Multivariable Calculus Summer Assignment

Hello and welcome to Multivariable Calculus, aka “Calc 3” at most research universities. In essence, Calc 3 is just Calc 1 with more than one independent variable. In other words, the course will revisit topics from Calc 1 & 2 (BC Calc is both) but in more dimensions than before. Please keep that in mind throughout the course. If you are ever confused on a topic, just try to think of how it worked with one variable.

(With this in mind, please note we will NOT have to talk about convergent or divergent series at any time in this course! As important as they are for making approximations in the realms of Physics and Engineering, we have no need to consider series of multiple variables. If you are interested in this topic, I can give you a Directed Reading project for it.)*

You will receive a copy of, in my biased opinion, the best calculus book ever: *Stewart’s Calculus 5th Edition*. Please handle this book with care. I love the 5th edition; I would hate to have to replace it with a newer edition.

Please complete this summer assignment and put in my mailbox in the main office by Thursday 8/29/24 at noon.

-Mr. White
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*Unless we get to Chapter 18 in the book, which isn’t technically part of this course. Then there might be some mention of series.

Section 1: Review of differential calculus

The following derivative problems are meant to remind you of the basic properties of derivatives.

Please hand in:
Chapter 3 Review Exercises: #1, 2, 3, 5, 9, 10, 13, 15, 16, 23, 40, 51, 56, 68, 69, 71, 87.
Chapter 4 Review Exercises: #1, 8, 9.

Chapter 3 Review Exercises

1–48 Calculate y'.

1. $y = (x^4 - 3x^2 + 5)^3$

2. $y = \cos(\tan x)$

3. $y = \sqrt{x} + \dfrac{1}{\sqrt[3]{x^4}}$

4. $y = \dfrac{3x - 2}{\sqrt{2x + 1}}$

5. $y = 2x\sqrt{x^2 + 1}$

6. $y = \dfrac{e^x}{1 + x^2}$

7. $y = e^{\sin 2\theta}$

8. $y = e^{-t}(t^2 - 2t + 2)$

9. $y = \dfrac{t}{1 - t^2}$

10. $y = \sin^{-1}(e^x)$

11. $y = xe^{-1/x}$

12. $y = x^r e^{sx}$

13. $y = \tan\sqrt{1 - x}$

14. $y = \dfrac{1}{\sin(x - \sin x)}$

15. $xy^4 + x^2y = x + 3y$

16. $y = \ln(\csc 5x)$

17. $y = \dfrac{\sec 2\theta}{1 + \tan 2\theta}$

18. $x^2 \cos y + \sin 2y = xy$

19. $y = e^{cx}(c \sin x - \cos x)$

20. $y = \ln(x^2 e^x)$

21. $y = e^{e^x}$

22. $y = \sec(1 + x^2)$

23. $y = (1 - x^{-1})^{-1}$

24. $y = 1/\sqrt[3]{x + \sqrt{x}}$

25. $\sin(xy) = x^2 - y$

26. $y = \sqrt{\sin\sqrt{x}}$

27. $y = \log_5(1 + 2x)$

28. $y = (\cos x)^x$

29. $y = \ln \sin x - \frac{1}{2}\sin^2 x$

30. $y = \dfrac{(x^2 + 1)^4}{(2x + 1)^3(3x - 1)^5}$

31. $y = x \tan^{-1}(4x)$

32. $y = e^{\cos x} + \cos(e^x)$

33. $y = \ln|\sec 5x + \tan 5x|$

34. $y = 10^{\tan \pi\theta}$

35. $y = \cot(3x^2 + 5)$

36. $y = \sqrt{t \ln(t^4)}$

37. $y = \sin\left(\tan\sqrt{1 + x^3}\right)$

38. $y = \arctan\left(\arcsin\sqrt{x}\right)$

39. $y = \tan^2(\sin\theta)$

40. $xe^y = y - 1$

41. $y = \dfrac{\sqrt{x + 1}\,(2 - x)^5}{(x + 3)^7}$

42. $y = \dfrac{(x + \lambda)^4}{x^4 + \lambda^4}$

43. $y = x \sinh(x^2)$

44. $y = \dfrac{\sin mx}{x}$

45. $y = \ln(\cosh 3x)$

46. $y = \ln\left|\dfrac{x^2 - 4}{2x + 5}\right|$

47. $y = \cosh^{-1}(\sinh x)$

48. $y = x \tanh^{-1}\sqrt{x}$

49. If $f(t) = \sqrt{4t + 1}$, find $f''(2)$.

50. If $g(\theta) = \theta \sin\theta$, find $g''(\pi/6)$.

51. Find y'' if $x^6 + y^6 = 1$.

52. Find $f^{(n)}(x)$ if $f(x) = 1/(2 - x)$.

53. Use mathematical induction to show that if $f(x) = xe^x$, then $f^{(n)}(x) = (x + n)e^x$.

54. Evaluate $\displaystyle\lim_{t\to 0} \frac{t^3}{\tan^3(2t)}$.

55–59 Find an equation of the tangent to the curve at the given point.

55. $y = 4\sin^2 x$, $(\pi/6, 1)$

56. $y = \dfrac{x^2 - 1}{x^2 + 1}$, $(0, -1)$

57. $y = \sqrt{1 + 4\sin x}$, $(0, 1)$

58. $x^2 + 4xy + y^2 = 13$, $(2, 1)$

59. $y = (2 + x)e^{-x}$, $(0, 2)$

60. If $f(x) = xe^{\sin x}$, find $f'(x)$. Graph f and f' on the same screen and comment.

61. (a) If $f(x) = x\sqrt{5 - x}$, find $f'(x)$.
(b) Find equations of the tangent lines to the curve $y = x\sqrt{5 - x}$ at the points $(1, 2)$ and $(4, 4)$.
(c) Illustrate part (b) by graphing the curve and tangent lines on the same screen.
(d) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f'.

62. (a) If $f(x) = 4x - \tan x$, $-\pi/2 < x < \pi/2$, find f' and f''.
(b) Check to see that your answers to part (a) are reasonable by comparing the graphs of f, f', and f''.

63. At what points on the curve $y = \sin x + \cos x$, $0 \leqslant x \leqslant 2\pi$, is the tangent line horizontal?

64. Find the points on the ellipse $x^2 + 2y^2 = 1$ where the tangent line has slope 1.

65. If $f(x) = (x - a)(x - b)(x - c)$, show that

$$\frac{f'(x)}{f(x)} = \frac{1}{x-a} + \frac{1}{x-b} + \frac{1}{x-c}$$

66. (a) By differentiating the double-angle formula

$$\cos 2x = \cos^2 x - \sin^2 x$$

obtain the double-angle formula for the sine function.

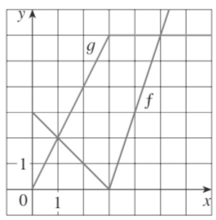
(b) By differentiating the addition formula

$$\sin(x + a) = \sin x \cos a + \cos x \sin a$$

obtain the addition formula for the cosine function.

67. Suppose that $h(x) = f(x)g(x)$ and $F(x) = f(g(x))$, where $f(2) = 3$, $g(2) = 5$, $g'(2) = 4$, $f'(2) = -2$, and $f'(5) = 11$. Find (a) $h'(2)$ and (b) $F'(2)$.

68. If f and g are the functions whose graphs are shown, let $P(x) = f(x)g(x)$, $Q(x) = f(x)/g(x)$, and $C(x) = f(g(x))$. Find (a) $P'(2)$, (b) $Q'(2)$, and (c) $C'(2)$.



69–76 ▥ Find f' in terms of g'.

69. $f(x) = x^2 g(x)$

70. $f(x) = g(x^2)$

71. $f(x) = [g(x)]^2$

72. $f(x) = g(g(x))$

73. $f(x) = g(e^x)$

74. $f(x) = e^{g(x)}$

75. $f(x) = \ln |g(x)|$

76. $f(x) = g(\ln x)$

77–79 ▥ Find h' in terms of f' and g'.

77. $h(x) = \dfrac{f(x)g(x)}{f(x) + g(x)}$

78. $h(x) = \sqrt{\dfrac{f(x)}{g(x)}}$

79. $h(x) = f(g(\sin 4x))$

81. At what point on the curve $y = [\ln(x + 4)]^2$ is the tangent horizontal?

82. (a) Find an equation of the tangent to the curve $y = e^x$ that is parallel to the line $x - 4y = 1$.

(b) Find an equation of the tangent to the curve $y = e^x$ that passes through the origin.

83. Find a parabola $y = ax^2 + bx + c$ that passes through the point $(1, 4)$ and whose tangent lines at $x = -1$ and $x = 5$ have slopes 6 and -2, respectively.

84. The function $C(t) = K(e^{-at} - e^{-bt})$, where a, b, and K are positive constants and $b > a$, is used to model the concentration at time t of a drug injected into the bloodstream.

(a) Show that $\lim_{t\to\infty} C(t) = 0$.

(b) Find $C'(t)$, the rate at which the drug is cleared from circulation.

(c) When is this rate equal to 0?

85. An equation of motion of the form $s = Ae^{-ct}\cos(\omega t + \delta)$ represents damped oscillation of an object. Find the velocity and acceleration of the object.

86. A particle moves along a horizontal line so that its coordinate at time t is $x = \sqrt{b^2 + c^2t^2}$, $t \geqslant 0$, where b and c are positive constants.

(a) Find the velocity and acceleration functions.

(b) Show that the particle always moves in the positive direction.

87. A particle moves on a vertical line so that its coordinate at time t is $y = t^3 - 12t + 3$, $t \geqslant 0$.

(a) Find the velocity and acceleration functions.

(b) When is the particle moving upward and when is it moving downward?

(c) Find the distance that the particle travels in the time interval $0 \leqslant t \leqslant 3$.

88. The volume of a right circular cone is $V = \pi r^2 h/3$, where r is the radius of the base and h is the height.

(a) Find the rate of change of the volume with respect to the height if the radius is constant.

(b) Find the rate of change of the volume with respect to the radius if the height is constant.

89. The mass of part of a wire is $x(1 + \sqrt{x})$ kilograms, where x is measured in meters from one end of the wire. Find the linear density of the wire when $x = 4$ m.

90. The cost, in dollars, of producing x units of a certain commodity is

Chapter 4 Review Exercises

1–6 Find the local and absolute extreme values of the function on the given interval.

1. $f(x) = 10 + 27x - x^3, \quad [0, 4]$

2. $f(x) = x - \sqrt{x}, \quad [0, 4]$

3. $f(x) = \dfrac{x}{x^2 + x + 1}, \quad [-2, 0]$

4. $f(x) = (x^2 + 2x)^3, \quad [-2, 1]$

5. $f(x) = x + \sin 2x, \quad [0, \pi]$

6. $f(x) = (\ln x)/x^2, \quad [1, 3]$

7–14 Evaluate the limit.

7. $\displaystyle\lim_{x \to 0} \frac{\tan \pi x}{\ln(1 + x)}$

8. $\displaystyle\lim_{x \to 0} \frac{1 - \cos x}{x^2 + x}$

9. $\displaystyle\lim_{x \to 0} \frac{e^{4x} - 1 - 4x}{x^2}$

10. $\displaystyle\lim_{x \to \infty} \frac{e^{4x} - 1 - 4x}{x^2}$

11. $\displaystyle\lim_{x \to \infty} x^3 e^{-x}$

12. $\displaystyle\lim_{x \to 0^+} x^2 \ln x$

13. $\displaystyle\lim_{x \to 1^+} \left(\frac{x}{x - 1} - \frac{1}{\ln x} \right)$

Section 2: Review of integral calculus

The following integrals are meant to remind you of the basic properties of integrals. No calculator for these (except to check your work).

Please hand in:

Chapter 5 Review Exercises: #9, 12, 15, 16, 21, 24, 26, 27, 28.

Chapter 6 Review Exercises: #3, 6, 7, 24.

No calculator for any of these problems!

Chapter 5 Review Exercises

9–38 Evaluate the integral, if it exists.

9. $\int_1^2 (8x^3 + 3x^2)\,dx$

10. $\int_0^T (x^4 - 8x + 7)\,dx$

11. $\int_0^1 (1 - x^9)\,dx$

12. $\int_0^1 (1 - x)^9\,dx$

13. $\int_1^9 \frac{\sqrt{u} - 2u^2}{u}\,du$

14. $\int_0^1 (\sqrt[4]{u} + 1)^2\,du$

15. $\int_0^1 y(y^2 + 1)^5\,dy$

16. $\int_0^2 y^2\sqrt{1 + y^3}\,dy$

17. $\int_1^5 \frac{dt}{(t - 4)^2}$

18. $\int_0^1 \sin(3\pi t)\,dt$

19. $\int_0^1 v^2\cos(v^3)\,dv$

20. $\int_{-1}^1 \frac{\sin x}{1 + x^2}\,dx$

21. $\int_0^1 e^{\pi t}\,dt$

22. $\int_1^2 \frac{1}{2 - 3x}\,dx$

23. $\int_2^4 \frac{1 + x - x^2}{x^2}\,dx$

24. $\int_1^{10} \frac{x}{x^2 - 4}\,dx$

25. $\int \frac{x + 2}{\sqrt{x^2 + 4x}}\,dx$

26. $\int \csc^2 3t\,dt$

27. $\int x^2 \sin x\,dx$

28. $\int x^3 \ln x\,dx$

Chapter 6 Review Exercises

1–6 Find the area of the region bounded by the given curves.

1. $y = x^2 - x - 6, \quad y = 0$

2. $y = 20 - x^2, \quad y = x^2 - 12$

3. $y = e^x - 1, \quad y = x^2 - x, \quad x = 1$

4. $x + y = 0, \quad x = y^2 + 3y$

5. $y = \sin(\pi x/2), \quad y = x^2 - 2x$

6. $y = \sqrt{x}, \quad y = x^2, \quad x = 2$

7–11 Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

7. $y = 2x, \; y = x^2$; about the x-axis

8. $x = 1 + y^2, \; y = x - 3$; about the y-axis

9. $x = 0, \; x = 9 - y^2$; about $x = -1$

10. $y = x^2 + 1, \; y = 9 - x^2$; about $y = -1$

11. $x^2 - y^2 = a^2, \; x = a + h$ (where $a > 0, h > 0$); about the y-axis

12–14 Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

12. $y = \cos x, \; y = 0, \; x = 3\pi/2, \; x = 5\pi/2$; about the y-axis

13. $y = x^3, \; y = x^2$; about $y = 1$

14. $y = x^3, \; y = 8, \; x = 0$; about $x = 2$

the base are isosceles right triangles with hypotenuse lying along the base.

24. The base of a solid is the region bounded by the parabolas $y = x^2$ and $y = 2 - x^2$. Find the volume of the solid if the cross-sections perpendicular to the x-axis are squares with one side lying along the base.

25. The height of a monument is 20 m. A horizontal cross-section at a distance x meters from the top is an equilateral triangle with side $x/4$ meters. Find the volume of the monument.

26. (a) The base of a solid is a square with vertices located at $(1, 0)$, $(0, 1)$, $(-1, 0)$, and $(0, -1)$. Each cross-section perpendicular to the x-axis is a semicircle. Find the volume of the solid.

(b) Show that by cutting the solid of part (a), we can rearrange it to form a cone. Thus compute its volume more simply.

27. A force of 30 N is required to maintain a spring stretched from its natural length of 12 cm to a length of 15 cm. How much work is done in stretching the spring from 12 cm to 20 cm?

28. A 1600-lb elevator is suspended by a 200-ft cable that weighs 10 lb/ft. How much work is required to raise the elevator from the basement to the third floor, a distance of 30 ft?