

Lesson 9-8

Monday, February 3, 2020 2:21 PM

Name _____



Solve & Share

What do you notice about the calculations below? Make a generalization about what you notice. Complete the remaining examples.

Sue's Equations

$$4 \div \frac{1}{3} = 12$$

$$8 \div \frac{1}{10} = 80$$

$$5 \div \frac{1}{4} = 20$$

$$12 \div \frac{1}{2} = \underline{\quad}$$

$$6 \div \frac{1}{100} = \underline{\quad}$$

Randy's Equations

$$\frac{1}{3} \div 4 = \frac{1}{12}$$

$$\frac{1}{10} \div 8 = \frac{1}{80}$$

$$\frac{1}{4} \div 5 = \frac{1}{20}$$

$$\frac{1}{2} \div 12 = \underline{\quad}$$

$$\frac{1}{100} \div 6 = \underline{\quad}$$

Generalization for Sue's Equations:
 Dividing a whole number by a unit fraction is the same as multiplying the whole number by the denominator. Ex. $12 \div \frac{1}{2} = 24$ $12 \times 2 = 24$

Generalization for Randy's Equations:
 Dividing a unit fraction by a whole number is the same as multiplying the unit fraction by the whole number written as a unit fraction. Ex. $\frac{1}{2} \div 12 = \frac{1}{24}$ $\frac{1}{2} \times \frac{1}{12} = \frac{1}{24}$

Math Practices and Problem Solving

Lesson 9-8

Repeated Reasoning

I can ...

notice repetition in calculations and describe a general method for dividing whole numbers and unit fractions.

Mathematical Practices MP.2, MP.3, MP.4, MP.6
MP.8 Also MP.8
Content Standards 5.NF.B.7a, 5.NF.B.7b, 5.NF.B.7c

Thinking Habits

Be a good thinker! These questions can help you.

- Are any calculations repeated?
- Can I generalize from examples?
- What shortcuts do I notice?



Look Back! **MP.8 Generalize** Test your general method by writing another pair of equations like Sue's and Randy's equations.

Sue

$$10 \div \frac{1}{32} = 320$$

$$10 \times 32 = 320$$

Randy

$$\frac{1}{5} \div 20 = \frac{1}{100}$$

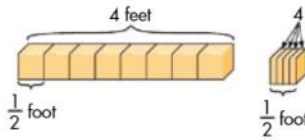
$$\frac{1}{5} \times \frac{1}{20} = \frac{1}{100}$$

How Do You Use Repeated Reasoning When Dividing Whole Numbers and Unit Fractions?

A

Ali partitioned a 4-foot board into $\frac{1}{2}$ -foot pieces. She counted 8 pieces.

Then she partitioned a $\frac{1}{2}$ -foot board into 4 equal pieces. Each piece was $\frac{1}{8}$ of a foot.



Study the equations below. What generalizations can you make? Explain.

$$4 \div \frac{1}{2} = 8 \quad \frac{1}{2} \div 4 = \frac{1}{8}$$

$$4 \times 2 = 8 \quad \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

What do I need to do?

I need to understand the equations and make generalizations about them.

B

How can I make a generalization from repeated reasoning?

I can

- look for things that repeat in a problem.
- test whether my generalization works for other numbers.

C

I see that

$$4 \div \frac{1}{2} = 4 \times 2 \text{ and } \frac{1}{2} \div 4 = \frac{1}{2} \times \frac{1}{4}$$

Check if the same relationship applies to other numbers.

$$10 \div \frac{1}{3} = 30 \text{ and } 10 \times 3 = 30$$

$$\frac{1}{3} \div 10 = \frac{1}{30} \text{ and } \frac{1}{3} \times \frac{1}{10} = \frac{1}{30}$$

Dividing a whole number by a unit fraction is the same as multiplying a whole number by the denominator of the unit fraction.

Dividing a unit fraction by a whole number other than zero is the same as multiplying the unit fraction by a unit fraction with the whole number as the denominator.

Here's my thinking...



Convince Me! **MP.8 Generalize** Marcus made the following generalization: $12 \div \frac{1}{5} = \frac{1}{12} \times \frac{1}{5}$. Is he correct? Explain.

No. In the generalizations the dividend stays the same.

$$12 \div \frac{1}{5} = 12 \times 5$$

★ Guided Practice

MP.8 Repeated Reasoning

Nathan has two 8-foot boards. He cuts one board into $\frac{1}{4}$ -foot pieces. He cuts the other board into $\frac{1}{2}$ -foot pieces.

- Write and solve a division equation to find how many $\frac{1}{4}$ -ft pieces can be cut from an 8-foot board. Explain your reasoning.

$$8 \div \frac{1}{4} = 32 \quad 8 \times 4 = 32$$

A whole number divided by a unit fraction is the same as multiplying the whole number by the denominator.

- Find how many $\frac{1}{2}$ -ft pieces can be cut from the 8-foot board. Can you repeat the method you used in Exercise 1 to solve this problem? Explain.

$$8 \div \frac{1}{2} = 16 \quad 8 \times 2 = 16$$

Use the same generalization.

Repeated reasoning can help you find a general method for solving problems that are the same type.



Answer 3 + 4

↓
Specific generalization

★ Independent Practice

MP.8 Repeated Reasoning

A landscaper's truck is filled with $\frac{1}{2}$ ton of gravel. The gravel is shared equally among 3 projects.

- Write and solve a division equation to find how much gravel each project will get. Explain your reasoning.

$$\frac{1}{2} \div 3 = \frac{1}{6} \quad \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

Dividing a unit fraction by a whole number is the same as multiplying the unit fraction by the whole number written as a unit fraction.

- Suppose another truck is filled with $\frac{1}{2}$ ton of gravel. Find how much gravel each project will get if the $\frac{1}{2}$ ton of gravel is shared equally among 8 projects. Can you repeat the method you used in Exercise 3 to solve this problem? Explain.

$$\frac{1}{2} \div 8 = \frac{1}{16} \quad \frac{1}{2} \times \frac{1}{8} = \frac{1}{16}$$

Remember, the method for dividing a whole number by a unit fraction is different from the method for dividing a unit fraction by a whole number.



*For another example, see Set F on page 578.

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Used the same generalization from Question 3.

Math Practices and Problem Solving

Common Core Performance Assessment

Pet Food

Karl has a cat and a dog. He buys one bag of cat food and one bag of dog food. How many $\frac{1}{4}$ -lb servings of cat food can he get from one bag? How many $\frac{1}{2}$ -lb servings of dog food can he get from one bag?

Pet Food	Bag Size
Fish	5 lb
Cat	12 lb
Dog	20 lb

5. **MP.3 Critique Reasoning** Karl thinks that he will be able to get more servings of dog food than cat food because the bag of dog food weighs more than the bag of cat food. Do you agree with his reasoning? Explain.

6. **MP.4 Model with Math** Write a division and a multiplication equation that Karl could use to find the number of servings of cat food in one bag.

7. **MP.8 Generalize** What generalization can you make that relates the division equation to the multiplication equation you wrote in Exercise 6?

8. **MP.8 Repeated Reasoning** Find how many servings of dog food are in one bag. Can you repeat the method you used in Exercise 6 to solve this problem? Explain.

When you use repeated reasoning, you notice repetition in calculations.

