Bergenfield High School
Bergenfield, New Jersey

Mathematics Department

Summer Course Work

in preparation for

Calculus Honors

Completion of this summer work
is required on the first day of the
2025-2026 school year.

Name: ______________________________________

Bergenfield Public Schools
Mathematics Department
80 South Prospect Avenue
Bergenfield, New Jersey
(201) 387-3850

June 2025

Dear Parents and Guardians:

We are excited again to present summer activities that the math teachers of Bergenfield High School have created. Enclosed are math activities designed to help your son or daughter practice the skills which they have already learned and are critical to success in this course. As you may be aware, studies have shown that students who do not practice or review during the summer months the material they have already mastered lose some of that mastery. Unfortunately, this then requires the next teacher to spend valuable teaching time reviewing. While certainly not the final answer, this packet of activities is designed to help your son or daughter retain his or her math skills and knowledge.

Like you, we want your child to enjoy a wonderful summer. That is why we have designed activities so that 20 to 30 minutes of work per week should be all that is required. We urge you to encourage your child to take this task seriously and complete it successfully. Together we can make a difference in your child's future. Now is the time to build on the foundation to help your child succeed on future standardized exams, placement tests, and even more importantly, assessments at a college level.

These activities will reinforce skills that were taught in previous courses**. This assignment should be completed and brought to the first day of the school in September.** Calculators are NOT to be used to complete this project except where noted. *Please read all directions carefully.*

We wish you a wonderful and safer summer.

Sincerely,

Robert Ragasa
Principal

Steven Neff
Supervisor of Mathematics

◆ Skill A Writing an equation of a line in point-slope form

Recall The point-slope form for an equation of a line is $y - y_1 = m(x - x_1)$.

◆ **Example**

Write an equation for the line through $(1, -1)$ and $(-1, 5)$

a. in point-slope form.
b. in slope-intercept form.

◆ **Solution**

a. First find m.

$$m = \frac{\text{difference in } y\text{-values}}{\text{difference in } x\text{-values}} = \frac{-1 - 5}{1 - (-1)} = \frac{-6}{2} = -3$$

Substitute the slope and one of the points into the point-slope equation.

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = -3(x - 1) \qquad \text{Use the point } (1, -1).$$

$$y + 1 = -3(x - 1) \qquad \text{Simplify.}$$

b. Rewrite the equation in the form $y = mx + b$.

$$y + 1 = -3(x - 1)$$

$$y + 1 = -3x + 3 \qquad \text{Distributive Property}$$

$$y = -3x + 2 \qquad \text{Subtract 1 from each side.}$$

Write an equation for each line in point-slope form.

7. containing $(4, -1)$ and with a slope of $\frac{1}{2}$ ______________________

8. crossing the x-axis at $x = -3$ and the y-axis at $y = 6$ ______________________

9. containing the points $(-6, -1)$ and $(3, 2)$ ______________________

Rewrite each equation in slope-intercept form.

10. the line from Exercise 7 ______________________

11. the line from Exercise 8 ______________________

12. the line from Exercise 9 ______________________

13. In what situations would you find it easier to use point-slope form, and in what situations would you find it easier to use slope-intercept form? ______________________

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◆ **Skill** B actoring trinomials by choosing factor pairs of the constant

Recall Another way to factor a trinomial, such as $x^2 - 5x - 6$, is to first make a list of the pairs of factors of the constant. Then choose the right combination to complete the factors of the trinomial.

◆ **Example**

Use the constant's factor pairs to factor $x^2 - 5x - 6$.

◆ **Solution**

List each pair of factors of -6 along with their sum.

Factors of -6	Sum of the factors
6 and -1	5
3 and -2	1
2 and -3	-1
1 and -6	-5

The sum of 1 and -6 is -5. Use the combination of 1 and -6 to form the factors. Thus, $x^2 - 5x - 6 = (x + 1)(x - 6)$.

Factor each trinomial. If the trinomial cannot be factored, write *prime*.

7. $x^2 - x - 2$ ____________

8. $x^2 + 3x - 4$ ____________

9. $x^2 + 4x + 3$ ____________

10. $x^2 - 4x + 3$ ____________

11. $x^2 + 2x - 8$ ____________

12. $x^2 + x - 20$ ____________

13. $x^2 + 2x - 15$ ____________

14. $x^2 - 3x + 10$ ____________

15. $x^2 - x - 12$ ____________

16. $x^2 + 6x + 8$ ____________

17. $x^2 - 20x + 36$ ____________

18. $x^2 + 2x - 24$ ____________

◆ **Skill** C Find the zeros of a polynomial function by factoring

Recall The zeros of a function are the values of x that make y equal to 0.

◆ **Example 1**

Find the zeros of the function $y = (x - 2)(x + 5)$.

◆ **Solution**

Let $y = 0$. Then use the Zero-Product Property to solve for x.

$(x - 2)(x + 5) = 0$

$(x - 2) = 0$ or $(x + 5) = 0$

$x = 2$ or $x = -5$

The zeros of $y = (x - 2)(x + 5)$ are 2 and -5.

Recall A quadratic polynomial can be factored into two binomials.

◆ **Example 2**

Solve the equation $x^2 - x - 6 = 0$.

◆ **Solution**

Since $x^2 - x - 6$ can be factored into $(x + 2)(x - 3)$, you can rewrite $x^2 - x - 6 = 0$ as $(x + 2)(x - 3) = 0$. Solve the equation $(x + 2)(x - 3) = 0$.

$x + 2 = 0$ or $x - 3 = 0$

$x = -2$ or $x = 3$

The solutions to $x^2 - x - 6 = 0$ are -2 and 3.

Solve by factoring.

11. $x^2 - 4x - 12 = 0$

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12. $x^2 - 6x + 9 = 0$

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13. $x^2 - 9x + 14 = 0$

____

14. $x^2 + 6x + 5 = 0$

____

15. $x^2 - 7x + 10 = 0$

____

16. $x^2 - 36 = 0$

____

17. $x^2 + 8x + 16 = 0$

____

18. $x^2 - x - 12 = 0$

____

19. $9x^2 - 1 = 0$

____

20. $4x^2 + 4x + 1 = 0$

____

◆ Skill D Using the quadratic formula to solve equations

Recall The solutions for a quadratic equation written in the form $ax^2 + bx + c = 0$, where $a \neq 0$, can be found by using the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

◆ **Example**

Use the quadratic formula to solve $x^2 - 8x + 15 = 0$ for x.

◆ **Solution**

For $x^2 - 8x + 15 = 0$, a is 1; b is -8, and c is 15. Substitute these values in the quadratic formula.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - (4)(1)(15)}}{(2)(1)}$$

$$= \frac{8 \pm \sqrt{64 - 60}}{2}$$

$$= \frac{8 \pm \sqrt{4}}{2}$$

$$= \frac{8 \pm 2}{2}$$

$x = 3$ or $x = 5$

The solutions are 3 and 5.

Use the quadratic formula to find the zeros of each function.

7. $y = x^2 + 2x - 8$

8. $y = 2x^2 - x - 15$

9. $y = 4x^2 - 8x + 3$

______________ ______________ ______________

◆ Skill E Using the discriminant to determine the number of solutions

Recall When a quadratic equation is written in the form $ax^2 + bx + c = 0$, where $a \neq 0$, the expression $b^2 - 4ac$ is called the *discriminant* of the quadratic formula.

If $b^2 - 4ac > 0$, there are two solutions.

If $b^2 - 4ac = 0$, there is one solution.

If $b^2 - 4ac < 0$, there are no real number solutions.

◆ **Example**

What does the discriminant tell you about $3x^2 - 2x + 9 = 0$?

◆ **Solution**

For $3x^2 - 2x + 9 = 0$, a is 3; b is -2, and c is 9.

Thus, $b^2 - 4ac = (-2)^2 - (4)(3)(9) = 4 - 108 = -104$

$-104 < 0$, so the equation $3x^2 - 2x + 9 = 0$ has no real solutions.

Give the value of each discriminant. What does the discrimant tell you about the function?

10. $y = 4x^2 + 4x + 1$

11. $y = x^2 + 5x + 4$

12. $y = x^2 + 5x + 8$

______________ ______________ ______________

◆ Skill F Writing and evaluating functions

Recall The value of $f(x) = x^2 + 5$ depends on the value of x.

◆ **Example 1**

Sarah uses an internet server which charges $12.50 per month plus $0.60 for each hour over 20 hours that she uses it during the month. Write this relation in function notation. How much will she be charged for using the service for 38 hours in April?

◆ **Solution**

Let h = number of hours over 20. Thus, the function is as follows.

$$f(h) = 12.50 + 0.60h$$
$$f(18) = 12.50 + 0.60(18) \quad \textit{where } h = 18$$
$$f(18) = 23.30$$

The charge for April will be $35.30.

◆ **Example 2**

If $g(x) = x^2 + 3x$, find $g(-5)$.

◆ **Solution**

$g(-5)$ means replace x with the value -5 and evaluate $g(x)$.

$$g(-5) = (-5)^2 + 3(-5)$$
$$= 25 - 15$$
$$= 10$$

Thus, $g(-5) = 10$.

Let $f(x) = 5 - \frac{2x}{3}$ and $g(x) = \frac{1}{2}x^2 + 3x$. Evaluate each function.

7. $f(6)$ ______________________

8. $f(0)$ ______________________

9. $f\left(\frac{1}{2}\right)$ ______________________

10. $g(1)$ ______________________

11. $g(-2)$ ______________________

12. $g\left(\frac{1}{2}\right)$ ______________________

13. $f(1) + g(0)$ ______________________

14. $g(4) - f(5)$ ______________________

15. $f(0) \cdot g(0)$ ______________________

16. $g(-6) \cdot f(-6)$ ______________________

◆ **Skill G** Using the four basic operations on functions to write new functions

Recall To write the sum, difference, product, or quotient of two functions, f and g, write the sum, difference, product, or quotient of the expressions that define f and g. Then simplify.

◆ **Example**

Let $f(x) = x^2 + 3x + 2$ and $g(x) = 5x - 1$. Write an expression for each function.

a. $(f + g)(x)$ **b.** $(f - g)(x)$ **c.** $(fg)(x)$ **d.** $\left(\frac{f}{g}\right)(x)$

◆ **Solution**

a. $(f + g)(x) = f(x) + g(x)$
$= (x^2 + 3x + 2) + (5x - 1)$
$= x^2 + 8x + 1$ *Combine like terms.*

b. $(f - g)(x) = f(x) - g(x)$
$= (x^2 + 3x + 2) - (5x - 1)$
$= x^2 + 3x + 2 - 5x + 1$
$= x^2 - 2x + 3$ *Combine like terms.*

c. $(fg)(x) = f(x) \cdot g(x)$
$= (x^2 + 3x + 2)(5x - 1)$
$= (x^2 + 3x + 2)(5x) + (x^2 + 3x + 2)(-1)$ *Distributive Property*
$= 5x^3 + 15x^2 + 10x - x^2 - 3x - 2$
$= 5x^3 + 14x^2 + 7x - 2$

d. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0$
$= \frac{x^2 + 3x + 2}{5x - 1}$, where $x \neq \frac{1}{5}$

Let $f(x) = 3x^2 + 2$, $g(x) = 2x - 1$, and $h(x) = x^2 + 5x$. Find each new function, and state any domain restrictions.

1. $(f + g)(x)$ ____________

2. $(f - h)(x)$ ____________

3. $(h - g)(x)$ ____________

4. $(gh)(x)$ ____________

5. $(hg)(x)$ ____________

6. $(f + h)(x)$ ____________

7. $\left(\frac{f}{g}\right)(x)$ ____________

8. $\left(\frac{h}{g}\right)(x)$ ____________

◆ Skill H Finding the composite of two functions

Recall To write an expression for the composite function $(f \circ g)(x)$, replace each x in the expression for f with the expression defining g. Then simplify the result.

◆ **Example**

Let $f(x) = 5x$ and $g(x) = 2x^2 - 3$. Find $(f \circ g)(2)$ and $(g \circ f)(2)$. Then write expressions for $(f \circ g)(x)$ and $(g \circ f)(x)$.

◆ **Solution**

$(f \circ g)(2)$: $\quad g(2) = 2(2)^2 - 3 = 5 \quad f(g(2)) = f(5) = 5(5) = 25$

Thus, $(f \circ g)(2) = 25$.

$(g \circ f)(2)$: $\quad f(2) = 5(2) = 10 \quad g(f(2)) = g(10) = 2(10)^2 - 3 = 197$

Thus, $(g \circ f)(2) = 197$.

To write expressions for $(f \circ g)(x)$ and $(g \circ f)(x)$, use the variable x instead of a particular number.

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(2x^2 - 3) \\ &= 5(2x^2 - 3) \\ &= 10x^2 - 15\end{aligned} \qquad \begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(5x) \\ &= 2(5x)^2 - 3 \\ &= 50x^2 - 3\end{aligned}$$

Let $f(x) = x^2 - 1$, $g(x) = 3x$, and $h(x) = 5 - x$. Find each composite function.

9. $(f \circ g)(x)$ ________________

10. $(g \circ f)(x)$ ________________

11. $(h \circ f)(x)$ ________________

12. $(h \circ g)(x)$ ________________

13. $(g \circ g)(x)$ ________________

14. $(h \circ h)(x)$ ________________

15. $(g \circ h)(4)$ ________________

16. $(f \circ f)(-3)$ ________________

17. $(f \circ (g \circ h))(1)$ ________________

18. $(g \circ (g \circ g))(5)$ ________________

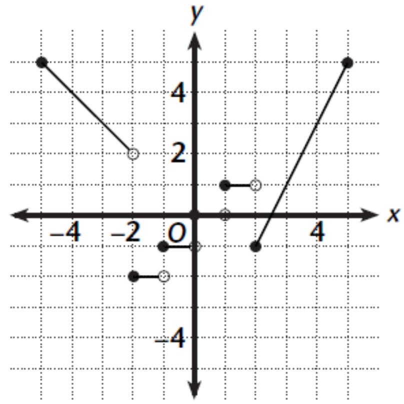
◆ Skill | I | Graphing piecewise, step, and absolute-value functions

Recall A piecewise function in x is a function defined by different expressions in x on different intervals for x.

◆ **Example**

Graph this piecewise function.

$$f(x) = \begin{cases} |x|, & \text{if } -5 \le x < -2 \\ [x], & \text{if } -2 \le x < 2 \\ 2x - 5, & \text{if } 2 \le x \le 5 \end{cases}$$



◆ **Solution**

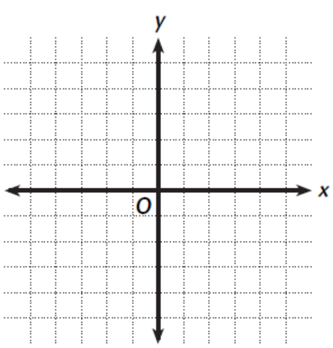
x	-5	-4	-3	-2.5
$y = \|x\|$	5	4	3	2.5

x	-2	-1.5	-1	-0.5	0	1
$y = [x]$	-2	-2	-1	-1	0	1

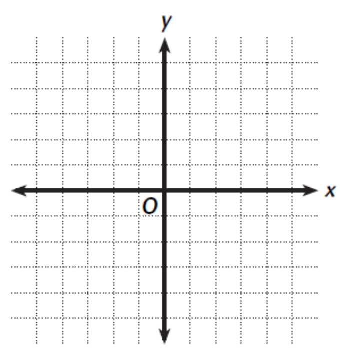
x	2	2.5	3	4	5
$y = 2x - 5$	2	0	1	3	5

Graph each function.

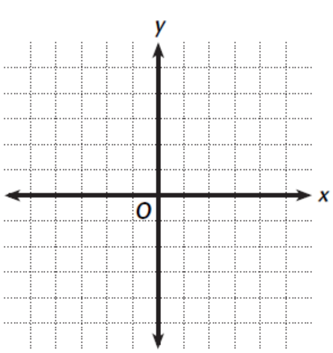
15. $f(x) = \begin{cases} x + 3, \text{ if } x < 0 \\ -2x + 5 \text{ if } x \ge 0 \end{cases}$



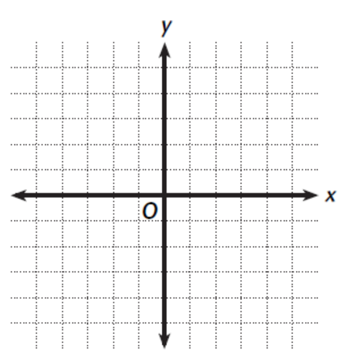
16. $f(x) = \begin{cases} \frac{1}{2}x \text{ if } -4 \le x \le 2 \\ 2x - 3 \text{ if } x > 2 \end{cases}$



17. $f(x) = \begin{cases} |x| \text{ if } x \le 1 \\ 2 - |x - 2| \text{ if } x > 1 \end{cases}$



18. $f(x) = \begin{cases} [x] \text{ if } -2 \le x \le 1 \\ \lceil x \rceil \text{ if } 1 < x \le 4 \end{cases}$



◆ **Skill J** Using logarithms to solve exponential equations

Recall The common logarithm, $\log_{10} x$, is usually written as $\log x$.

◆ **Example**

Solve each equation.

a. $3^x = 81$ **b.** $5^x = 75$ **c.** $7^{x+1} = 150$

◆ **Solution**

a. $3^x = 81$

Since 81 is a power of 3, use powers of 3.

$3^x = 3^4$

$x = 4$ *One-to-One Property of Exponential Functions*

b. $5^x = 75$

Since 75 is **not** a power of 5, use logarithms to solve this equation.

$\log 5^x = \log 75$

$x \log 5 = \log 75$ *Power Property of Logarithms*

$$x = \frac{\log 75}{\log 5}$$

$x \approx 2.68$

Check: $5^{2.68} \approx 75$

c. $7^{x+1} = 150$

$\log 7^{x+1} = \log 150$

$(x + 1)\log 7 = \log 150$

$$x + 1 = \frac{\log 150}{\log 7}$$

$$x = \frac{\log 150}{\log 7} - 1$$

$x \approx 1.57$

Solve each equation. Round your answers to the nearest hundredth.

1. $7^x = 80$ ______

2. $5^x = 10$ ______

3. $6^x = 1296$ ______

4. $4^{x+1} = 100$ ______

5. $2^{x-3} = 25$ ______

6. $3^{x+4} = 27$ ______

7. $6^{2x-7} = 216$ ______

8. $5^{3x-1} = 49$ ______

9. $10^{x+5} = 125$ ______

◆ Skill K Finding logarithms with bases other than 10 (You will need a calculator.)

Recall The equation $x = \log_b y$ is equivalent to $b^x = y$.

◆ **Example 1**

Find $\log_3 40$ using your calculator.

◆ **Solution**

Since calculators do not work in base 3, you can change this problem to a base 10 logarithm problem.

$$x = \log_3 40 \quad \rightarrow \quad 3^x = 40$$

$$\log 3^x = \log 40 \qquad \textit{base 10 logarithms}$$

$$x \log 3 = \log 40$$

$$x = \frac{\log 40}{\log 3}$$

$$x \approx 3.36 \qquad \textit{Use a calculator.}$$

Recall The change-of-base formula is $\log_b x = \frac{\log_a x}{\log_a b}$, where a can be any permissible logarithmic base.

◆ **Example 2**

Find $\log_5 68$ using your calculator.

◆ **Solution**

To find $\log_5 68$, use logarithms with base 10. That is, use $a = 10$. Then you can use a calculator's built-in base 10 logarithms.

$$x = \log_5 68 \quad \Rightarrow \quad x = \frac{\log 68}{\log 5}$$

$$x \approx 2.62$$

Evaluate each logarithmic expression to the nearest hundredth.

10. $\log_6 18$ __________

11. $\log_5 100$ __________

12. $\log_2 400$ __________

13. $\log_8 512$ __________

14. $\log_{10} 215$ __________

15. $\log_{\frac{1}{2}} 24$ __________

16. $\log_{13} 110$ __________

17. $\log_{2.5} 76$ __________

18. $\log_{\frac{1}{16}} 329$ __________

◆ Skill L Identifying parent functions in transformations

Recall Transformations of a function are indicated by the addition or subtraction of constants from the variable term or from the entire function or by multiplication or division of the variable term by a constant.

◆ **Example**

In the following equations, identify the parent function:

a. $y = -3|x - 2| + 5$ **b.** $y = -(x + 3)^2 - 4$

◆ **Solution**

a. Identify the additions, multiplications, subtractions, or divisions that occur. If the addition of 5 is removed, the equation becomes $y = -3|x - 2|$. If the multiplication by -3 is removed, the equation becomes $y = |x - 2|$. Finally, if the subtraction of 2 is removed, the equation becomes $y = |x|$. This is the absolute-value parent function.

b. Start with $y = -(x + 3)^2 - 4$ and remove the additions and subtractions, starting with the subtraction of 4 outside the parentheses. This leaves $y = -(x + 3)^2$. Then remove the negative sign preceding the parentheses, leaving $y = (x + 3)^2$. Finally, remove the addition of 3 within the parentheses, producing the function $y = x^2$. This is the quadratic parent function.

Identify the parent function for each of the following:

1. $y = -2|x + 1| - 4$ ______________________________

2. $y = 3(x - 1)^2 - 2$ ______________________________

3. $y = 3 \cdot 2^{-x} + 1$ ______________________________

4. $y = -3(x + 2) - 4$ ______________________________

5. $y = \frac{3}{x} + 2$ ______________________________

6. $y = \frac{3}{x + 2}$ ______________________________

7. $y = 3x^2 - 4$ ______________________________

8. $y = -2(x - 1)^2$ ______________________________

◆ Skill M Understanding the effect of order on combining transformations

Recall To determine the order of transformations to a function, reverse the order of operations. Addition or subtraction indicates a vertical translation; multiplication or division indicates a vertical stretch; addition or subtraction within parentheses or within absolute-value symbols indicates a horizontal translation.

◆ **Example**
Describe the various transformations included in the equation $y = 2|x - 1| + 3$.

◆ **Solution**
The first operation to consider is the addition of 3. This affects the parent function by translating it vertically 3 units up. The second operation, multiplication by 2, stretches the translated function by a factor of 2. The third operation, subtraction of 1, translates the stretched function horizontally 1 unit to the right. Thus, the parent function, $y = |x|$, has been shifted 1 unit to the right, stretched by a factor of 2, and then shifted 3 units up.

Describe the transformations of the parent functions included in each equation.

9. $y = -3|x + 2| - 3$ ______________________________

____

____

10. $y = 2(x - 3)^2 + 1$ ______________________________

____

____

11. $y = 4|x - 1| + 2$ ______________________________

____

____

12. $y = 4 \cdot 2^x - 2$ ______________________________

____

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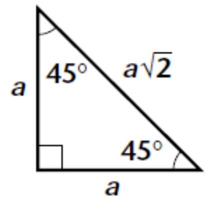
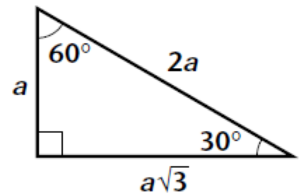
◆ **Skill** N Solving 45-45-90 and 30-60-90 triangles

Recall

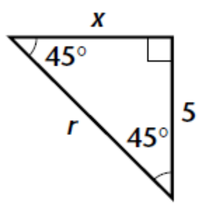




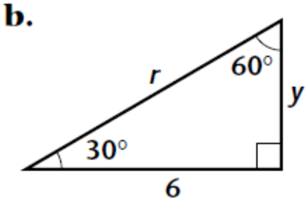
◆ **Example**

Find the lengths of the other 2 sides in each right triangle.

a.



b.



◆ **Solution**

a. $a = 5$

$x = a = 5$

$r = a\sqrt{2} = 5\sqrt{2}$

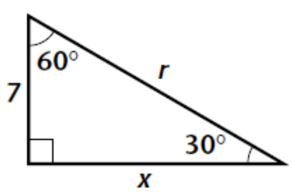
b. $a\sqrt{3} = 6 \rightarrow a = \frac{6}{\sqrt{3}}$ (or $2\sqrt{3}$)

$y = a = \frac{6}{\sqrt{3}}$ (or $2\sqrt{3}$)

$r = 2a = \frac{12}{\sqrt{3}}$ (or $4\sqrt{3}$)

Find the missing side lengths in each right triangle.

1.



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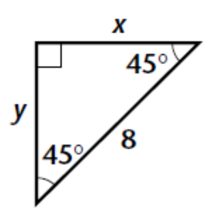
2.



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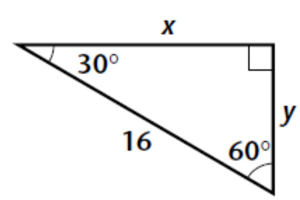
3.



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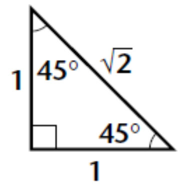
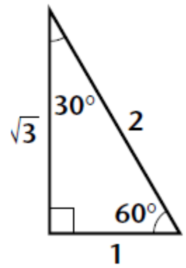
◆ Skill O Finding exact values of the trigonometric functions for an angle whose reference angle is 30°, 45°, or 60°

Recall

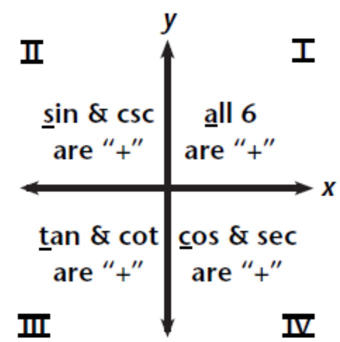




θ	30°	45°	60°
$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

A **reference angle** is the positive acute angle between the terminal side of a given angle and the *x*-axis.

One mnemonic for remembering which functions are **positive** in each quadrant is "All students take calculus."



◆ Example

Find each exact value.

a. sin 315° **b.** cos 240° **c.** tan 210°

◆ Solution

a. 315° is in Quadrant IV, where sine is negative. The reference angle is 45°.

$$\sin 315° = -\sin 45° = -\frac{1}{\sqrt{2}}$$

b. 240° is in Quadrant III, where cosine is negative. The reference angle is 60°.

$$\cos 240° = -\cos 60° = -\frac{1}{2}$$

c. 210° is in Quadrant III, where tangent is positive. The reference angle is 30°.

$$\tan 210° = \tan 30° = \frac{1}{\sqrt{3}}$$

Find each trigonometric value. Give exact answers.

4. sin 120° ________ **5.** cos 330° ________ **6.** tan 225° ________ **7.** cos 150° ________

8. sin 240° ________ **9.** sin 150° ________ **10.** tan 315° ________ **11.** cos 225° ________

◆ **Skill** P Finding exact values for the trigonometric functions of an angle measured in radians

◆ **Example**

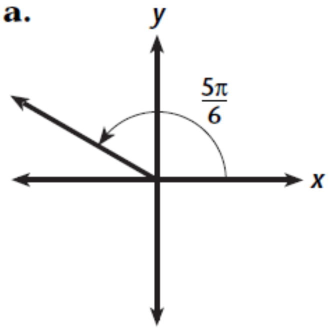
Give the exact value of each expression where the angle measures are in radians.

a. $\cos \frac{5\pi}{6}$ **b.** $\tan \frac{4\pi}{3}$ **c.** $\sin\left(-\frac{3\pi}{2}\right)$

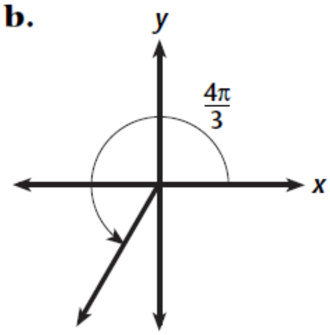
◆ **Solution**

Use reference angles.

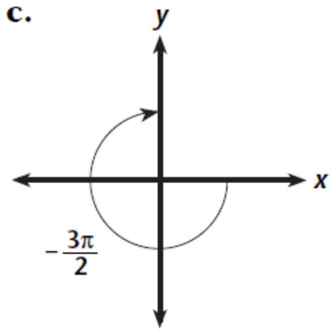
a.



b.



c.



$\frac{5\pi}{6}$ radians $= 150°$

$\cos 150° = -\cos 30° = -\frac{\sqrt{3}}{2}$

$\frac{4\pi}{3}$ radians $= 240°$

$\tan 240° = \tan 60° = \sqrt{3}$

$-\frac{3\pi}{2}$ radians $= -270°$

$\sin(-270°) = \sin 90° = 1$

Evaluate each expression. Give exact answers.

16. $\sin \frac{3\pi}{4}$ ____________

17. $\cos \frac{2\pi}{3}$ ____________

18. $\tan \frac{5\pi}{6}$ ____________

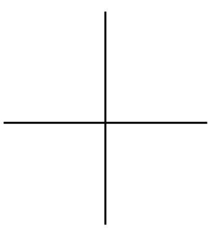
19. $\cos\left(-\frac{7\pi}{6}\right)$ ____________

20. $\tan\left(-\frac{\pi}{4}\right)$ ____________

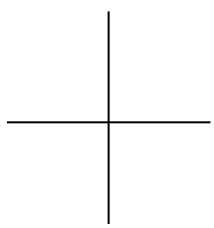
21. $\sin \pi$ ____________

Draw Each Angle in standard form below:

16.



17.



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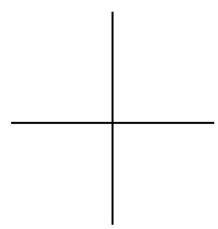

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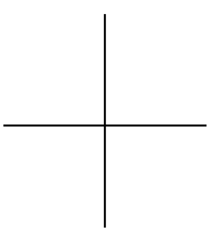
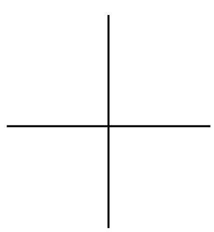


20.



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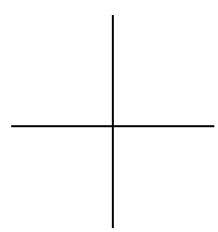
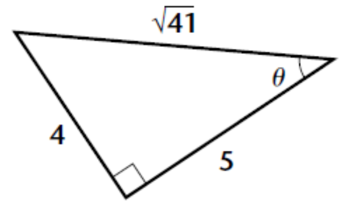


◆ Skill Q Finding the trigonometric functions of an acute angle

Recall The hypotenuse is the longest side in a right triangle and is opposite the right angle.

◆ **Example**

Refer to the triangle shown at right and give values for $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, $\sec\theta$, and $\csc\theta$.



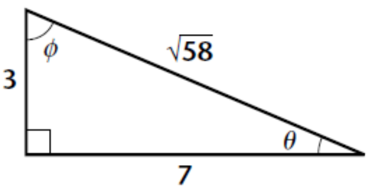
◆ **Solution**

The hypotenuse (hyp.) has a length of $\sqrt{41}$.
The leg opposite (opp.) θ has a length of 4.
The leg adjacent (adj.) to θ has a length of 5.

$\sin\theta = \frac{\text{opp.}}{\text{hyp.}} = \frac{4}{\sqrt{41}}$ $\quad \csc\theta = \frac{\text{hyp.}}{\text{opp.}} = \frac{\sqrt{41}}{4}$ $\quad \cos\theta = \frac{\text{adj.}}{\text{hyp.}} = \frac{5}{\sqrt{41}}$

$\sec\theta = \frac{\text{hyp.}}{\text{adj.}} = \frac{\sqrt{41}}{5}$ $\quad \tan\theta = \frac{\text{opp.}}{\text{adj.}} = \frac{4}{5}$ $\quad \cot\theta = \frac{\text{adj.}}{\text{opp.}} = \frac{5}{4}$

Refer to the triangle at right to find each value. Give exact answers.



1. $\sin\theta$ ____________ **2.** $\cos\theta$ ____________

3. $\tan\theta$ ____________ **4.** $\csc\theta$ ____________

5. $\sec\theta$ ____________ **6.** $\cot\theta$ ____________ **7.** $\sin\phi$ ____________

8. $\cos\phi$ ____________ **9.** $\tan\phi$ ____________ **10.** $\csc\phi$ ____________

11. Draw Right Triangle ABC given the following properties:

BC = 20
AB = 12

12. Find each value (make sure to simplify your ratio)

a. sin B = d. sin C =

b. cos B = e. cos C =

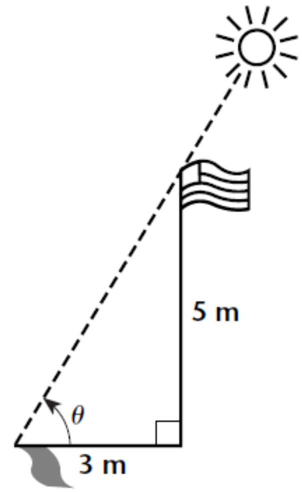
c. tan B = f. tan C =

◆ **Skill R** Applying inverse trigonometric functions

Recall $\sin\theta = \frac{\text{opp.}}{\text{hyp.}}$ $\cos\theta = \frac{\text{adj.}}{\text{hyp.}}$ $\tan\theta = \frac{\text{opp.}}{\text{adj.}}$

◆ **Example**

At a certain time of the day, the 5 meter flagpole shown at right casts a shadow that is 3 meters long. What is the angle of elevation of the sun at this time?



◆ **Solution**

Since 3 meters is the length of the side **adjacent** to θ and 5 meters is the length of the side **opposite** θ, use the tangent function.

$\tan\theta = \frac{5}{3}$

$\theta = \tan^{-1}\left(\frac{5}{3}\right)$

This last equation states that θ is the angle that has a tangent of $\frac{5}{3}$.

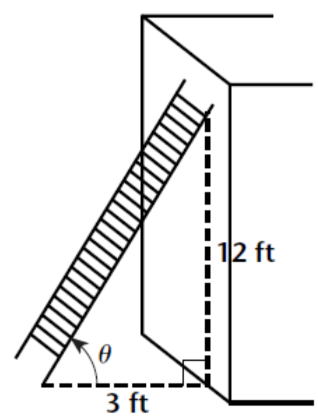
$\theta \approx 59°$ *Use calculator in **degree** mode.*

Find the measure of each angle to the nearest whole degree.

13. Find the measure of the smallest angle in a right triangle with sides of 3, 4, and 5 centimeters. ________________

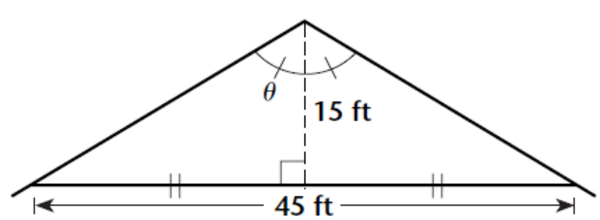
14. What is the angle between the bottom of the ladder and the ground as shown at right?

____



15. Find the angle at the peak of the roof as shown at right.

____



16. The hypotenuse of a right triangle is 3 times as long as the shorter leg. Find the measure of the angle between the shorter leg and the hypotenuse. ________________

◆ Skill S Asymptotes

Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

(a) Determine the vertical asymptotes for the function.
(b) Determine the x and y –intercepts

(1) $f(x)=\frac{1}{x^2}$ (2) $f(x)=\frac{x^2}{x^2-4}$ (3) $f(x)=\frac{2+x}{x^2(1-x)}$

Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

(a) Determine all Horizontal Asymptotes
(b) Determine the x and y –intercepts

(4) $f(x)=\frac{x^2-2x+1}{x^3+x-7}$ (5) $f(x)=\frac{5x^3-2x^2+8}{4x-3x^3+5}$ (6) $f(x)=\frac{4x^5}{x^2-7}$