AP Calculus AB Summer Work!

Welcome to Calc AB! This year for summer work, you will be reviewing previously taught concepts from Algebra 1, Geometry, Algebra 2, and Pre-Calculus. These are items that you need to know coming into the class and will not be reviewed past the first two days of class. If you do not know these skills, this class will be extremely difficult for you.

You will have a review assessment within the first week of class.

If you have questions, please feel free to reach out to me (royv@holliston.k12.ma.us)

This packet is due on the first day of school. No late submissions will be accepted.

Enjoy your summer and can't wait to have you in class!

-Ms. Roy

The following formulas and identities will help you complete this packet. You are expected to know ALL of these for the course.

LINES

Slope-intercept: $y = mx + b$

Point-slope: $y - y_1 = m(x - x_1)$

Standard: $Ax + By = C$

Horizontal line: $y = b$ (slope $= 0$)

Vertical line: $x = a$ (slope $=$ undefined)

Parallel → same slope

Perpendicular → opposite reciprocal slopes

QUADRATICS

Standard: $y = ax^2 + bx + c$

Vertex: $y = a(x - h)^2 + k$

Intercept: $y = a(x - p)(x - q)$

Parabola opens: up if $a > 0$
down if $a < 0$

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EXPONENTIAL PROPERTIES

$x^a \cdot x^b = x^{a+b}$ $\qquad (xy)^a = x^a y^a$

$\frac{x^a}{x^b} = x^{a-b}$ $\qquad \sqrt[n]{x^m} = x^{m/n}$

$x^0 = 1 \ (x \neq 0)$ $\qquad \left(\frac{x}{y}\right)^a = \frac{x^a}{x^b}$

$x^{-n} = \frac{1}{x^n}$

In general, it is fine to have negative exponents in your answers!

LOGARITHMS

$y = \log_a x$ is equivalent to $a^y = x$

$\log_b(mn) = \log_b m + \log_b n$

$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b(m^p) = p \log_b m$

TRIGONOMETRIC IDENTITIES

$\csc x = \frac{1}{\sin x}$ $\qquad \sec x = \frac{1}{\cos x}$ $\qquad \cot x = \frac{1}{\tan x}$ $\qquad \tan x = \frac{\sin x}{\cos x}$ $\qquad \cot x = \frac{\cos x}{\sin x}$

$\sin^2 x + \cos^2 x = 1$ $\qquad \tan^2 x + 1 = \sec^2 x$ $\qquad 1 + \cot^2 x = \csc^2 x$

$\sin(2x) = 2 \sin x \cos x$ $\qquad \cos(2x) = \cos^2 x - \sin^2 x$ **or** $1 - 2\sin^2 x$ **or** $2\cos^2 x - 1$

You are expected to know the general shape, domain, and range of each parent function in the table.

"Parent" functions mean no transformations have been applied. Transformation (shifting, stretching, compressing, or reflecting) may change the domain or range.

Linear functions (x) **D:** $(-\infty, \infty)$ **R:** $(-\infty, \infty)$	**Quadratic** functions (x^2) **D:** $(-\infty, \infty)$ **R:** $[\mathbf{0}, \infty)$
Odd-degree polynomials (x^n for **odd** $\boldsymbol{n}$) **D:** $(-\infty, \infty)$ **R:** $(-\infty, \infty)$	**Even-degree** polynomials (x^n for **even** $\boldsymbol{n}$) **D:** $(-\infty, \infty)$ **R:** $[\mathbf{0}, \infty)$
Exponential functions (a^x, e^x) **D:** $(-\infty, \infty)$ **R:** $(\mathbf{0}, \infty)$ Horizontal asymptote: $y = 0$	**Logarithmic** functions $(\log_a x, \ln x)$ **D:** $(\mathbf{0}, \infty)$ **R:** $(-\infty, \infty)$ Vertical asymptote: $x = 0$
Sinusoidal functions $(a \sin x, a \cos x)$ **D:** $(-\infty, \infty)$ **R:** $[-\boldsymbol{a}, \boldsymbol{a}]$	**Tangent** function $(\tan x)$ **D:** $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{(2n+1)\pi}{2}, \frac{(2n+3)\pi}{2}\right)$ **R:** $(-\infty, \infty)$ Vertical asymptotes: $x = \frac{(2n+1)\pi}{2}$
Reciprocal functions $\left(\frac{1}{x}\right)$ **D:** $(-\infty, \mathbf{0}) \cup (\mathbf{0}, \infty)$ **R:** $(-\infty, \mathbf{0}) \cup (\mathbf{0}, \infty)$ Vertical asymptote: $x = 0$ Horizontal asymptote: $y = 0$	**Absolute value** functions $(\lvert x \rvert)$ **D:** $(-\infty, \infty)$ **R:** $[\mathbf{0}, \infty)$
Even root functions ($\sqrt{x}$, $\sqrt[n]{x}$ for even n) **D:** $[\mathbf{0}, \infty)$ **R:** $[\mathbf{0}, \infty)$	**Odd root** functions ($\sqrt[3]{x}$, $\sqrt[n]{x}$ for odd $n > 1$) **D:** $(-\infty, \infty)$ **R:** $(-\infty, \infty)$

NO CALCULATOR!!!

Given $f(x) = x^2 - 2x + 5$, find the following.

1. $f(-2) =$	2. $f(x+2) =$	3. $f(x+h) =$

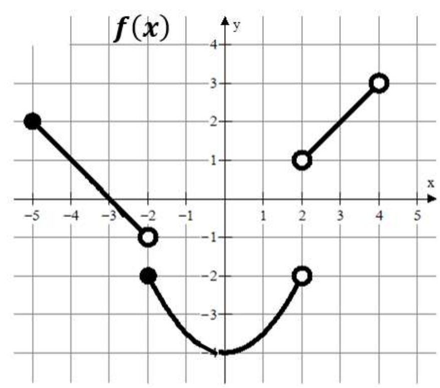
Use the graph $f(x)$ to answer the following.

4. $f(0) =$ $\quad$ $f(4) =$

$f(-1) =$ $\quad$ $f(-2) =$

$f(2) =$ $\quad$ $f(3) =$

$f(x) = 2$ when x = ? $\quad$ $f(x) = -3$ when x = ?

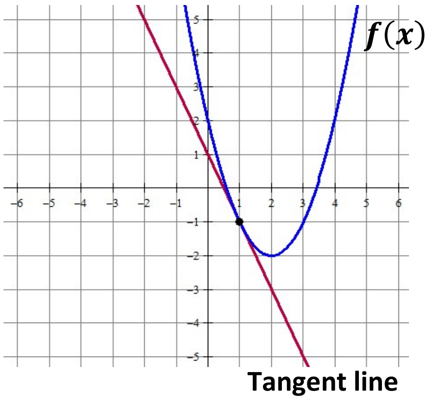


Write the equation of the line meets the following conditions. Use point-slope form.
$y - y_1 = m(x - x_1)$

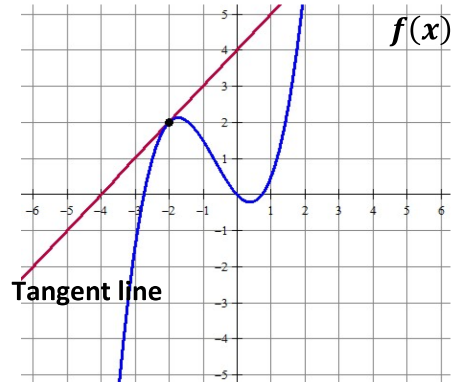
5. slope = 3 and $(4, -2)$	6. $m = -\frac{3}{2}$ and $f(-5) = 7$	7. $f(4) = -8$ and $f(-3) = 12$

Write the equation of the tangent line in point slope form. $y - y_1 = m(x - x_1)$

8. The line tangent to $f(x)$ at $x = 1$



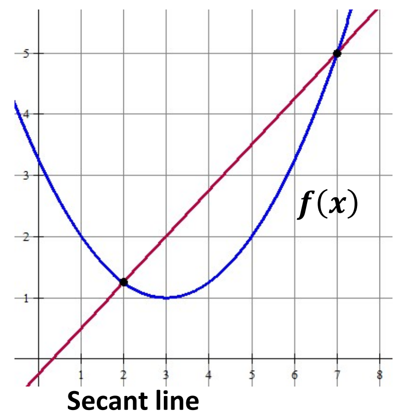
9. The line tangent to $f(x)$ at $x = -2$



MULTIPLE CHOICE! Remember slope = $\frac{y_2 - y_1}{x_2 - x_1}$

10. Which choice represents the slope of the secant line shown?

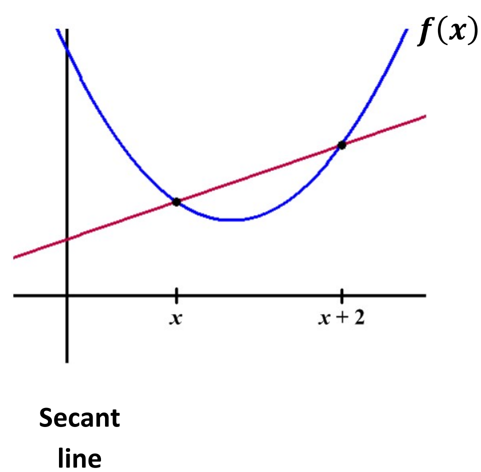
A) $\frac{7-2}{f(7)-f(2)}$ B) $\frac{f(7)-2}{7-f(2)}$ C) $\frac{7-f(2)}{f(7)-2}$ D) $\frac{f(7)-f(2)}{7-2}$



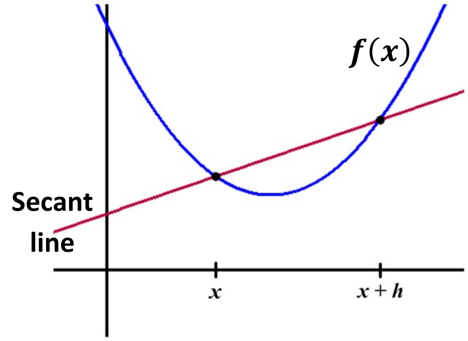
11. Which choice represents the slope of the secant line shown?

A) $\frac{f(x)-f(x+2)}{x+2-x}$ B) $\frac{f(x+2)-f(x)}{x+2-x}$ C) $\frac{f(x+2)-f(x)}{x-(x+2)}$

D) $\frac{x+2-x}{f(x)-f(x+2)}$



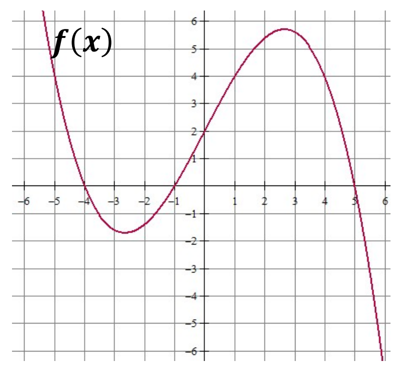
12. Which choice represents the slope of the secant line shown?



A) $\frac{f(x+h)-f(x)}{x-(x+h)}$ B) $\frac{x-(x+h)}{f(x+h)-f(x)}$ C) $\frac{f(x+h)-f(x)}{x+h-x}$

D) $\frac{f(x)-f(x+h)}{x+h-x}$

13. Which of the following statements about the function $f(x)$ is true?

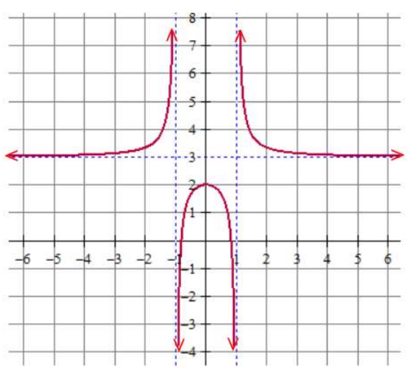


I. $f(2) = 0$
II. $(x+4)$ is a factor of $f(x)$
III. $f(5) = f(-1)$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.

14.



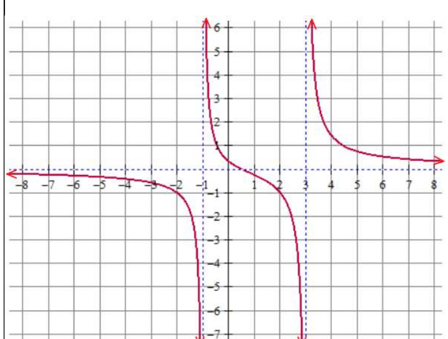
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

15.



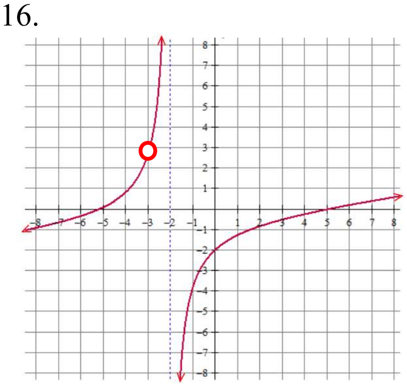
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

16.



Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

MULTIPLE CHOICE!

17. Which of the following functions has a vertical asymptote at $x = 4$?

(A) $\frac{x+5}{x^2-4}$

(B) $\frac{x^2-16}{x-4}$

(C) $\frac{4x}{x+1}$

(D) $\frac{x+6}{x^2-7x+12}$

(E) None of the above

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18. Consider the function: $(x) = \frac{x^2-5x+6}{x^2-4}$. Which of the following statements is true?

I. $f(x)$ has a vertical asymptote of $x = 2$
II. $f(x)$ has a vertical asymptote of $x = -2$
III. $f(x)$ has a horizontal asymptote of $y = 1$

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II and III

Rewrite the following using rational exponents. Example: $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$

19. $\sqrt[5]{x^3} + \sqrt[5]{2x}$	20. $\sqrt{x+1}$	21. $\frac{1}{\sqrt{x+1}}$
22. $\frac{1}{\sqrt{x}} - \frac{2}{x}$	23. $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$	24. $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$

Write each expression in radical form and positive exponents. Example: $x^{-\frac{2}{3}} + x^{-2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x^2}$

25. $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$	26. $\frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$	27. $3x^{-\frac{1}{2}}$
28. $(x+4)^{-\frac{1}{2}}$	29. $x^{-2} + x^{\frac{1}{2}}$	30. $2x^{-2} + \frac{3}{2}x^{-1}$

Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.		
31. $\sin\frac{\pi}{6}$	32. $\cos\frac{\pi}{4}$	33. $\sin 2\pi$
34. $\tan\pi$	35. $\sec\frac{\pi}{2}$	36. $\cos\frac{\pi}{6}$
37. $\sin\frac{\pi}{3}$	38. $\sin\frac{3\pi}{2}$	39. $\tan\frac{\pi}{4}$
40. $\csc\frac{\pi}{2}$	41. $\sin\pi$	42. $\cos\frac{\pi}{3}$
43. Find x where $0 \le x \le 2\pi$, $\sin x = \frac{1}{2}$	44. Find x where $0 \le x \le 2\pi$, $\tan x = 0$	45. Find x where $0 \le x \le 2\pi$, $\cos x = -1$

Solve the following equations. Remember $e^0 = 1$ and $\ln 1 = 0$.		
46. $e^x + 1 = 2$	47. $3e^x + 5 = 8$	48. $e^{2x} = 1$
49. $\ln x = 0$	50. $3 - \ln x = 3$	51. $\ln(3x) = 0$
52. $x^2 - 3x = 0$	53. $e^x + xe^x = 0$	54. $e^{2x} - e^x = 0$

Solve the following trig equations where $0 \le x \le 2\pi$.

55. $\sin x = \frac{1}{2}$	56. $\cos x = -1$	57. $\cos x = \frac{\sqrt{3}}{2}$
58. $2\sin x = -1$	59. $\cos x = \frac{\sqrt{2}}{2}$	60. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$
61. $\tan x = 0$	62. $\sin(2x) = 1$	63. $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$

For each function, determine its domain and range.

Function	**Domain**	**Range**
64. $y = \sqrt{x-4}$		
65. $y = (x-3)^2$		
66. $y = \ln x$		
67. $y = e^x$		
68. $y = \sqrt{4-x^2}$		

Simplify.

69. $\frac{\sqrt{x}}{x}$	70. $e^{\ln x}$	71. $e^{1+\ln x}$

72. $\ln 1$	73. $\ln e^7$	74. $\log_3 \frac{1}{3}$
75. $\log_{1/2} 8$	76. $\ln \frac{1}{2}$	77. $27^{\frac{2}{3}}$
78. $\left(5a^{2/3}\right)\left(4a^{3/2}\right)$	79. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$	80. $\left(4a^{5/3}\right)^{3/2}$

If $\boldsymbol{f(x) = \{(3,5),(2,4),(1,7)\}}$ $\quad \boldsymbol{g(x) = \sqrt{x-3}}$
$\boldsymbol{h(x) = \{(3,2),(4,3),(1,6)\}}$ $\quad \boldsymbol{k(x) = x^2 + 5}$ **, then determine each of the following.**

81. $(f+h)(1)$	82. $(k-g)(5)$	83. $f\big(h(3)\big)$
84. $g\big(k(7)\big)$	85. $h(3)$	86. $g\big(g(9)\big)$

87. $f^{-1}(4)$	88. $k^{-1}(x)$
89. $k\big(g(x)\big)$	90. $g(f(2))$

For #9-16, solve each equation for x. Note that some equations with have a specific value, but most will have a solution in terms of other variables. (For example: $x = \frac{a+b}{c}$ may be a solution.)

9. $x^2 + 3x = 8x - 6$

10. $\frac{2x-5}{x+y} = 3 - y$

11. $3xy + 6x - xz = 12$

12. $A = ax + bx$

13. $cx = vx$

14. $r = t - x(z - y)$

15. $\frac{3+x}{5-x} = 6 + y$

16. $\frac{y+2}{4-x} = 4(2 - z)$

For #17-22, solve each quadratic by factoring.

17. $x^2 - 4x - 12 = 0$

____

18. $x^2 - 6x + 9 = 0$

____

19. $x^2 - 9x + 14 = 0$

____

20. $x^2 - 36 = 0$

____

21. $9x^2 - 1 = 0$

____

22. $4x^2 + 4x + 1 = 0$

____

For #23-27, evaluate the following knowing that $f(x) = 5 - \frac{2x}{3}$ and $g(x) = \frac{1}{2}x^2 + 3x$.

23. $f\left(\frac{1}{2}\right) =$ ____________

24. $g(-2) =$ ____________

25. $f(1) + g(0) =$ ____________

26. $f(0) \cdot g(0) =$ ____________

27. $\frac{g(-6)}{f(-6)} =$ ____________

For #28-35, use $f(x) = x^2 - 1$, $g(x) = 3x$, and $h(x) = 5 - x$ to find each composite function.

28. $f(g(x)) =$ ____________________

29. $g(f(x)) =$ ____________________

30. $f(f(4)) =$ ____________________

31. $g(h(-4)) =$ ____________________

32. $f\left(g(h(1))\right) =$ ____________________

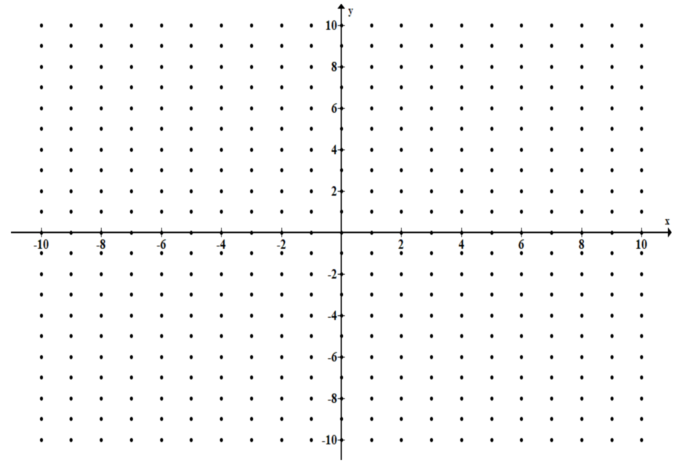
33. $f(g(x-1)) =$ ____________________

34. $g(f(x^3)) =$ ____________________

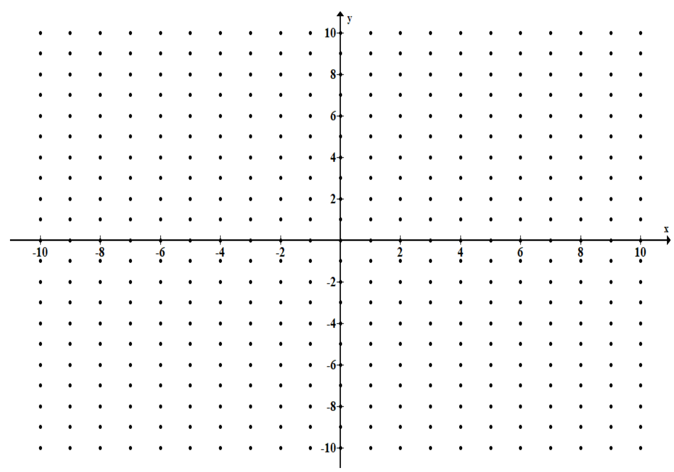
35. $\frac{f(x+h)-f(x)}{h} =$ ____________________

For #36-38, graph each piecewise function.

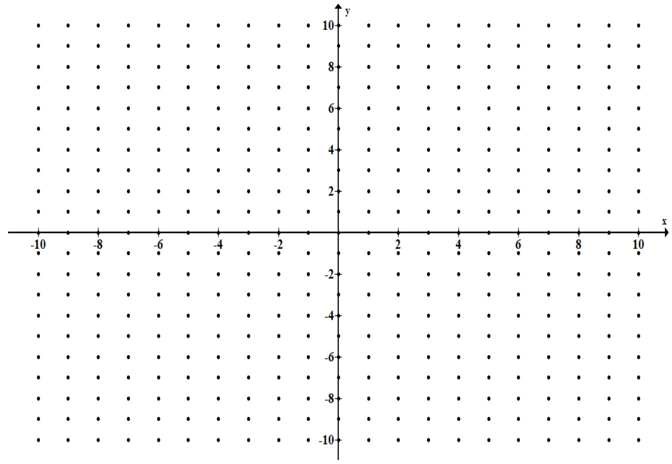
36. $f(x) = \begin{cases} x+3 & ; \ x < 0 \\ -2x+5 & ; \ x \geq 0 \end{cases}$



37. $g(x) = \begin{cases} \frac{1}{2}x & ; \ -4 \leq x \leq 2 \\ 2x-3 & ; \ x > 2 \end{cases}$



38. $h(x) = \begin{cases} |x| & ; \; x \leq 1 \\ 2 - |x-2| & ; \; x > 1 \end{cases}$



For #39-43, solve each exponential equation and round answers to the nearest thousandth. Some equations can be solved by writing each side as the same base while others will require a logarithm.

39. $5^x = \frac{1}{5}$ ____________

40. $6^x = 1296$ ____________

41. $6^{2x-7} = 216$ ____________

42. $5^{3x-1} = 49$ ____________

43. $10^{x+5} = 125$ ____________

For #44-47, simplify each expression without the use of a calculator. The exponential properties on page 2 of this packet will help.

44. $e^{\ln 4} =$ ____________

45. $e^{2\ln 3} =$ ____________

46. $\ln e^9 =$ ____________

47. $5\ln e^3 =$ ____________

For #48-53, solve each exponential or logarithmic equation by hand. Round answers to the nearest thousandth.

48. $e^x = 34$ ____________

49. $3e^x = 120$ ____________

50. $e^x - 8 = 51$ ____________

51. $\ln x = 2.5$ ____________

52. $\ln(3x - 2) = 2.8$ ____________

53. $2\ln(e^x) = 5$ ____________

For #54-66, find the <u>exact</u> value of the expression using the Unit Circle. To be clear, "exact" answer means no decimals!

54. $\sin 120° =$ ________

55. $\cos \frac{11\pi}{6} =$ ________

56. $\tan 225° =$ ________

57. $\sin\left(-\frac{2\pi}{3}\right) =$ ________

58. $\sin 150° =$ ________

59. $\tan \frac{7\pi}{4} =$ ________

60. $\csc\left(\frac{\pi}{4}\right) =$ ________

61. $\sec(-210°) =$ ________

62. $\cot\left(\frac{5\pi}{4}\right) =$ ________

63. $\sin\left(\frac{9\pi}{4}\right) =$ ________

64. $\sec\left(-\frac{\pi}{4}\right) =$ ________

65. $\tan\left(-\frac{4\pi}{3}\right) =$ ________

66. $\cos\left(\frac{8\pi}{3}\right) =$ ________

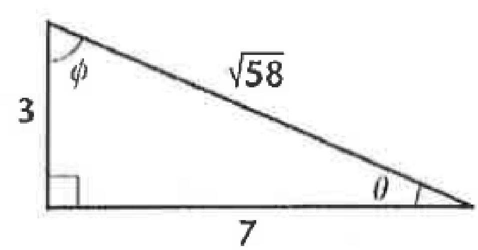
For #67-70, evaluate each trigonometric expression using the right triangle provided. You do <u>NOT</u> need to rationalize the denominator.

67. $\sin\theta =$ ________

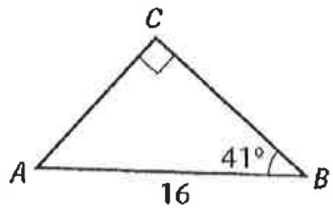
68. $\cos\theta =$ ________

69. $\tan\phi =$ ________

70. $\sec\phi =$ ________



71. Solve the triangle, rounding all angles and sides to the nearest thousandth. ("Solving a triangle" means to find all missing sides and angles.)



$m\angle A =$ ___________

$AC =$ ___________

$CB =$ ___________

For #72-79, evaluate each inverse trigonometric function using the Unit Circle. Write all answer in radians, not degrees. Do not use a calculator.

72. $\sin^{-1}\left(\frac{1}{2}\right) =$ _______

73. $\sin^{-1}(-1) =$ _______

74. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$ _______

75. $\tan^{-1}(\sqrt{3}) =$ _______

76. $\tan^{-1}(-1) =$ _______

77. $\tan\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right) =$ _______

78. $\sin\left(\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right) =$ ______

79. $\sin^{-1}(\cos(0)) =$ ________

80. Explain how the graph of $f(x)$ and its inverse, $f^{-1}(x)$, compare.

____

____

For #81-83, find the inverse of each function.

81. $g(x) = \frac{5}{x-2}$

$g^{-1}(x) =$ ______________________

82. $f(x) = \frac{x^2}{3}$

$f^{-1}(x) =$ ______________________

83. $y = \sqrt{4-x} + 1$

$y^{-1} =$ ______________________

84. If the graph of $f(x)$ has the point $(2,7)$, then what is one point on the graph of $f^{-1}(x)$? ____________

For #85-89, write each inequality in interval notation. For example, $x > 3$ becomes $(3, \infty)$.

85. $1 < x \leq 10$ ____________________

86. $x < 0$ **or** $x \geq 4$ ____________________

87. $x \geq -2$ ____________________

88. $x \geq 4$ **and** $x > 10$ ____________________

89. $x > 5$ **or** $x < 7$ ____________________

For #90-99, find the domain and range of each function. Write answers in interval notation. Confirm your answer by graphing the function on your calculator. The parent functions on page 3 of this packet will help.

90. $f(x) = \sqrt{x+5}$

D: ____________________ R: ____________________

91. $g(x) = x^2 - 5$

D: ____________________ R: ____________________

92. $y(t) = \frac{1}{t+7}$

D: ____________________ R: ____________________

93. $h(x) = \frac{5}{x^2+1}$

D: ____________________ R: ____________________

94. $f(x) = \sqrt{x^2+5}$

D: ____________________ R: ____________________

95. $g(t) = t^3 + 2t - 7$

D: ____________________ R: ____________________

96. $h(x) = 3\sin(\pi x) - 1$

D: ____________________ R: ____________________

97. $y(x) = \sqrt[5]{2x+3}$

D: ____________________ R: ____________________

98. $f(x) = -3e^{2x} + 5$

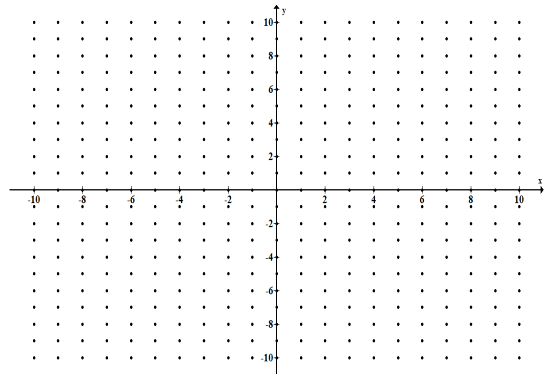
D: ____________________ R: ____________________

99. $g(t) = \log_4(x-2) + 1$

D: ____________________ R: ____________________

The remaining exercises are more challenging and specifically from concepts and skills covered in Pre-Calculus. You must show all work to earn credit.

103. State the domain and range of $f(x) = \frac{2x^2-6x-20}{x^3-2x^2-15x}$



D: ______________________ R: ______________________

104. Consider the function $f(x) = \frac{e^x}{\log x - x^3}$.

a. Use your calculator to find the relative maximum and minimum y-value of $f(x)$.

min = _______________ max = _________________

b. State the domain of $f(x)$ in interval notation.

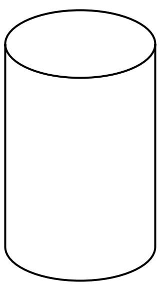
D: ____________________________

c. State when the function is increasing and decreasing. Write in interval notation.

increasing: ____________________________________ decreasing: ________________________________

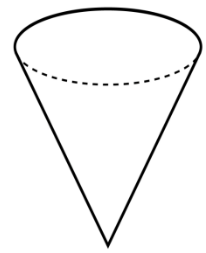
105. A rectangular sheet of tin measures 20 inches by 12 inches. Suppose you cut a square out of each corner and fold up the sides to make an open-topped box. What size square should you cut out in order to maximize the volume of the box? Show all work to earn credit.

106. You have been asked to design a cylindrical can that will hold 1000 cubic centimeters. What dimensions (height and radius) will use the least amount of material?



$r =$ ______________ $h =$ _______________

107. An inverted conical reservoir has a height of 10 inches and a base diameter of 12 inches. It is slowly being filled with water. Write an expression for the volume of the water in terms of its…



Use similar triangles

a. radius

$V(r) =$ ____________________

b. height

$V(h) =$ ____________________

108. Evaluate the following limits algebraically.

a. $\lim_{x \to 1} e^{x^3 - x} =$ ________

b. $\lim_{x \to -3} \dfrac{x^2 - 9}{x^2 + 2x - 3} =$ ________

c. $\lim_{x \to 5^+} \dfrac{x + 5}{x - 5} =$ ________

d. $\lim_{h \to 0} \dfrac{(h - 1)^3 + 1}{h} =$ ________

e. $\lim_{x \to 0} \dfrac{\sqrt{4 + x} - 2}{x} =$ ________

f. For constant c, $\lim_{x \to c} x =$ ________

g. For constants a and c, $\lim_{x\to a} c =$ __________

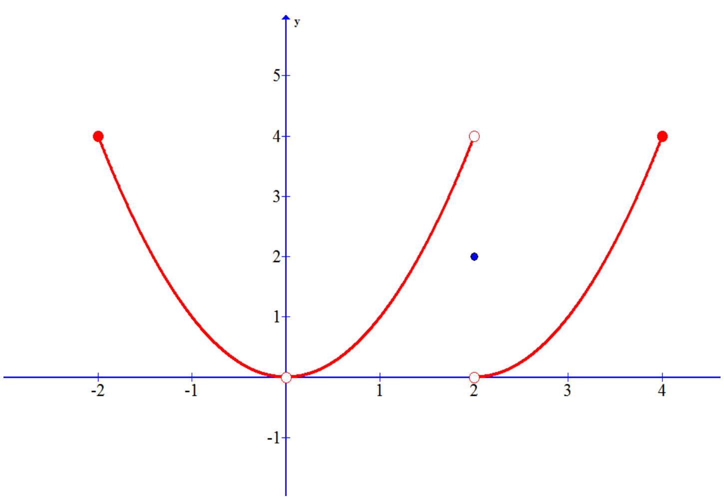
h. $\lim_{h\to 0} \frac{h}{\sqrt{x+h}-\sqrt{x}} =$ __________

i. $\lim_{x\to\infty} \frac{3x^3+5x^2-7x}{8x^3-13} =$ __________

j. $\lim_{v\to 4^+} \frac{4-v}{|4-v|} =$ __________

k. $\lim_{x\to\infty} \frac{\sqrt{x^2-9}}{2x-6} =$ __________

109. Use the graph of the function to answer the following questions. Be as specific as possible



a. $f(2) =$ __________

b. $\lim_{x\to 2^+} f(x) =$ __________

c. $\lim_{x\to 2^-} f(x) =$ __________

d. $\lim_{x\to 2} f(x) =$ __________

e. $f(0) =$ __________

f. $\lim_{x\to 0} f(x) =$ __________