Bergenfield High School
Bergenfield, New Jersey

Mathematics Department

Summer Course Work

in preparation for

AP Calculus BC

Completion of this summer work
is required on the first day of the
2025-2026 school year.

Bergenfield Public Schools
Mathematics Department
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June 2025

Dear Parents and Guardians:

Attached are the summer curriculum review materials for AP *Calculus BC*. This packet was prepared by the Bergenfield High School Math department and contains topics that reflect content learned in prerequisite courses. These materials must be completed and brought to class on the first day of school in September.

Your child is required to complete this packet over the summer. A test based on the material in the packet will be given to your child during the second week of school. It will count as the first test of the year and the grade will be determined as follows:

Completion of the packet on time will count 20% of the grade
Performance on the test will count 80% of the grade.

Thank you for your cooperation.

Sincerely,

Robert Ragasa	Steve Neff
Principal	Director of Mathematics

2.4

Find each derivative.

(a) $h(t) = 2t\cos t + t^2 \sin t$ (b) $f(x) = 2x^2 \cot x$ (c) $f(x) = \dfrac{\tan x}{\sin x + 1}$

5. Find the equation(s) of the tangent line(s) to the graph of $y = \dfrac{x+1}{x-1}$ that are parallel to the line $2y + x = 6$.

3.7

5. Find the linear approximation of the function $f(x) = \sqrt{1-x}$ at $a = 0$ and use it to approximate the numbers $\sqrt{0.9}$ and $\sqrt{0.99}$. Illustrate by sketching a graph of f and the tangent line.

2.6

6. If $g(5)=-3$, $g'(5)=6$, $h(5)=3$, and $h'(5)=-2$, find $f'(5)$ (if possible) for each of the following. If it is not possible, state what additional information is required.

(a) $f(x)=\dfrac{g(x)}{h(x)}$ (b) $f(x)=g(h(x))$ (c) $f(x)=g(x)h(x)$

_____15. If $g(x)=(1-x)^3(4x+1)$, then $g'(x)=$

(A) $-12(1-x)^2$

(B) $(1-x)^2(1+8x)$

(C) $(1-x)^2(1-16x)$

(D) $3(1-x)^2(4x+1)$

(E) $(1-x)^2(16x+7)$

2.7

2. Find $\dfrac{dy}{dx}$ at the indicated point, then find the equation of the indicated line at the point.

(a) $y^2=\dfrac{x^2-4}{x^2+4}$ at $(2,0)$, tangent line (b) $(x+y)^3=x^3+y^3$ at $(-1,1)$, normal line

_____ 11. Find the equation of the tangent line to the graph of $y^2 - xy - 12 = 0$ at the point $(1,4)$.

(A) $3y = 2x + 10$

(B) $3y + 2x = 10$

(C) $y = 4x$

(D) $7y = 4x + 24$

(E) $7y + 4x = 24$

2.8

5. If $f(x) = x^5 + 2x^3 + x - 1$ and $f(g(x)) = x = g(f(x))$ find

(a) $f(1)$

(b) $g'(3)$.

2.9

Find the derivative.

1. $y = e^{2x^2+2x}$
2. $y = 6^{2x}$
3. $y = \sin^2 x + 2^{\sin x}$
4. $y = xe^2 - e^{x^2}$

3.1

5. Find the absolute extrema of *f* on the given interval.

(a) $f(x) = 2x^3 - 3x^2 - 12x + 1,\ [-2,3]$

(b) $f(x) = (x^2 - 1)^3,\ [-1,2]$

3.2

_____ 3. Determine if the function $f(x) = x^3 - x - 1$ satisfies the hypothesis of the MVT on $[-1, 2]$. If it does, find all possible values of c satisfying the conclusion of the MVT.

(A) $-\frac{1}{2}$

(B) $-1,\ 1$

(C) 0

(D) 1

(E) hypothesis not satisfied

3.3

_____ 4. The derivative of a function f is given for all x by $f'(x) = (2x^2 + 4x - 16)(1 + g^2(x))$ where g is some unspecified function. At which value(s) of x will f have a local maximum?
Note: $g^2(x) = (g(x))^2$

(A) $x = -4$

(B) $x = 4$

(C) $x = -2$

(D) $x = 2$

(E) $x = -4,\ 2$

3.4

_____ 3. The x-coordinates of the points of inflection of the graph of $y = x^5 - 5x^4 + 3x + 7$ are

(A) 0 only (B) 1 only (C) 3 only (D) 0 and 3 (E) 0 and 1

_____ 4. Which of the following conditions would enable you to conclude that the graph of f has a point of inflection at $x = c$?

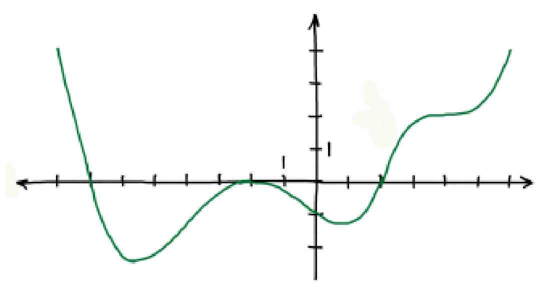
(A) There is a local max of f' at $x = c$ (B) $f''(c) = 0$ (C) $f''(c)$ does not exist

(D) The sign of f' changes at $x = c$ (E) f is a cubic polynomial and $c = 0$

_____ 13. Let f be the function defined by $f(x) = 2x^3 - 3x^2 - 12x + 18$. On which of the following intervals is the graph of f both increasing and concave down?

(A) $(-\infty, -1)$ (B) $\left(-1, \frac{1}{2}\right)$ (C) $(-1, 2)$ (D) $\left(\frac{1}{2}, 2\right)$ (E) $(2, \infty)$

_____ 14.



The graph of $f'(x)$

The figure above shows the graph of f', the derivative of function f, for $-8 < x < 6$. Of the following, which best describes the graph of f on the same interval?

(A) 1 local minimum, 1 local maximum, and 3 inflection points

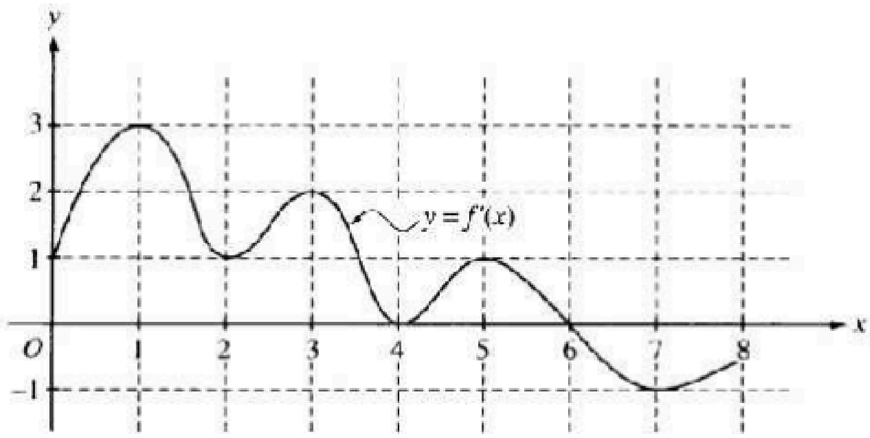
(B) 1 local minimum, 1 local maximum, and 4 inflection points

(C) 2 local minima, 1 local maximums, and 2 inflection points

(D) 2 local minima, 1 local maximum, and 4 inflection points

(E) 2 local minima, 2 local maxima, and 3 inflection points

3.5



The graph above shows $f'(x)$ for some function $f(x)$ on $[0,8]$.

_____ 2. How many points of inflection does the graph of f have on $[0,8]$?

(A) Two

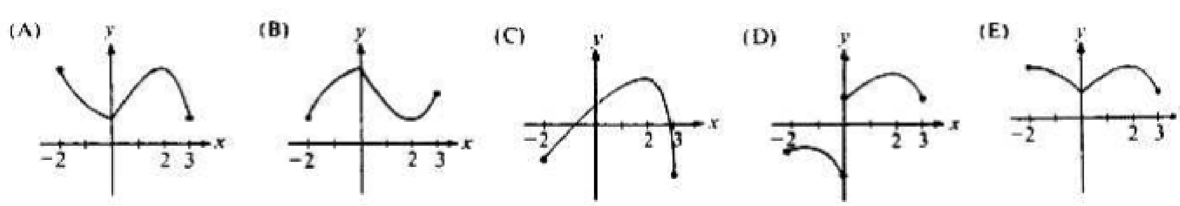
(B) Three

(C) Four

(D) Five

(E) Six

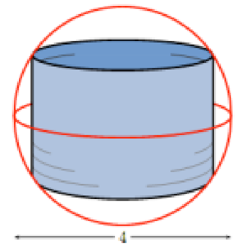
_____ 7. Let f be a function that is continuous on the closed interval $[-2,3]$ such that $f'(0)$ does not exist, $f'(2)=0$, and $f''(x)<0$ for all x except $x=0$. Which of the following could be the graph of f.



3.6

3. A right circular cylinder is inscribed in a sphere with diameter 4cm as shown. If the cylinder is open at both ends, find the largest possible surface area of the cylinder.

(A) $A = 8 \text{ cm}^2$ (B) $A = 16 \text{ cm}^2$ (C) $A = 16\pi \text{ cm}^2$
(D) $A = 2 \text{ cm}^2$ (E) $A = 8\pi \text{ cm}^2$ (F) $A = 4\pi \text{ cm}^2$
(G) $A = 4 \text{ cm}^2$ (H) $A = 2\pi \text{ cm}^2$ (I) $A = 2 \text{ cm}^2$



4. The point on the curve $y = \sqrt{2x}$ that is nearest the point $(1,4)$ occurs at the point

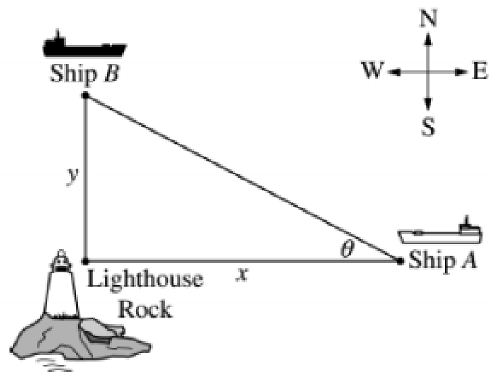
(A) $(1,\sqrt{2})$ (B) $(2,2)$ (C) $(4,2\sqrt{2})$ (D) $(0,0)$ (E) $(8,4)$

3.8

4. A point is moving on the graph of $5x^3 + 6y^3 = xy$. When the point is at $\left(\frac{1}{11},\frac{1}{11}\right)$, its y-coordinate is increasing at a speed of 5 units per second. What is the speed of the x-coordinate at that time and in which direction is the x-coordinate moving?

(A) 8 units/sec, increasing x (B) $\frac{17}{2}$ units/sec, decreasing x (C) $\frac{17}{2}$ units/sec, increasing x
(D) $\frac{33}{4}$ units/sec, decreasing x (E) 8 units/sec, decreasing x (F) $\frac{35}{4}$ units/sec, decreasing x

6. (2002B AB6) Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure at right.



(a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

(b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

(c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.

4.1

6. Evaluate the following:

(a) $\int\left(\sqrt{x^3}+2x+1\right)dx$

(b) $\int\left(\frac{x^3+2x-3}{x^4}\right)dx$

(c) $\int\left(2t^2-1\right)^2 dt$

(d) $\int\left(\theta^2+\sec^2\theta-\csc\theta\cot\theta\right)d\theta$

(e) $\int\left(\frac{\cos x}{1-\cos^2 x}\right)dx$

(f) $\int\left(\cos x+3^x\right)dx$

4.2

x	0	1	2	3	4	5	6
$f(x)$	9.3	9.0	8.3	6.5	2.3	–7.6	–10.5

6. The table above gives the values of a function obtained from an experiment. Use them to estimate $\int_0^6 f(x)\,dx$ using **three equal subintervals** using

(a) right endpoints (REP)

(b) left endpoints (LEP)

(c) midpoints (MDPT)

(d) the trapezoidal rule (TRAP)

4.3

1. (Calculator Permitted) What is the average value of $f(x)=\cos x$ on the interval $[1,5]$?

(A) –0.990 (B) –0.450 (C) –0.128 (D) 0.412 (E) 0.998

2. If the average value of the function f on the interval $[a,b]$ is 10, then $\int_a^b f(x)\,dx =$

(A) $\frac{10}{b-a}$ (B) $\frac{f(a)+f(b)}{10}$ (C) $10b-10a$ (D) $\frac{b-a}{10}$ (E) $\frac{f(a)+f(b)}{20}$

3. (Calculator Permitted) Let $f'(x)=\ln(2+\sin x)$. If $f(3)=4$, then $f(5)=$

(A) 0.040 (B) 0.272 (C) 0.961 (D) 4.555 (E) 6.667

5. What is the linearization of $f(x)=\int_\pi^x \cos^3 t\,dt$ at $x=\pi$?

(A) $y=-1$ (B) $y=-x$ (C) $y=\pi$ (D) $y=x-\pi$ (E) $y=\pi-x$

6. (Calculator Permitted) The area of the region enclosed between the graph of $y=\sqrt{1-x^4}$ and the x-axis is

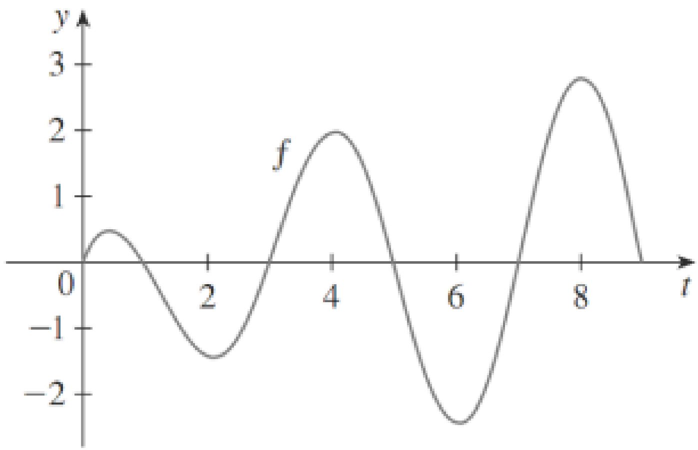
(A) 0.886 (B) 1.253 (C) 1.414 (D) 1.571 (E) 1.748

9. Find $\frac{dy}{dx}$

(a) $y = \int_{-\pi}^{x} \frac{2-\sin t}{3+\cos t}\,dt$

(b) $y = \int_{x}^{7} \sqrt{2m^4 + m + 1}\,dm$

17. Let $g(x) = \int_{0}^{x} f(t)\,dt$, where f is the function whose graph is given below.



(a) At what values of x do the local maximum and local minimum of g occur? Justify.

(b) Where does g attain its absolute maximum value?

(c) On what approximate intervals is g concave downward?

(d) Sketch the graph of g.

4.4

1. Find the most general function f such that $f''(x) = 9\cos 3x$

(A) $f(x) = -3\sin x + Cx^2 + D$ (B) $f(x) = -\cos 3x + Cx + D$ (C) $f(x) = -3\cos 3x + Cx^2 + D$
(D) $f(x) = \sin x + Cx + D$ (E) $f(x) = 3\sin 3x + Cx + D$

4. Evaluate the definite integral $\int_0^1 (4-2x)e^{8x-2x^2}\,dx$

(A) $\frac{1}{2}(e^6 - 1)$ (B) $\frac{1}{2}(e^{-6} - 1)$ (C) $e^{-6} + 1$ (D) $\frac{1}{2}(e^6 + 1)$ (E) $e^6 - 1$

5. Evaluate $\int_0^{\frac{\pi}{4}} \frac{2e^{\tan x} + 5}{\cos^2 x}\,dx$

(A) $2e + 3$ (B) $2e$ (C) $2e - 3$ (D) e (E) $e + 5$

8. Evaluate $\int_0^1 \frac{6x}{1+x^2}\,dx$

(A) $\frac{3}{2}$ (B) 3 (C) 6 (D) $3\ln 2$ (E) $\frac{3}{2}\ln 2$

6.3

2. (Calculator Permitted) Let R be the region in the first quadrant bounded by the graph of $y = 8 - x^{3/2}$, the x-axis, and the y-axis. Which of the following gives the best approximation of the volume of the solid generated when R is revolved about the x-axis?

(A) 60.3 (B) 115.2 (C) 225.4 (D) 319.7 (E) 361.9

3. Let R be the region enclosed by the graph of $y = x^2$, the line $x = 4$, and the x-axis. Which of the following gives the best approximation of the volume of the solid generated when R is revolved about the y-axis.

(A) 64π (B) 128π (C) 256π (D) 360 (E) 512

4. Let R be the region enclosed by the graphs of $y = e^{-x}$, $y = e^{x}$, and $x = 1$. Which of the following gives the volume of the solid generated when R is revolved about the x-axis?

(A) $\int_0^1 \left(e^x - e^{-x}\right) dx$ (B) $\int_0^1 \left(e^{2x} - e^{-2x}\right) dx$ (C) $\int_0^1 \left(e^x - e^{-x}\right)^2 dx$

(D) $\pi \int_0^1 \left(e^{2x} - e^{-2x}\right) dx$ (E) $\pi \int_0^1 \left(e^x - e^{-x}\right)^2 dx$

7. (Calculator Permitted) Let R be the region bounded by the graphs of $y = \sqrt{x}$, $y = e^{-x}$, and the y-axis.

(a) Find the area of R.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a semicircle whose diameter runs from the graph of $y = \sqrt{x}$ to the graph of $y = e^{-x}$. Find the volume of this solid.

5.1

5. ('93-AB42) (**Calculator permitted**) A puppy weighs 2.0 pounds and birth and 3.5 pounds two months later. If the weight of the puppy during the first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

(A) 4.2 pounds (B) 4.6 pounds (C) 4.8 pounds (D) 5.6 pounds (E) 6.5 pounds

For problems 9 – 18, find the general solution to the following differential equations, then find the particular solution using the initial condition. **You may have to factor and/or rewrite the expression in order to separate your *x*-factors and *y*-factors.**

9. $\frac{dy}{dx} = \frac{x}{y}$, $y(1) = -2$

10. $\frac{dy}{dx} = -\frac{x}{y}$, $y(4) = 3$

11. $\frac{dy}{dx} = \frac{y}{x}$, $y(2) = 2$

12. $\frac{dy}{dx} = 2xy$, $y(0) = -3$

13. $\frac{dy}{dx} = xy + 5x + 2y + 10$, $y(0) = -1$

14. $\frac{dy}{dx} = \cos^2 y$, $y(0) = 0$