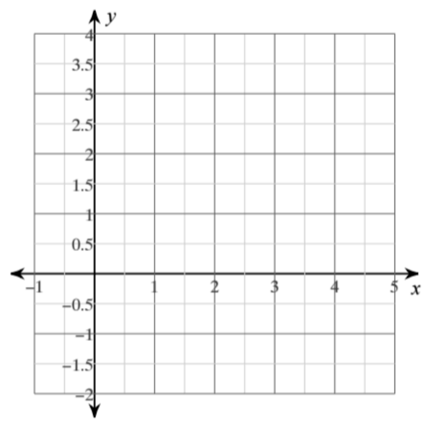
6) $y = -3|x| + 2$



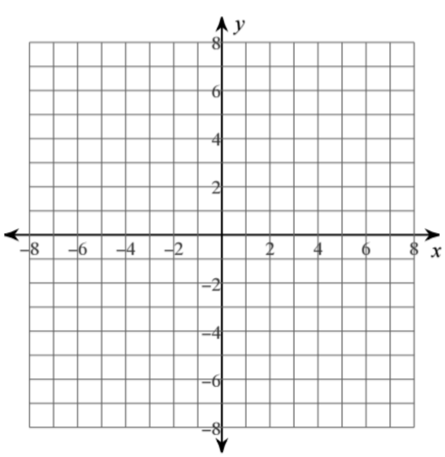
7) $y = (x + 4)^2 + 1$



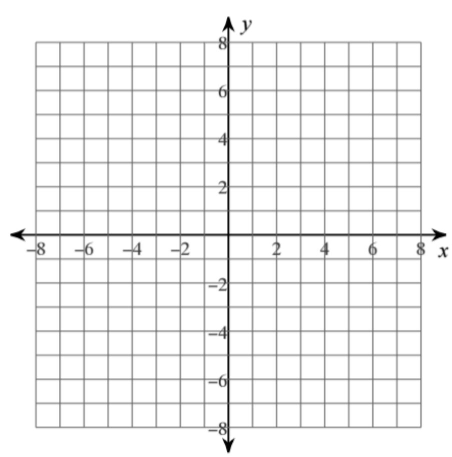
8) $y = -(x - 2)^2 + 3$



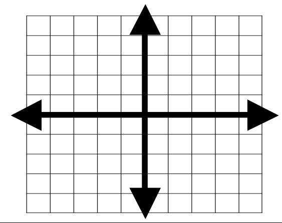
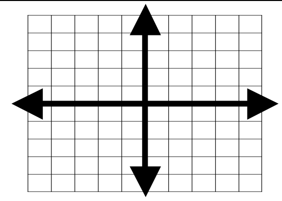
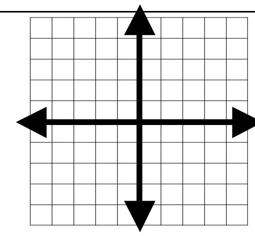
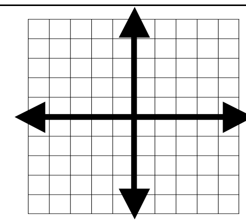
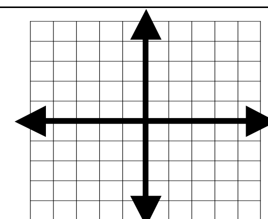
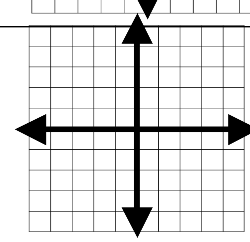
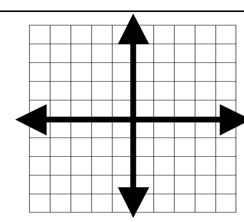
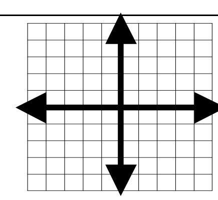
9) $y = \sqrt[3]{x} + 4$



10) $y = 2 + 3\sqrt[3]{x - 3}$



LT 11 More Practice #1 Use transformations to graph each function (no calculator). State Domain & Range.

Equation	Description of Transformation & Domain and Range	Graph
1a) $y = x^2 + 1$		
b) $y = \lvert x + 3 \rvert$		
c) $y = \sqrt{x - 2} - 1$		
d) $y = -(x - 4)^2$		
e) $y = (x + 3)^2 - 1$		
f) $y = \sqrt[3]{x} + 3$		
g) $y = -\sqrt{x} - 3$		
h) $y = -(x + 2)^2 + 1$		

LT 11 More Practice #2 7.8

Practice 7-8 *(modified)* Graphing Square Root and Other Radical Functions

Use transformations to graph each function without a calculator. State Domain and Range.

1. $y = -\sqrt{x+2}$
2. $y = \sqrt{x-3}$
3. $y = 3\sqrt{x} + 1$
4. $y = -\sqrt{x} - 1$
5. $y = \sqrt{x-4} + 2$
6. $y = 2\sqrt{x+1} - 3$
7. $y = \sqrt{x+2} - 6$
8. $y = -\sqrt{x-2} + 3$
9. $y = -2\sqrt{x-3} + 3$
10. $y = \sqrt{x+3} - 2$
11. $y = \sqrt{x-1} - 5$
12. $y = -5\sqrt{x-2} + 5$
13. $y = -\sqrt{x+1} - 4$
14. $y = -\sqrt{x-1} + 2$
15. $y = -4\sqrt{x-1} + 3$
16. $y = \sqrt{x-2} + 1$
17. $y = \sqrt{x+2} - 2$
18. $y = -2\sqrt{x-1} + 2$
19. $y = \sqrt{x+1} + 4$
20. $y = \sqrt{x-3} + 3$
21. $y = 3\sqrt{x+1} - 2$
22. $y = \sqrt{x-1} - 1$
23. $y = \sqrt{x+3} - 3$
24. $y = 4\sqrt{x+4} - 1$
25. $y = \sqrt{x-2} - 4$
26. $y = \sqrt{x+2} + 1$
27. $y = -3\sqrt{x-2} + 3$
28. $y = |x+2| - 1$
29. $y = -2|x-3|$
30. $y = 3|x-2| - 4$
31. $y = -|x| + 6$
32. $y = 5|x-1| - 2$
33. $y = |x+4| - 7$
34. $y = (x-1)^2 + 4$
35. $y = 3(x+1)^2 - 5$
36. $y = -2(x-1)^2 + 4$
37. $y = 2(x-3)^2 - 4$
38. $y = (x-3)^2 - 8$
39. $y = \sqrt[3]{x-1}$
40. $y = \sqrt[3]{x+2} - 3$
41. $y = 4\sqrt[3]{x+1} - 2$
42. $y = -\sqrt[3]{x} + 2$
43. $y = 2\sqrt[3]{x-3}$
44. $y = -2\sqrt[3]{x+3} - 1$

LT 11 More Practice #3:

Use transformations to graph each function without a calculator. State Domain and Range.

Lesson 7-8 **Graph each function.**

50. $y = \sqrt{x}$

51. $y = \sqrt{x} - 1$

52. $y = \sqrt{x} + 3$

53. $y = \sqrt{x+3}$

54. $y = 4\sqrt{x}$

55. $y = \frac{3}{4}\sqrt{x}$

56. $y = 2\sqrt{x-5} + 2$

57. $y = \sqrt[3]{x+1}$

58. $y = \sqrt[3]{x-2} - 3$

LT 11 More Practice #4 7-8 Book Page 417

Use transformations to graph each function without a calculator. State Domain and Range.

Graph each function.

1. $y = \sqrt{x} + 1$ **2.** $y = \sqrt{x} - 2$ **3.** $y = \sqrt{x} - 4$ **4.** $y = \sqrt{x} + 5$

5. $y = \sqrt{x - 3}$ **6.** $y = \sqrt{x + 1}$ **7.** $y = \sqrt{x + 6}$ **8.** $y = \sqrt{x - 4}$

Graph each function.

9. $y = 3\sqrt{x}$ **10.** $y = -0.25\sqrt{x}$ **11.** $y = \frac{1}{3}\sqrt{x}$

12. $y = -\sqrt{x - 1}$ **13.** $y = -5\sqrt{x + 2}$ **14.** $y = -0.75\sqrt{x} + 3$

15. $y = -\sqrt{x - 3} + 2$ **16.** $y = \frac{1}{4}\sqrt{x + 2} - 1$ **17.** $y = 3\sqrt{x + 1} + 4$

Graph each function.

18. $y = \sqrt[3]{x + 5}$ **19.** $y = \sqrt[3]{x - 4}$ **20.** $y = \sqrt[3]{x + 2} - 7$

21. $y = -\sqrt[3]{x + 3} - 1$ **22.** $y = 2\sqrt[3]{x - 6} - 9$ **23.** $y = \frac{1}{2}\sqrt[3]{x - 1} + 3$