College and Career Readiness Standards for Mathematics

Mastering the Most Challenging Math Standards with Rigorous Instruction

Learning environments that help all students succeed with mathematics.

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Mark is a Professor of Education at California State University, Fullerton and a highly regarded scholar and teacher of K–12 mathematics education. He is best known for his collaborative work with educators in developing strategies that help students understand mathematics concepts, supporting instruction around new standards, and addressing issues of equity in mathematics education. He is also known for his research on middle grades mathematics teaching and learning; equity, discourse, and technology in mathematics education; and preparation of teachers of mathematics. With over 40 publications to his credit, Mark has served on the National Council of Teachers of Mathematics (NCTM) Board of Directors and Executive Committee. His awards and key positions include: Board of Directors, Executive Committee, NCTM; Department Chair and Professor, Education, CSU Fullerton; National Board Certified Teacher. Mark is an author of the Ready® Mathematics program.

Introduction

New content standards for mathematics such as the Common Core State Standards (CCSS) are designed to help today's students build a solid foundation of knowledge and skills in preparation for the world they will enter and eventually lead.

The new standards emphasize the concepts behind the calculations and how to reason about complex problems. It is no longer sufficient for students just to know basic algorithms and facts, given that the machines around us do millions of calculations every second. Instead, today's young people need to reason and analyze in order to make data-based decisions and develop creative approaches to the non-routine problems of the 21st century.

> *When educators focus on the most challenging areas first, they are able to **maximize their time, accelerate progress,** and create learning environments in which all students can succeed with mathematics.*

This paper provides insight into the four most challenging areas in the math standards: **measurement**, **modeling**, **fractions**, and **statistics**.

When educators focus on the most challenging areas first, they are able to maximize their time, accelerate progress, and create learning environments in which all students can succeed with mathematics.

Impact of the New Math Standards

The new standards emphasize both knowledge and practice, recognizing that students must be able to use math as a tool for understanding the world in order to leave school ready for college or career. In today's world, students are unlikely to encounter neatly ordered problems to be solved with predetermined rules, since these are routinely taken care of by computers. Instead, they will be called on to tackle complex situations that require reasoning, sense-making, and perseverance.

As a result, learning environments must change to focus on discourse, reasoning, and problem solving so that students can develop deep, coherent content knowledge and proficiency with rich practices of mathematical thinking. These skills are essential for the 21st century, and should be taught from kindergarten on (*Brengard, 2015*).

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The Most Challenging Math Standards

The data from over 750,000 students in the *i-Ready®* diagnostics show that the most challenging Common Core math standards are related to geometric measurement, modeling problem situations, fractions, and statistics.* These are the key topics educators must pay attention to, focusing their time where it matters most. The most challenging standards require students to develop new conceptual understandings or to recognize new relationships among mathematical ideas, tasks that become more difficult if new content is not well connected to prior knowledge or experiences.

For example, when students are learning to use standard units of linear measure, they must understand no fewer than six important concepts that are described in detail later in the "Mastering Measurement" section. If they lack an understanding of these concepts, they will carry with them shallow, incomplete, and often incorrect ideas as they move on to more advanced content. It is essential that students develop their own understanding through meaningful activities and discourse that allow them to own the concepts and use them as building blocks for future learning.

Research confirms that students are more likely to use their skills accurately and flexibly when they understand the concepts behind the computations, becoming more efficient learners who are better able to recall and retain knowledge over time (*Baroody, Bajwa, & Eiland, 2009; Fuson, Kalchman, & Bransford, 2005*).

*Independent research demonstrated that i-Ready® was found to have strong correlations to the 2013 New York State Assessment (correlations ranged from 0.77–0.85 across grades and subjects).

Preparing Students to Tackle Difficult Math Standards

Today we as educators benefit from extensive research on how to support meaningful learning for all students. In particular, studies have shown that we must stop seeing math ability as unevenly distributed and give learners the message that greater effort leads to greater learning, to help them develop more productive behaviors (*Dweck, 2006, 2008*).

Students need learning environments that emphasize coherence, reasoning, sense-making, and the relationship among mathematics concepts and procedures. They need classrooms in which there is time and space to work individually, in small groups, and as a whole class, engaged in mathematical practices that support meaningful learning. They also need resources and tools to help them explore mathematical ideas.

In the past, teachers generally presented math examples that students mimicked as best they could, with little or no discussion of how procedures worked, few opportunities to engage in non-routine problems requiring time and thought, and with limited resources (beyond a calculator) to explore mathematics.

Because the new math content standards require that students learn with meaning and develop proficiency with mathematical practices, teachers must be able to revisit their own mathematical knowledge while, at the same time, designing and implementing new learning activities for students.

Teachers and administrators must recognize that students are not the only ones learning and that students' input can help teachers deepen their own understanding of mathematics. Administrators are called upon to facilitate professional development driven by teacher input, organize schedules that promote collaboration, and model intellectual curiosity by increasing their own understanding of mathematics.

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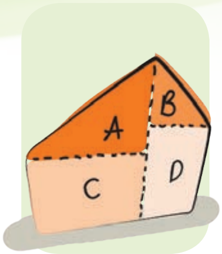


Mastering Measurement

According to the *i-Ready* research, measurement is one of the most challenging math domains for students. On the surface, measurement involves just three skills: identifying what is to be measured, choosing a unit of measure, and determining the number of units.

Six new concepts

However, what makes measurement challenging is that in order to perform these skills proficiently and meaningfully, students must understand six new concepts (*Cross, Woods, & Schweingruber, 2009*):

1. **Conservation of length or other measurable quality,** understanding that an object's length does not change when moved
2. **Transitivity of measurement,** understanding that the measures of two objects can be compared to a third object
3. **Equal partitioning,** understanding that an object can be sliced into segments of the same size or unit without changing its overall length
4. **Unit and unit iteration,** understanding that measurement requires using identical copies of a standard length
5. **Origin,** understanding that a measurement of length is a distance from a fixed starting point, including from zero
6. **Relationship between abstract number (quantity) and physical measurement characteristic (e.g., length),** understanding that a numerical measurement is a representation of an object's length

Progression

As explained in "Developing Measurement Concepts within Context," students' proficiency with linear measurement typically follows a "progression from identification of the attribute and use of informal measurement through the use of formal units, with application to contexts being the final stage" (*MacDonald & Lowrie, 2011*).

Measurement: Early Concepts and Skills

Progression enables students to build proficiency with linear measurement.

Kindergarten

- *Describe lengths (K.MD.A.1)*
- *Compare lengths (K.MD.A.2)*

Grade 1

- *Order objects by length (1.MD.A.1)*
- *Compare lengths (1.MD.A.1)*
- *Understand length measurement (1.MD.A.2)*

Grade 2

- *Understand length and measurement tools (2.MD.A.1)*
- *Measure length (2.MD.A.1)*
- *Understand measurement with different units (2.MD.A.2)*
- *Understand how to estimate with length (2.MD.A.3)*
- *Compare lengths (2.MD.A.4)*
- *Add and subtract lengths (2.MD.B.5)*
- *Represent numbers as lengths on a number line (2.MD.B.6)*

We see this progression in the Common Core State Standards for mathematics:

- **Kindergarteners** learn to describe measurable attributes of objects and compare measurable attributes of two objects, addressing the concept of conservation.
- **First graders** learn to compare two objects to a third to determine relative length (transitivity) and begin to use unit iteration to determine lengths.
- **Second graders** learn to use standard units and tools of linear measurement.

Learning progressions are important in every topic that students find challenging, since the standards were intentionally developed with coherence within and across grade levels. One program designed to build knowledge based on learning progressions is *Ready Mathematics*; each teacher resource book includes a visual map of key progressions and lesson-specific connections to the progressions.

Classroom Activities

Measurement should not be taught simply as a skill to be mimicked. Teachers must build understanding of measurement concepts starting with familiar contexts and experiences. For example, to establish the concept of unit and unit iteration, have students first measure lengths using informal units. They might take a bunch of paperclips (as a unit) and line them up end-to-end to determine the length of the side of their desk. Challenge them to think about how to measure with just one copy of a unit (e.g., one paperclip). This leads them to understand unit iteration. A great follow up is to ask students to make their own measurement tools (rulers) based on a unit of their choosing. Then have students compare measures of the same items using their different measuring tools. This will lead to discussions about the difficulty of making comparisons when units are different. In other words, non-standard units can be used to show the need for standard units.

Building the foundation for later grades

Students' early learning about linear measurement in grades K–2 sets the foundation for later work with two-dimensional measurement of area in grades 3–5 and three-dimensional measurement in grades 5–7. Students in the upper grades are expected to develop a flexible approach to solving measurement problems, including decomposing figures, choosing appropriate units, and making reasonable estimates. Understanding each of the six key concepts of measurement is essential to students' success with the later standards.

Students' early learning about linear measurement in grades K–2 sets the foundation for later work...

In the CCSS, the culmination for measurement comes in grade 8 when students learn about geometric relationships on the coordinate plane using transformations. Much of what students need to know about measurement in order to engage productively with standard 8.G.A.2 goes back to their early learning about measuring length. To be ready for this standard, students must understand concepts of origin, unit, conservation of length, and transitivity. After all, the coordinate plane formed by x and y axes is, at its foundation, two perpendicular number lines. So early learning about measurement matters.

Classroom Activities

Once students start using standard units, you can stretch their understanding of origin and conservation by requiring them to measure beyond the end of the ruler or other tool, or to begin measuring at a non-zero location. This will help them understand that the measure of a specific length is a characteristic of the object itself, not simply a number on a measurement tool.

As you design early measurement activities, build on students' everyday experiences and interests and make their thinking the center of classroom discussions. For example, ask students to measure various parts of their body with yarn and find the lengths in centimeters. Then ask groups of students to compare the measurements of two students and make posters for the class math wall.

To help students understand the relationship between perimeter (as a measure of distance in linear units) and area (as a measure of two-dimensional space having square units), ask students to explore, with a partner, how many different rectangles can be formed with a given perimeter but yielding different areas. *Ready Mathematics* Grade 4 Lesson 26 uses the context of maximizing the size of a rectangular pen given a fixed length of fencing to engage students in such an exploration. After writing down their observations, a whole class discussion is a good way to check for understanding.

To continually reinforce measurement concepts and skills in an informal but productive way, try occasional estimation tasks that get students thinking and talking. For older students struggling with measurement, it may be helpful to assess and revisit their understanding of the six key measurement concepts. Two articles, "Measurement of Length: How Can We Teach It Better?" (*Kamii, 2006*) and "Assessing Children's Understanding of Length Measurement" (*Bush, 2009*) offer assessment items that target these concepts.

Hands-On Activity

Use geoboards to understand finding perimeter and area

Materials: geoboards, rubber bands

- Have students build rectangles on the geoboard and then find the area and perimeter. For example, what is the perimeter and area of a rectangle with width of 4 and length of 6?
- Have students use the geoboard to find as many different combinations of length and width that will produce a given area of a rectangle. For example, if a rectangle has an area of 36 square units, what are all the possible combinations of length and width for the rectangle?
- Have students use the geoboard to find as many different combinations of length and width that will produce a given perimeter of a rectangle. For example, if a rectangle has a perimeter of 24 units, what are all the possible combinations of length and width for the rectangle?

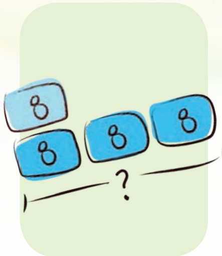
Challenge Activity

Provide the following problem situation for students to solve.

A farmer has 100 feet of fencing to build an enclosed, rectangular pen for his animals.

- In order to get the largest possible area to keep his animals, what would be the length and width of animal pen he could make? [10 feet by 10 feet]
- What would be the dimensions of the largest rectangular pen he could make with 169 feet of fence? [13 feet by 13 feet]
- What generalizations can you make about getting the largest possible rectangular area with a given perimeter? [The largest rectangular area will be a square.]

Mastering Modeling



The challenge of helping students make sense of modeling with mathematics begins with the word itself. Although in everyday language the word "model" refers to a small-scale replica of a real object, in mathematics a model is a representation of a real-world object or situation.

A familiar example might be weather forecasts, which are based on mathematical models that involve many elements to predict what might happen. For complex phenomena like weather, there is no one correct model. Many different models with varying degrees of accuracy are constantly being refined and revised. The match between the real world and a mathematical model can range from exact, as in a model of combining nine books with 15 books, to impressionistic, as in models for weather forecasts that give best estimates based on those variables that can be measured and the mathematical relationships among them.

Modeling is especially challenging when students have only experienced math as abstract rules and procedures or as contrived problems to be solved with a recently learned algorithm. In order to become proficient, students must regularly engage in modeling that offers opportunities to apply their learning. They need to look at real-world situations and think about what elements can be represented mathematically. Then they can collect data, generate mathematical expressions, check the data against the original situation, and make revisions.

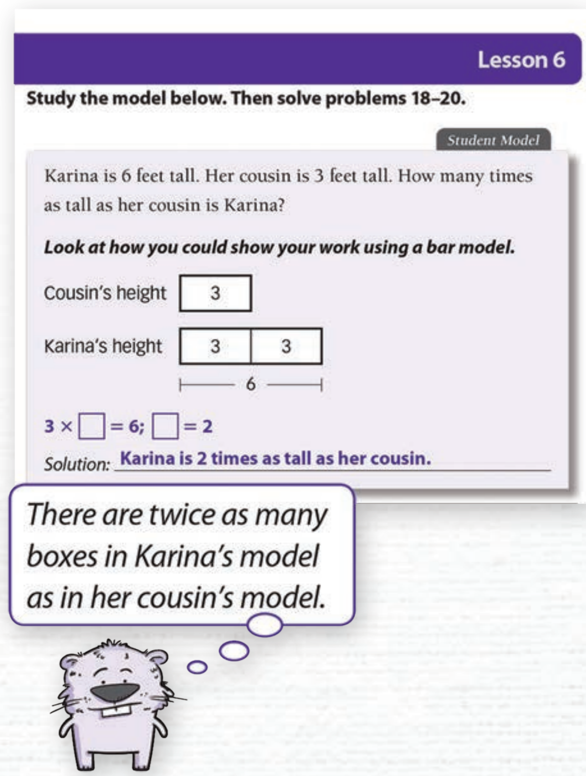
Modeling steps

Modeling involves several steps that may or may not be sequential:

- **Identify** a problem situation
- **Make a representation** of one or more elements of the situation
- **Create a mathematical expression**
- **Compare results** or predictions from the mathematical model with the real situation
- **Make revisions** to the model if needed

Classroom Activities

For primary grade students, teachers can suggest strategies for representing simple situations. For example, *Ready Mathematics* Grade 4 Lesson 6 asks students to use bar diagrams to represent contextual problems requiring multiplication or division. Students create mathematical expressions and are asked to explain how each element of their diagram and expression connects to the original context, thereby reinforcing mathematical sense making.



Bar diagrams can be a useful tool for students as they engage in modeling tasks into middle school, allowing them to generate visual representations from which they can figure out mathematical relationships.

The example below comes from a seventh grade task. Notice how the bar diagrams help to organize students' thinking and scaffold the writing of mathematical expressions.

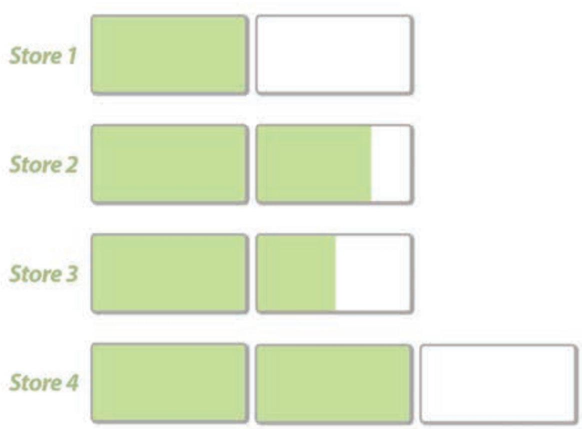
The activities on the Thinking Blocks website help students learn to use rectangular bar diagrams to represent and solve problems. When working with modeling problems, always ask students to explain how their work—the visual model, mathematical model, and final answer—relates to the original problem context. This will strengthen the habit of looking for connections and checking that the methods and results make sense.

Four different stores are having a sale on shirts. The signs below show the discounts available at each of the four stores.

Store 1: Two for the price of one
Store 2: Buy one, get 25% off the second
Store 3: Buy two, get 50% off the second
Store 4: Three for the price of two

Which store is offering the best deal? Prove it!

Solution: Look at how this might be represented visually.



		Each shirt costs:
Store 1	→ $1 \times \text{price} = 2$ shirts →	$\frac{1}{2} \times$ price OR 0.5 price
Store 2	→ $1.75 \times \text{price} = 2$ shirts →	$\frac{1.75}{2} \times$ price OR 0.875 price
Store 3	→ $1.5 \times \text{price} = 2$ shirts →	$\frac{1.5}{2} \times$ price OR 0.75 price
Store 4	→ $2 \times \text{price} = 3$ shirts →	$\frac{2}{3} \times$ price OR $0.6\overline{6}$ price

Solution: **Order the fraction of the price you pay for a shirt at each store from least to greatest:**

$0.5 < 0.66 < 0.75 < \overline{0}.875$

The lowest price per shirt is at Store 1, so it has the best deal.

A strategy to use at all grade levels is to invite students to come up with modeling problems. When challenged to create their own problems, students are able to exercise more creativity while deepening their understanding of math concepts, relationships, and skills.

When challenged to create their own problems, students are able to exercise more creativity while deepening their understanding of math concepts, relationships, and skills.

Modeling also lends itself to cross-disciplinary problems. Mosa Mack Science Detective offers explorations for grades 4–8 aligned to current science, math, and English standards. An example from NCTM's Mathematics Teaching in the Middle School, "Preserving Pelicans with Models that Make Sense," asks students to examine the issue of pelican population using visual and mathematical models, offering interdisciplinary connections with science standards.

As students become more comfortable with modeling, encourage them to identify their own problems based on familiar situations that may not have an obvious solution and solve these problems using modeling. For a real world case of this, see the article "Posing Problems that Matter: Investigating School Overcrowding." When sixth-grade students complained that their urban middle school was overcrowded, the teacher developed a unit on area, perimeter, and similar figures, and students were challenged to prove to the school board that their school was, in fact, too crowded.

As students move into high school, modeling tasks become less well defined and mathematical representations move from arithmetic to algebraic. However, the modeling steps remain the same. The more students become comfortable with these in elementary and middle grades, the more ready they will be for later modeling problems.

Mastering Fractions

A 2010 report, "[Developing Effective Fractions Instruction for Kindergarten through 8th Grade](),"found that half of eighth graders could not order three fractions from least to greatest.

The authors identified several recommendations for improving fraction instruction (*Siegler et. al., 2010*):

- Build on students' informal understanding of **sharing and proportionality** to develop initial fraction concepts.
- Help students recognize that **fractions are numbers and that they expand the number system beyond whole numbers**. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.
- Help students understand **why procedures for computations with fractions make sense**.

These recommendations help students develop "fraction sense" by understanding key concepts and how they relate to prior knowledge of number and operations, and how they are reflected in the design of the learning progressions for fraction sense and fraction operations in the Common Core math standards.

Knowledge of learning progressions lets teachers think about concepts, strategies, and models that will be used over several grades as students develop their understanding.

Early work in grades 1 and 2 with fraction concepts—although not with fractions written as numbers—involves visually identifying halves, thirds, and fourths using geometric figures such as rectangles and circles. Before students are asked to consider operations with fractions, they must understand what fractions represent conceptually and how fractions fit into and extend the set of whole numbers. When planning for instruction, knowledge of learning progressions lets teachers think about concepts, strategies, and models that will be used over several grades as students develop their understanding.

Essential to strong fraction sense is learning to think about unit fractions with a numerator of 1 as building blocks for other fractions, based on an understanding that "1/n" measures one part of a whole that is cut into "n" equal-sized parts. This extends students' understanding that whole numbers are made up of units—ones, tens, and hundreds—and their knowledge of measurement concepts.

Classroom Activities

Check out the Math Maniac blog for fun games that encourage discussion and help students understand that fractional pieces do not have to be congruent in order to be equivalent.

Use the *Ready Mathematics* lessons on grade 3 fraction standards to start with unit fractions and add visual representations to help make sense of fraction relationships. For example, in Grade 3 Lesson 16, students explore the concept of equivalent fractions with fraction strips (rectangular area model) and number lines (linear model). Ask students to make their own set of fractions strips using a blank template to make fractions tangible then probe their understanding with queries such as, "How many 1/5s make one whole?" and "Prove in two ways which is larger, 2/5 or 2/3."

The goal of all of this early work is to have students develop a more coherent understanding of the meaning of fractions, so that rather than seeing 3/5 as "3 over 5" they think of it as "three one-fifths" or "1/5 + 1/5 + 1/5." Instead of teaching tricks or procedures to compare fractions, students should be encouraged to use sense-making strategies such as those outlined in "Laying the Foundation for Success in Algebra," which includes "fraction tents" that can be used for a fraction clothesline (number line) ordering activity.

Once students recognize fractions as numbers with meaning, introduce them to operations with fractions. While standard algorithms for fraction operations are often efficient, they should not be taught to students in the absence of sense-making and understanding why they work. Ideally, students' own thinking will form the basis for what they come to know as standard algorithms. In addition to Common Core aligned materials such as *Ready Mathematics*, check out the Rational Number Project for research-based activities that support learning fraction operations with understanding.

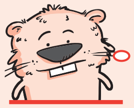
Fractions: Making sense of fraction relationships

Focus on **Math Concepts**

Lesson 16 **Part 1: Introduction**

Understand Equivalent Fractions

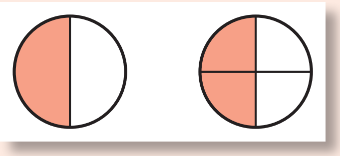
CCSS
3.NF.A.3a



How can two different fractions be equal?

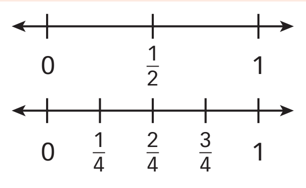
Two fractions can be equal if they show the same amount of the whole.

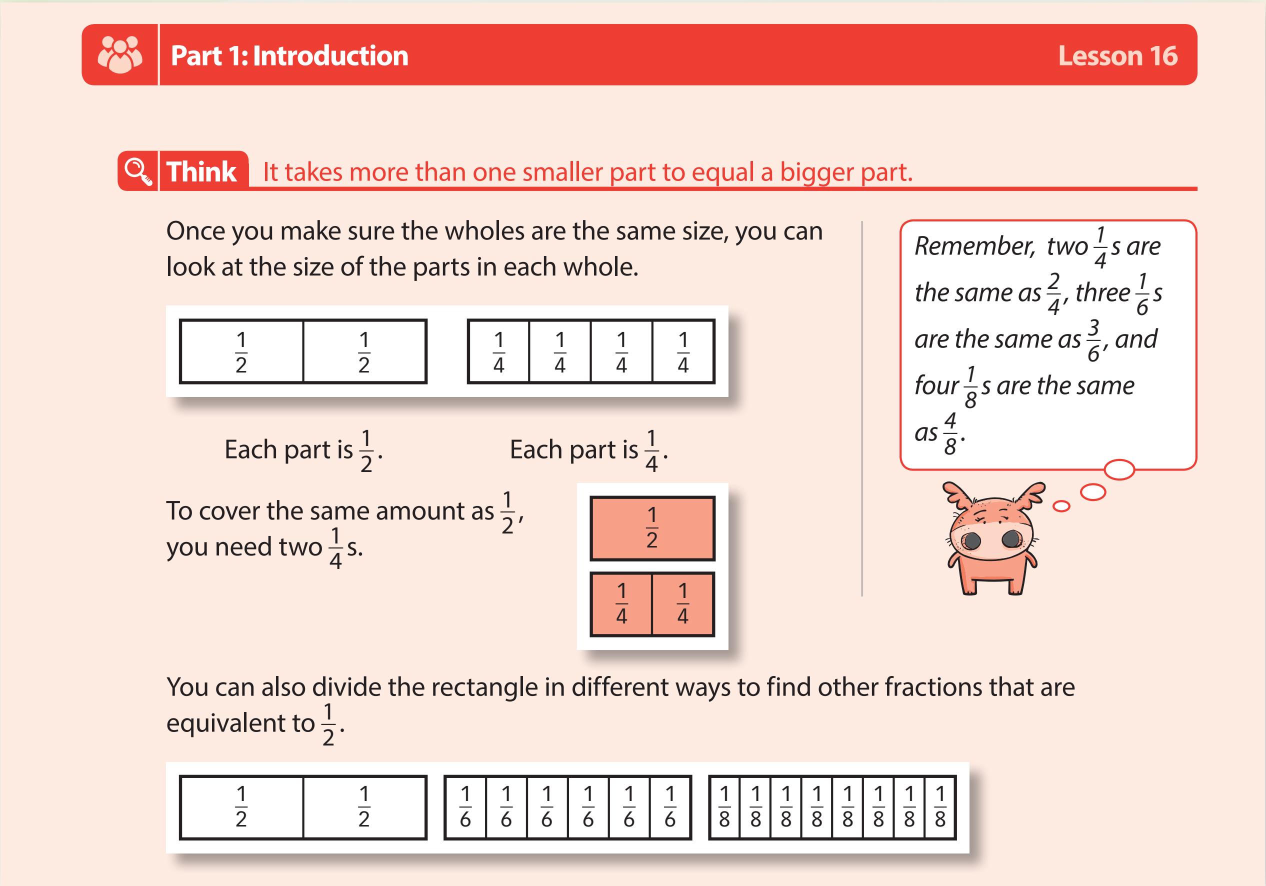
These are called **equivalent fractions**. The same area in each of the circles is shaded. Each circle is divided into a different number of parts. So, the fractions used to name the parts are different.



$\frac{1}{2}$ $\frac{2}{4}$

You can also see equivalent fractions using a number line. $\frac{1}{2}$ and $\frac{2}{4}$ are located at the same point on the number line. This shows they are equivalent.





Part 1: Introduction Lesson 16

Think It takes more than one smaller part to equal a bigger part.

Once you make sure the wholes are the same size, you can look at the size of the parts in each whole.

Each part is $\frac{1}{2}$. Each part is $\frac{1}{4}$.

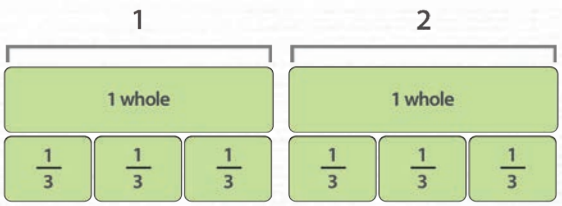
To cover the same amount as $\frac{1}{2}$, you need two $\frac{1}{4}$s.

Remember, two $\frac{1}{4}$s are the same as $\frac{2}{4}$, three $\frac{1}{6}$s are the same as $\frac{3}{6}$, and four $\frac{1}{8}$s are the same as $\frac{4}{8}$.

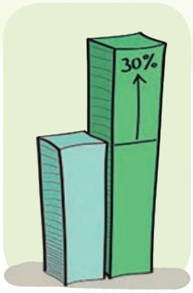
You can also divide the rectangle in different ways to find other fractions that are equivalent to $\frac{1}{2}$.

Give students opportunities to develop their own ways of reasoning about fraction operations, typically starting with visual models and then moving to abstract numerical representations. Hone your ability to elicit and extend students' thinking and engage them in error-analysis tasks aimed at addressing common mistakes or misconceptions. These activities get students communicating with and about fractions and also help to develop their academic language.

One of my favorite topics to teach is division by a fraction. It is a joy to see the light come on when learners understand why division by a fraction between 0 and 1 "makes things bigger." To start this investigation, ask students what situation is represented by the expression 2 ÷ 1/3. First dividing a whole number by a unit fraction lets students focus on the meaning of the expression and worry less about the quantities involved. One way to think about this is to use a measurement model for division here (how many 1/3s are in 2?). For example, this might be in the context of a recipe that calls for 1/3c of sugar and you ask how many batches can be made if you have 2 cups of sugar.



Approaching fractions by unitizing gives students not only a powerful way to make sense of this challenging topic but also prepares them well for work with algebraic expressions that will similarly require them to understand and work with units (e.g., see the table in Daro, Moscher, & Corcoran, p. 42).



Mastering Statistics

In the middle grades, students are introduced to Common Core statistical reasoning that involves four processes: **formulating questions** that can be answered with data; **designing and using a plan** to collect relevant data; **analyzing** the data with appropriate methods; and, **interpreting results** and drawing valid conclusions that relate to the questions posed.

Data displays such as picture graphs and line plots are not new. For many years, students have learned to look for specific values such as "the greatest value" or "the least value" and make simple comparisons within a sample of univariate data (data with one measurable quality). But now, beginning in sixth grade, students must learn to do the following:

- Examine, represent, and interpret the characteristics of distributions of data such as measures of center, measures of spread, and overall shape.
- Use these characteristics together with the data context to answer statistical questions.
- Apply and extend these understandings to explore and analyze bivariate data representing two categories or measurable qualities—a much more challenging activity.

Statistics students need to understand the concepts of variability and center, since these form the foundation for their later explorations involving bivariate data and probabilistic models. If data never varied (we always had exact measurements and quantities), there would be no use for statistics. We would simply want the exact value! For example, if the height of all sixth graders was 150 cm and you were asked to find the mean, median, mode, and range, you would quickly realize there is no need for these since the data do not vary. So *variability* is the first big concept of statistics.

Mean, median, and mode are all measures of center and provide different perspectives on the center of a data set. Perhaps the most poorly understood is the mean, since students are often given a procedure—add up all values and divide by the number of values—instead of help with understanding the concept. If students learn the mean as an algorithm without first understanding the concept as a measure of center, they often have difficulty making meaningful connections between their calculations and the data context.

As with the other challenging standards, the key to supporting student success in statistics is to give them time to make sense of new concepts and relationships.

When starting out with statistics, students tend to see all data sets as comprised of individual values representing a whole population from which they can find the middle (center, density), spread, and skewness. This reflects a perspective that all data is self-contained (e.g., we surveyed everyone in the class). Moving into 7th and 8th grade, students need to learn to see data sets as just one sample from a larger distribution for which they may never know the "actual" mean. This perspective on statistics uses probability-based models (think normal distribution) to gauge how well a sample, or set of samples, represents an overall distribution or how one distribution compares to another.

For instance, back to thinking about height of 6th graders, if we compared a distribution of heights for 6th graders to a distribution of heights for 8th graders, the shapes might look very similar (meaning similar spread and density) but the "center" likely has shifted (different mean, median).

Also, starting in 8th grade and into high school students will move from working with univariate data to bivariate data. They will need to be fluent in working with coordinate graphs to plot bivariate data and increasingly complex mathematical models that represent patterns in data (from linear with lines of best fit to non-linear).

Classroom Activities

To help students make sense of variability, start by discussing the difference between a statistical question and a non-statistical question that lacks variability. *Ready Mathematics* Grade 6 Lesson 26 offers a set of activities and prompts designed for this purpose. The goal is to help students recognize questions involving variability. As an extension, students can be asked to rank a set of questions in order of greatest to least variability, which sets them up well for later work with measures of variability including the mean average deviation (MAD).

Statistics: Making sense of variability

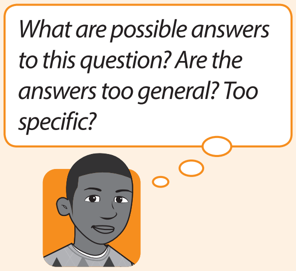
Part 1: Introduction **Lesson 26**

Think How do I write statistical questions?

What statistical question could Sasha ask if she was interested in knowing what school sport sixth graders like to watch the most?

Look at the questions Sasha wrote. "What is your favorite sport to watch?" is too general. Someone's favorite sport to watch might not be a school sport. There may be too many varying answers.

"What was the last sports game you watched at this school?" is too specific. Depending on the time of year or what home game was most recent, there may not be enough variability.



To collect data on what school sport sixth graders like to watch the most, Sasha could ask:

"Which school sport are you most interested in watching? Circle one from the list below."

Then Sasha could list all the school sports at her school.

Possible responses would be one of the listed sports. The varying answers would help Sasha draw conclusions about which sports sixth graders at her school most like to watch.

Now you'll have a chance to think more about statistical questions and the data they help collect.

When working with data sets—which should be a frequent activity—have students create displays based on individual data points and displays that represent groups of data such as histograms, box plots, and stem and leaf plots. Then ask them to describe the shape of the data, what they see visually and what that means in relation to the data context, including the shape of the data and any characteristics such as typical or common values. Early on, allow students to use whatever language they wish, since the goal is to reinforce the idea that the representations have meaning and to generate curiosity about possible meanings.

Visual Model

Use counters to find the mean of a set of data.

Materials: counters, data sets

Write the following list on the board: 3, 7, 1, 3, 6. Give students the following instructions:

- Use counters to make stacks that match the data.
- Now move some of the counters so that all 5 stacks have an equivalent number of counters.
- Ask: *How many counters are in each stack?* [4]. The set of data can be described by the number 4. It is the mean, or average of the data set.

Repeat the activity as needed with the following data sets: (3, 4, 2, 5, 1, 3); (6, 7, 4, 3); (7, 8, 4, 11, 5).

Challenge students to collect their own data by surveying classmates and then use the counters to find the mean.

To help grade school students understand the mean, use the concepts of fair sharing and balance point. Fair sharing takes a large collection of values, puts them all together, and redistributes them evenly, and the mean indicates how much each person will get.

Acting out a fair share scenario can help students connect the concept to the procedure for finding the mean. You can also draw on the article "The Mean as Balance Point" to help students think about the mean as the value around which the data are balanced, which nicely extends to making sense of the mean average deviation in later grades.

Excellent activities about Common Core data and statistics standards are available in the blog post "Statistical Reasoning in the Middle School." The *Mathematics Assessment Project* offers an activity involving relationships among mean, median, mode, and range to gauge and reinforce students' fluency with these topics. The *Statistics Education Web* provides many curated lesson plans aligned with GAISE standards and Common Core. The *GAISE Reports* offer a framework for developing statistical reasoning across the four processes and three developmental levels from A to C. And the *Interactivate* website offers several free apps to explore statistics including univariate and bivariate data.

Conclusion

To address the most challenging math standards identified by *i-Ready Diagnostic*, it is important to understand the reasons why students struggle and to think about ways that promote greater success. Consistent themes in teaching the difficult standards include:

- Promote understanding of the concepts behind the calculations
- Design sequences of learning activities that promote coherence
- Make connections within and between topics and across grade levels
- Elicit student ideas to drive reasoning and sense-making
- Deepen student knowledge through questioning and investigation
- Use well-designed assessment to inform instructional decisions

To ensure success with the new standards, administrators and other educational leaders must support teachers' ongoing professional learning. New standards-aligned curriculum materials, although important, are not enough. Structures must be in place and resources must be available to respond to teachers' needs as lifelong learners.

As teachers work to support student success with the more rigorous math standards, their own knowledge is changing and growing. And they are not working in isolation. Many innovative and caring teachers are collaborating locally and nationally to share their experiences with new instructional strategies, concept-oriented lessons, and tasks that challenge students to think deeply about math. It is truly an exciting time to be a mathematics educator!

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