**NCDPI Unpacked Content
with
OCS Priority Standards Identified**

MS NC Math 1
Reveal Math

2022 Alignment

North Carolina Math 1 Standards

Number	Algebra	Functions	Geometry

Number

The real number system
Extend the properties of exponents
NC.M1.N-RN.2

Algebra

Overview
Seeing structure in expressions
Interpret the structure of expressions
NC.M1.A-SSE.1a
NC.M1.A-SSE.1b
Write expressions in equivalent forms to solve problems
NC.M1.A-SSE.3

Perform arithmetic operations on polynomials
Perform arithmetic operations on polynomials
NC.M1.A-APR.1
Understand the relationship between zeros and factors of polynomials
NC.M1.A-APR.3

Creating Equations
Create equations that describe numbers or relationships
NC.M1.A-CED.1
NC.M1.A-CED.2
NC.M1.A-CED.3
NC.M1.A-CED.4

Algebra

Reasoning with equations and inequalities
Understand solving equations as a process of reasoning and explain the reasoning
NC.M1.A-REI.1
Solve equations and inequalities in one variable
NC.M1.A-REI.3
NC.M1.A-REI.4
Solve systems of equations
NC.M1.A-REI.5
NC.M1.A-REI.6
Represent and solve equations and inequalities graphically
NC.M1.A-REI.10
NC.M1.A-REI.11
NC.M1.A-REI.12

Functions

Overview
Interpreting Functions
Understand the concept of a function and use function notation
NC.M1.F-IF.1
NC.M1.F-IF.2
NC.M1.F-IF.3
Interpret functions that arise in applications in terms of a context
NC.M1.F-IF.4
NC.M1.F-IF.5
NC.M1.F-IF.6
Analyze functions using different representations
NC.M1.F-IF.7
NC.M1.F-IF.8a
NC.M1.F-IF.8b
NC.M1.F-IF.9

Building Functions
Build a function that models a relationship between two quantities
NC.M1.F-BF.1a
NC.M1.F-BF.1b
NC.M1.F-BF.2

Linear, Quadratics and Exponential Models
Construct and compare linear and exponential models to solve problems
NC.M1.F-LE.1
NC.M1.F-LE.3
Interpret expressions for functions in terms of the situations they model
NC.M1.F-LE.5

Geometry

Overview
Expressing geometric properties with equations
Use coordinates to prove simple geometric theorems algebraically
NC.M1.G-GPE.4
NC.M1.G-GPE.5
NC.M1.G-GPE.6

Statistics & Probability

Overview
Interpreting Categorical and Quantitative Data
Summarize, represent, and interpret data on a single count or measurement variable
NC.M1.S-ID.1
NC.M1.S-ID.2
NC.M1.S-ID.3
Summarize, represent, and interpret data on two categorical and quantitative variables
NC.M1.S-ID.6a
NC.M1.S-ID.6b
NC.M1.S-ID.6c
Interpret linear models
NC.M1.S-ID.7
NC.M1.S-ID.8
NC.M1.S-ID.9

Standards for Mathematical Practice

Practice	Explanation
1. Make sense of problems and persevere in solving them.	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
2. Reason abstractly and quantitatively.	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
3. Construct viable arguments and critique the reasoning of others.	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
4. Model with mathematics.	Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
6. Attend to precision.	Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
7. Look for and make use of structure.	Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.
8. Look for and express regularity in repeated reasoning.	Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

MS Math 1 Instructional Blueprint

UNIT	CONCEPT	DURATION	OCS PRIORITY STANDARD(S)	REVEAL MODULE	REVEAL LESSON(S)
1	Equations & Introduction to Functions	30-34 days	NC.M1.A-SSE.1 NC.M1.A-CED.1 NC.M1.A-REI.10 NC.M1.F-IF.4 NC.M1.F-IF.9	Math 1: Module 1	1, 2, 3, 4
				Math 1: Module 2	1, 2, 3, 4, 7
				Math 1: Module 6	1, 2
				Math 1: Module 3	1, 2, 3, 4, 5, 6
2	Linear Functions	18-22 days	NC.M1.A-CED.2 NC.M1.A-REI.10 NC.M1.F-IF.7 NC.M1.F-IF.9 NC.M1.G-GPE.5 NC.M1.S-ID.6	Math 1: Module 4	1, 2, 3, 5
				Math 1: Module 5	1, 2, 3, 4, 5
				Math 1: Module 6	5
3	Introduction to Exponential Functions	16-18 days	NC.M1.A-CED.2 NC.M1.A-REI.10 NC.M1.F-IF.4 NC.M1.F-IF.7 NC.M1.F-IF.9	Math 1: Module 8	1, 3, 5, 6
				Math 2: Module 10	1, 2, 3, 4, 5, 6, 7
4	Introduction to Quadratic Functions & Equations	18-22 days	NC.M1.A-SSE.1 NC.M1.F-IF.4 NC.M1.F-IF.7 NC.M1.G-GPE.6	Math 2: Module 11	1, 2, 3, 4, 5, 6, 7
				Math 2: Module 12	1, 3, 4, 8
5	Systems of Equations & Inequalities	9-13 days	NC.M1.A-REI.5	Math 1: Module 7	1, 2, 3, 4, 5
6	Descriptive Statistics	10-14 days	NC.M1.S-ID.3	Math 1: Module 9	1, 2, 3, 4, 5, 6, 7
7	Tools of Geometry	11-15 days	NC.M1.G-GPE.4 NC.M1.G-GPE.6	Math 1: Module 10 Apex: Unit 8, Lessons 5,6	1, 2, 3, 4, 5, 6, 7

OCS Math 1 Priority Standards

ALGEBRA	

NC.M1.A-SSE.1	Interpret expressions that represent a quantity in terms of its context.
	  a. Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.
	  b. Interpret a linear, exponential, or quadratic expression made of multiple parts as a combination of entities to give meaning to an expression.

NC.M1.A-CED.1	Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.
NC.M1.A-CED.2	Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities.
NC.M1.A-REI.5	Explain why replacing one equation in a system of linear equations by the sum of that equation and a multiple of the other produces a system with the same solutions.
NC.M1.A-REI.10	Understand that the graph of a two variable equation represents the set of all solutions to the equation.
NC.M1.F-IF.4	Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.
NC.M1.F-IF.7	Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior.
NC.M1.F-IF.9	Compare key features of two functions (linear, quadratic, or exponential) each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).
GEOMETRY	

NC.M1.G-GPE.4	Use coordinates to solve geometric problems involving polygons algebraically
	  • Use coordinates to compute perimeters of polygons and areas of triangles and rectangles.
	  • Use coordinates to verify algebraically that a given set of points produces a particular type of triangle or quadrilateral.

NC.M1.G-GPE.5	Use coordinates to prove the slope criteria for parallel and perpendicular lines and use them to solve problems.
	  • Determine if two lines are parallel, perpendicular, or neither.
	  • Find the equation of a line parallel or perpendicular to a given line that passes through a given point.

NC.M1.G-GPE.6	Use coordinates to find the midpoint or endpoint of a line segment.
STATISTICS & PROBABILTY	
NC.M1.S-ID.3	Examine the effects of extreme data points (outliers) on shape, center, and/or spread.

NC.M1.S-ID.6	Represent data on two quantitative variables on a scatter plot and describe how the variables are related.
	  a. Fit a least squares regression line to linear data using technology. Use the fitted function to solve problems.
	  b. Assess the fit of a linear function by analyzing residuals.
	  c. Fit a function to exponential data using technology. Use the fitted function to solve problems.

Number – The Real Number System

NC.M1.N-RN.2
Extend the properties of exponents.
Rewrite algebraic expressions with integer exponents using the properties of exponents.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite	**Connections**
• Using the properties of exponents to create equivalent numerical expressions (8.EE.1)	*The following SMPs can be highlighted for this standard.* 7 – Look for and make use of structure 8 – Look for and express regularity in repeated reasoning
Connections	**Disciplinary Literacy**
• Use operations to rewrite polynomial expressions (NC.M1.A-APR.1)	*New Vocabulary – index* Students should be able to justify their steps in rewriting algebraic expressions.

Mastering the Standard	
Clarification	**Checking for Understanding**

Students extend the properties of integer exponents learned in middle school with numerical expressions to algebraic expressions.
 	
The process of "simplifying square roots" and leaving them in radical form is not an expectation for NC Math 1 students. In NC Math 2, students will extend the properties of exponents to rewriting exponential expressions with rational exponents as radical expressions.
	Students should be able to use the properties of exponents to write expression into equivalent forms.
	    Indicator: Rewrite the following with positive exponents:
	      a. $(8x^{-4}y^3)(-2x^5y^{-6})^2$
	      b. $\dfrac{(3m^2p^{-2}q)^3}{9m^{-3}q^3}$
	
	Students should be able to use the new skills of applying the properties of exponents with skills learned in previous courses.
	    Indicator: Simplify: $\sqrt{25m^{14}p^2t^4}$
	    *Note: In 8th grade, students learned to evaluate the square roots of perfect squares and the cube root of perfect cubes. In NC Math 1, students extend this skill to algebraic expressions. When addressing a problem like this in NC Math 1, students should be taught to rewrite the expression using the **properties of exponents** and then use inverse operations to rewrite. For example, $\sqrt{m^{14}} = \sqrt{(m^7)^2} = m^7$. (Some students may notice that $\sqrt{m^{14}}$ must have a positive value while m^7 can have a negative value. $\sqrt{m^{14}} = \lvert m^7 \rvert$ This discussion is an extension above the expectation of this standard in Math 1.)*
	NC Math 1, the limitation from 8th grade of evaluating square roots of perfect squares and cube root of perfect cubes still applies.

Instructional Resources	
Additional Tasks	**Additional Resources**
Raising to the Zero and Negative Power (Illustrative Mathematics)	

Algebra, Functions & Function Families

NC Math 1	NC Math 2	NC Math 3
Functions represented as graphs, tables or verbal descriptions in context		
Focus on comparing properties of linear function to *specific* non-linear functions and rate of change. • Linear • Exponential • Quadratic	**Focus on properties of quadratic functions and an introduction to inverse functions through the inverse relationship between quadratic and square root functions.** • Quadratic • Square Root • Inverse Variation	**A focus on more complex functions** • Exponential • Logarithm • Rational functions w/ linear denominator • Polynomial w/ degree $\leq$ three • Absolute Value and Piecewise • Intro to Trigonometric Functions
A Progression of Learning of Functions through Algebraic Reasoning		

The conceptual categories of Algebra and Functions are inter-related. Functions describe situations in which one quantity varies with another. The difference between the Function standards and the Algebra standards is that the Function standards focus more on the characteristics of functions (e.g. domain/range or max/min points), function definition, etc. whereas the Algebra standards provide the computational tools and understandings that students need to explore specific instances of functions. As students progress through high school, the coursework with specific families of functions and algebraic manipulation evolve. Rewriting algebraic expressions to create equivalent expressions relates to how the symbolic representation can be manipulated to reveal features of the graphical representation of a function.

Note: The Numbers conceptual category also relates to the Algebra and Functions conceptual categories. As students become more fluent with their work within particular function families, they explore more of the number system. For example, as students continue the study of quadratic equations and functions in Math 2, they begin to explore the complex solutions. Additionally, algebraic manipulation within the real number system is an important skill to creating equivalent expressions from existing functions.

Algebra – Seeing Structure in Expressions

OCS Priority Standard: NC.M1.A-SSE.1a

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

    a.   Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.

Concepts and Skills

Pre-requisite
- Identify parts of an expression using precise vocabulary (6.EE.2b)   - Interpret numerical expressions written in scientific notation (8.EE.4)   - For linear and constant terms in functions, interpret the rate of change and the initial value (8.F.4)
Connections
- Creating one and two variable equations (NC.M1.A-CED.1, NC.M1.A-CED.2, NC.M1.A-CED.3)   - Interpreting part of a function to a context (NC.M1.F-IF.2, NC.M1.F-IF.4, NC.M1.F-IF5, NC.M1.F-IF.7, NC.M1.F-IF.9)   - Interpreting changes in the parameters of a linear and exponential function in context (NC.M1.F-LE.5)

The Standards for Mathematical Practices

Connections
The following SMPs can be highlighted for this standard. 2 – Reason abstractly and quantitatively. 4 – Model with mathematics 7 – Look for and make use of structure.
Disciplinary Literacy
New Vocabulary: Quadratic term, exponential term

Mastering the Standard

Clarification	**Checking for Understanding**
The A-SSE standards require students: - to write expressions in equivalent forms to reveal key quantities in terms of its context. - to choose and use appropriate mathematics to analyze situations. Part (a) of this standard examines each individual term, coefficient, factor and/or constant in the expression. For example, the *linear expression* $mx + b$ has two <u>terms</u> (mx and b) where m is the <u>coefficient</u> of the linear term mx, and b is a <u>constant</u> term. For *quadratic expressions*, students are expected to recognize the terms of a quadratic expression in standard form as quadratic, linear and constant terms or factors the factors of a quadratic expression in factored form. Finally, for *exponential expressions*, students should recognize the factors, the base, and exponent(s) in the expression. Students extend beyond simplifying expressions to interpreting the components of an algebraic expression.	Students should recognize that in the expression $2x + 1$, "2" is the coefficient, "2" and "x" are factors, and "1" is a constant, as well as "$2x$" and "1" being terms of the binomial expression. Also, a student recognizes that in the expression $4(3)^x$, 4 is the coefficient, 3 is the factor, and x is the exponent. Using real-world context examples, the nature of algebraic expressions can be explored. **Indicator**: The height *(in feet)* of a balloon filled with helium can be expressed by $5 + 6.3s$ where s is the number of seconds since the balloon was released. Identify and interpret the terms and coefficients of the expression. **Indicator:** The expression $-4.9t^2 + 17t + 0.6$ describes the height in meters of a basketball t seconds after it has been thrown vertically into the air. Interpret the terms and coefficients of the expression in the context of this situation. **Indicator:** The expression $35000(0.87)^t$ describes the cost of a new car t years after it has been purchased. Interpret the terms and coefficients of the expression in the context of this situation.

Instructional Resources	
Additional Tasks	**Additional Resources**
Delivery Trucks (Illustrative Mathematics)	Interpreting Algebraic Expressions (Mathematics Assessment Project – FAL)

Algebra – Seeing Structure in Expressions

OCS Priority Standard: NC.M1.A-SSE.1b

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

    b.   Interpret a linear, exponential, or quadratic expression made of multiple parts as a combination of entities to give meaning to an expression.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite	**Connections**
- Interpret a sum, difference, product, and quotient as a both a whole and as a composition of parts (6.EE.2b) - Understand that rewriting expressions into equivalent forms can reveal other relationships between quantities (7.EE.2) - Interpret numerical expressions written in scientific notation (8.EE.4)	*The following SMPs can be highlighted for this standard.* 2 – Reason abstractly and quantitatively. 4 – Model with mathematics 7 – Look for and make use of structure.
Connections	**Disciplinary Literacy**
- Factor to reveal the zeros of functions and solutions to quadratic equations (NC.M1.A.SSE.3) - Creating one and two variable equations (NC.M1.A-CED.1, NC.M1.A-CED.2, NC.M1.A-CED.3) - Interpreting part of a function to a context (NC.M1.F-IF.2, NC.M1.F-IF.4, NC.M1.F-IF5, NC.M1.F-IF.7, NC.M1.F-IF.9) - Interpreting changes in the parameters of a linear and exponential function in context (NC.M1.F-LE.5)	*New Vocabulary: exponential expression, quadratic expression*

Mastering the Standard	
Clarification	**Checking for Understanding**

The A-SSE standards require students:
   - to write expressions in equivalent forms to reveal key quantities in terms of its context.   - to choose and use appropriate mathematics to analyze situations.
Part (b) of the standard expects students to identify parts of an expression as a single quantity and interpret the parts in terms of their context. For example, in the expression $x(x - 5)$, $(x - 5)$ represents a factor of the entire expression even though it is a binomial expression, in and of its self, with two terms. Additionally, in many contexts, these quantities have meaning in context of a problem. For example, if the expression $x(x - 5)$ represents the area of a rectangle, then the expression $(x - 5)$ represents the shorter side of the rectangle.
	Students should understand that working with different forms of an expressions often reveals key information from a context.
	
	Indicator: The expression $20(4x) + 500$ represents the cost in dollars of the materials and labor needed to build a square fence with side length x feet around a playground. According to this context, how should the following parts of expression be interpreted?
	    a) $4x$
	    b) $20(4x)$
	    c) $80x$
	
	Indicator: A rectangle has a length that is 2 units longer than the width. If the width is increased by 4 units and the length increased by 3 units, write two equivalent expressions for the area of the rectangle, and interpret the quantities and terms in context.
	    Solution: *The area of the rectangle is* $(x + 5)(x + 4) = x^2 + 9x + 20$. *Students should recognize* $(x + 5)$ *as the length of the modified rectangle and* $(x + 4)$ *as the width. Students can also interpret* $x^2 + 9x + 20$ *as the sum of the three areas (a square with side length x, a rectangle with side lengths* 9 *and x, and another rectangle with area* 20 *that have the same total area as the modified rectangle.*
	
	Indicator: Given that income from a concert is the price of a ticket times each person in attendance, consider the equation $I = 4000p - 250p^2$ that represents income from a concert where p is the price per ticket. What expression could represent the number of people in attendance?

Solution: *The equivalent factored form, $p(4000 - 250p)$, shows that the income can be interpreted as the price times the number of people in attendance based on the price charged. Students recognize $(4000 - 250p)$ as a single quantity for the number of people in attendance.*

Indicator: The expression $10{,}000(1.055)^n$ is the amount of money in an investment account with interest compounded annually for *n* years. Determine the initial investment and the annual interest rate.
Note: *The factor of 1.055 can be rewritten as $(1 + 0.055)$, revealing the growth rate of 5.5% per year.*

Instructional Resources	
Tasks	**Additional Resources**
	FAL: Generating Polynomials from Patterns (Math Assessment Project)

Algebra – Seeing Structure in Expressions

NC.M1.A-SSE.3

Write expressions in equivalent forms to solve problems.

Write an equivalent form of a quadratic expression by factoring, where a is an integer of the quadratic expression, $ax^2 + bx + c$, to reveal the solutions of the equation or the zeros of the function the expression defines.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite	**Connections**
● Factoring and expanding linear expressions with rational coefficients (7.EE.1) ● Understand that rewriting expressions into equivalent forms can reveal other relationships between quantities (7.EE.2)	*The following SMPs can be highlighted for this standard.* 4 – Model with mathematics 7 – Look for and make use of structure.
Connections	**Disciplinary Literacy**
● Interpreting the factors in context (NC.M1.A-SSE.1b) ● Understanding the relationship between factors, solutions, and zeros (NC.M1.A-APR.3) ● Solving quadratic equations (NC.M1.A-REI.4) ● Rewriting quadratic functions into different forms to show key features of the function (NC.M1.F-IF.8a)	*New Vocabulary: quadratic expression, zeros, linear factors* Students should be able to compare and contrast the zeros of a function and the solutions of a function.

Mastering the Standard

Clarification	Checking for Understanding

Students factor a quadratic expression in the form of $ax^2 + bx + c$ where a is an integer to reveal the constant and linear factors.
 	
 Students use the linear factors of a quadratic function to explain the meaning of the zeros of quadratic functions and the solutions to quadratic equations in a real-world problem.
	Students should understand that the reasoning behind rewriting quadratic expressions into factored form is to reveal different key features of a quadratic function, namely the zeros/x-intercepts.
	
	 Indicator: The expression $-4x^2 + 8x + 12$ represents the height of a coconut thrown from a person in a tree to a basket on the ground where x is the number of seconds.
	      a. Rewrite the expression to reveal the linear factors.
	      b. Identify the zeroes and intercepts of the expression and interpret what they mean in regard to the context.
	      c. How long is the ball in the air?
	
	 Indicator: Part A: Three equivalent equations for $f(x)$ are shown below.
	 $f(x) = -2x^2 + 24x - 54$ a) Select the form that reveals the zeros of $f(x)$ without changing the form of the equation.
	 $f(x) = -2(x - 3)(x - 9)$ Explain why this form is the appropriate choice.
	 $f(x) = -2(x - 6)^2 + 18$ b) Select from the listed values of x for which $f(x) = 0$: $-54, -18, -9, -6, -3, 0, 3, 6, 9, 18, 54$
	  (From the *Smarter Balanced Assessment Consortium*)
	 Students should understand that the reasoning behind rewriting quadratic expressions into factored form is to reveal the solutions to quadratic equations.
	 Indicator: The community garden was originally planned to be a square. However, to better fit the space, the length was increased, and the width decreased. The area of the garden is now represented by $x^2 + 10x - 24$. A fence is placed around the garden. What is the relationship between the length and width of the fence.

Instructional Resources

Tasks	Additional Resources
[Graphs of Quadratic Functions](#) (Illustrative Mathematics)	

Algebra – Arithmetic with Polynomial Expressions

NC.M1.A-APR.1
Perform arithmetic operations on polynomials.
Build an understanding that operations with polynomials are comparable to operations with integers by adding and subtracting quadratic expressions and by adding, subtracting, and multiplying linear expressions.

Concepts and Skills
Pre-requisite
- Add, subtract, factor and expand linear expressions (7.EE.1)   - Understand that rewriting expressions into equivalent forms can reveal other relationships between quantities (7.EE.2)
Connections
- Rewrite expressions using the properties of exponents (NC.M1.N-RN.2)   - Understanding the process of elimination (NC.M1.A-REI.5)   - Rewrite a quadratic function to reveal key features (NC.M1.F-IF.8a)   - Building functions to model a relationship (NC.M1.F-BF.1b)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard. 2 – Reason abstractly and quantitatively 7 – Look for and make use of structure
Disciplinary Literacy
New Vocabulary: polynomial, quadratic expression Students should be able to compare operations with polynomials to operations with integers.

| Mastering the Standard	
Clarification	**Checking for Understanding**

Students connect their knowledge of integer operations to polynomial operations.
 	
At the NC Math 1 level, students are only responsible for the following operations: - adding and subtracting quadratic expressions   - adding, subtracting, and multiplying linear expressions
This standard is connected to NC.M1.F-BF.1b, where students use adding, subtracting, and multiplying functions to build a new function.
	Students should be able to rewrite polynomial expressions using the properties of operations.
	    Indicator: Write at least two equivalent expressions for the area of the circle with a radius of $5x - 2$ kilometers.
	
	    Indicator: Rewrite the following expressions with as few terms as possible:
	a. $(4x + 3) - (2x + 1)$
	b. $(x^2 + 5x - 9) + 2x(4x - 3)$
	
	    Indicator: The area of a trapezoid is found using the formula $A = \frac{1}{2}h(b_1 + b_2)$, where A is the area, h is the height, and b_1 and b_2 are the lengths of the bases.
	
	What is the area of the above trapezoid?
	a. $A = 4x + 2$
	b. $A = 4x + 8$
	c. $A = 2x^2 + 4x - 21$
	d. $A = 2x^2 + 8x - 42$
	
	    $b_1 = x - 3$
	    $h = 4$
	    $b_2 = x + 7$
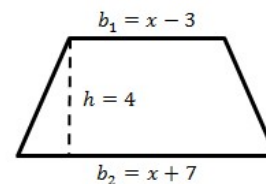	
	(NCDPI Math I released EOC #33)

Indicator: A town council plans to build a public parking lot. The outline below represents the proposed shape of the parking lot.

a. Write an expression for the area, in square feet, of this proposed parking lot. Explain the reasoning you used to find the expression.

x yd.

x – 5 yd.

2*x* – 5 yd.

2*x* + 15 yd.

b. The town council has plans to double the area of the parking lot in a few years. They plan to increase the length of the base of the parking lot by p yards, as shown in the diagram below.

x yd.

x – 5 yd.

2*x* – 5 yd.

2*x* + 15 yd.

p yd.
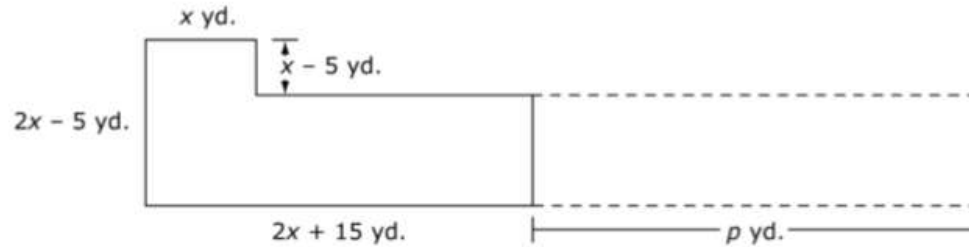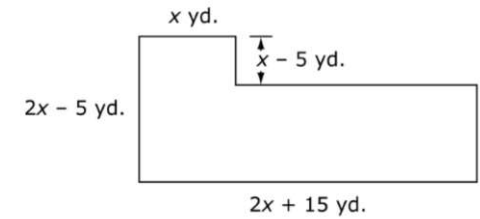
Write an expression in terms of *x* to represent the value of *p*, in feet. Explain the reasoning you used to find the value of *p*.

Indicator: A cardboard box as a height of *x*, a width that is 3 units longer than the height, and a length that is 2 units longer than the width. Write an expression in terms of *x* to represent the volume of the box.

Instructional Resources	
Tasks	**Additional Resources**
	FAL: Generating Polynomials from Patterns (Math Assessment Project)

Algebra – Arithmetic with Polynomial Expressions

NC.M1.A-APR.3

Understand the relationship between zeros and factors of polynomials.
Understand the relationships among the factors of a quadratic expression, the solutions of a quadratic equation, and the zeros of a quadratic function.

Concepts and Skills
Pre-requisite
• Understand that is the product is zero, at least one of the factors is zero (3.OA.7)
Connections
• Factor quadratic expressions to reveal zeros of functions and solutions to equations (NC.M1.A-SSE.3)   • Justify the steps in solving a quadratic equation (NC.M1.A-REI.1)   • Solving quadratic equations (NC.M1.A-REI.4)   • Factor quadratic functions to reveal key features (NC.M1.F-IF.8)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard.   2 – Reason abstractly and quantitatively   7 – Look for and make use of structure
Disciplinary Literacy
New Vocabulary: quadratic expression, quadratic equation, quadratic function, zeroes, linear factors, roots   Students should be able to compare solutions functions to solutions of equations.

Mastering the Standard

| Clarification	Checking for Understanding

The focus of this standard is for students to understand that the multiplicative property of zero can be used with linear factors to solve a quadratic equation. Students should be able to explain why each factor is set to zero and how this corresponds to the zeros of the function.
 	
 This standard should be taught with NC.M1.A-SSE.3 and NC.M1.A-REI.1.
 	
 Students can find the solutions of a factorable quadratic equation and use the roots to sketch its x −intercepts on the graph.
	Students should be able to explain how they go from factored form to identifying the zeros of the function.
	    Indicator: Given the function $y = 2x^2 + 6x − 3$, list the zeroes of the function and sketch its graph.
	
	    Indicator: Sketch the graph of the function $f(x) = (x + 5)^2$. How many zeros does this function have? Explain.
	
	    Note: It is a common error for students to assume that the solution or zero of linear factor, $(x − b)$, will always be the opposite of the constant term, b. If this is noticed, be sure to include examples in which $a \neq 1$.
	
	 Indicator: Which of the following has the largest x-intercept?
	   A) $f(x) = x^2 + 4x − 12$ C) $h(x) = (x − 1)^2 − 4$
	   B) $g(x) = (x + 2)(x − 5)$ D)
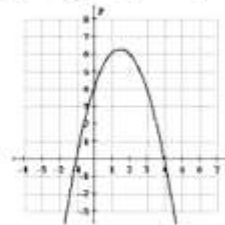	
	 Students should understand the relationship between zeros/solutions and the quadratic expression.
	    Indicator: If the zeros of a function are $x = 2$ and $x = 7$, what was the function? Could there be more than one answer?
	
	    Indicator: Based on the graph below, which of the following functions could have produced the graph?
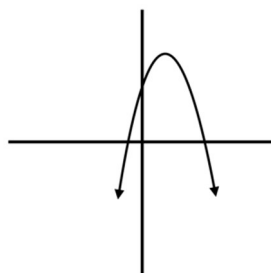
a. $f(x) = (x + 2)(x + 6)$
b. $f(x) = (x - 2)(x + 6)$
c. $f(x) = (2 - x)(6 - x)$
d. $f(x) = (2 + x)(6 - x)$

Indicator: For the following equation, identify the solutions and the factors for its corresponding quadratic function:
$$x^2 - 13x = 30$$

Instructional Resources	
Tasks	**Additional Resources**
	Match My Parabola (Desmos)

Algebra – Creating Equations

OCS Priority Standard: NC.M1.A-CED.1
Create equations that describe numbers or relationships.
Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.

Concepts and Skills
Pre-requisite
• Create two-step linear equations and inequalities from a context (7.EE.4)
Connections
• Interpret parts of an expression in context (NC.M1.A-SSE.1a,b)   • Justify a chosen solution method and each step of a that process (NC.M1.A-REI.1)   • Solve linear and quadratic equations and linear inequalities (NC.M1.A-REI.3, NC.M1.A-REI.4)   • Solve linear, exponential and quadratic equations using tables and graphs (NC.M1.A-REI.11)   • Represent the solutions of linear inequalities on a graph (NC.M1.A-REI.12)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard.   4 – Model with mathematics   7 – Look for and make use of structure
Disciplinary Literacy
New Vocabulary: exponential function, quadratic function   Students should be able to describe the origins of created equations and inequalities and demonstrate its relation to the context.

Mastering the Standard	
Clarification	**Checking for Understanding**

Students create equations and inequalities in one-variable and use them to solve problems.
 	
 In Math I, focus on linear, quadratic, and exponential contextual situations that students can use to create equations and inequalities in one variable and use them to solve problems. It is also important to note that equations can be created from an associated function when a given value is substituted in for either the independent or dependent variable.
 After the students have created an equation, they can use other representations to assist in solving problems, such as graphs and tables.
 	
 For quadratic and exponential inequalities, the focus of this standard is to create the inequality and use that inequality to solve a problem. Solving these inequalities algebraically is *not* part of the standard. Once a student has the inequality, the student can use a table or graph to find a solution to the problem.
	Students should be able to create an equation from a function and use the equation to solve problems.
	 Indicator: A government buys x fighter planes at z dollars each, and y tons of wheat at w dollars each. It spends a total of B dollars, where $B = xz + yw$. In (a)–(c), write an equation whose solution is the given quantity.
	    a. The number of tons of wheat the government can afford to buy if it spends a total of $100 million, wheat costs $300 per ton, and it must buy 5 fighter planes at $15 million each.
	    b. The price of fighter planes if the government bought 3 of them, in addition to 10,000 tons of wheat at $500 a ton, for a total of $50 million.
	    c. The price of a ton of wheat, given that a fighter plane costs 100,000 times as much as a ton of wheat, and that the government bought 20 fighter planes and 15,000 tons of wheat for a total cost of $90 million.
	      (https://www.illustrativemathematics.org/content-standards/HSA/CED/A/1/tasks/580)
	
	 Indicator: A ball thrown vertically upward at an initial velocity of v_0 ft/sec rises a distance d feet in t seconds, given by $d = 6 + v_0 t - 16t^2$. Write an equation whose solution is:
	    a. The time it takes a ball thrown at a speed of 88 ft/sec to rise 20 feet.
	    b. The speed with which the ball must be thrown to rise 20 feet in 2 seconds.
	      (https://www.illustrativemathematics.org/content-standards/HSA/CED/A/2/tasks/437)
	
	 Students should be able to create equations from various representations, such as verbal descriptions, and use them to solve problems.

For quadratic inequalities, students can think of its related quadratic function. As the quadratic inequality is compared to zero, the zeros of its related function become important, as they become the boundaries of the solutions for the inequality. From this knowledge students can either use test points in the various regions to determine the correct solution, or the students can use reasoning with the coefficient of the quadratic term to know the solution region. For example, if asked to solve the following inequality $x^2 + x - 9 \leq 3$.

$$x^2 + x - 9 \leq 3$$
$$x^2 + x - 12 \leq 0$$
$$(x - 3)(x + 4) \leq 0$$

When $x = -4$ or 3 the expression is equal to 0. If the equation is thought of as a function and we observe that the quadratic term is positive, we can conclude that the output of the function will be less than zero between zeros.
Hence, $-4 \leq x \leq 3$ represents the solution to the equation.

Students in Math 1 are not responsible for using interval notation to represent a solution. They are to write answers to these inequalities using inequality notation.

Indicator: Mary and Jeff both have jobs at a baseball park selling bags of peanuts. They get paid $12 per game and $1.75 for each bag of peanuts they sell. Create equations, that when solved, would answer the following questions:
   a.   How many bags of peanuts does Jeff need to sell to earn $54?
   b.   How much will Mary earn if she sells 70 bags of peanuts at a game?
   c.   How many bags of peanuts does Jeff need to sell to earn at least $68?

Indicator: Phil purchases a used truck for $11,500. The value of the truck is expected to decrease by 20% each year. When will the truck first be worth less than $1,000?

Indicator: Suppose a friend tells you she paid a total of $16,368 for a car, and you'd like to know the car's list price (the price before taxes) so that you can compare prices at various dealers. Find the list price of the car if your friend bought the car in:
   a.   Arizona, where the sales tax is 6.6%.
   b.   New York, where the sales tax is 8.25%.
   c.   A state where the sales tax is r.
                    (https://www.illustrativemathematics.org/content-standards/HSA/CED/A/1/tasks/582)

Indicator: Fishing Adventures rents small fishing boats to tourists for day-long fishing trips. Each boat can only carry 1200 pounds of people and gear for safety reasons. Assume the average weight of a person is 150 pounds. Each group will require 200 lbs of gear for the boat plus 10 lbs of gear for each person.
   a. Create an inequality describing the restrictions on the number of people possible in a rented boat. Graph the solution set.
   b. Several groups of people wish to rent a boat. Group 1 has 4 people. Group 2 has 5 people. Group 3 has 8 people. Which of the groups, if any, can safely rent a boat? What is the maximum number of people that may rent a boat?
                    (https://www.illustrativemathematics.org/content-standards/tasks/643)

Students should be able to create inequalities and use those inequalities to solve problems. (Students are not expected to solve quadratic and exponential inequalities algebraically. Students should use technology, tables and graphs to solve problems.)
   Indicator: Stephen wants to create a landscaping feature in the shape of a parallelogram in his yard. Stephen has 200 square feet of mulch available for the project. To be most pleasing to the eye, he decides that he wants the length of the parallelogram to be 3 more than twice the width, measured in feet. If Stephen intends to cover the entire landscape feature in mulch, what can the width of the parallelogram be?

   Indicator: Susanna heard some exciting news about a well-known celebrity. Within a day she told 4 friends who hadn't heard the news yet. By the next day each of those friends told 4 other people who also hadn't yet heard the news. By the next day each of those people told four more, and so on.
   a.   Assume the rumor continues to spread in this manner. Let N be the function that assigns to d the number of people who hear the rumor on the dth day. Write an expression for N(d).
   b.   On which day will at least 100,000 people hear the rumor for the first time?

c. How many people will hear the rumor for the first time on the 20th day?
d. Is the answer to (c) realistic? Explain your reasoning.
e. Create an inequality that could be used to determine when there will be greater than 200,000 people that have heard the rumor.

Indicator: A ball thrown vertically upward at an initial velocity of 88 ft/sec rises d feet in t seconds, given by $d = 6 + 88t - 16t^2$.

a. Write an inequality whose solution represents when the ball would be at least 78 ft above the ground.
b. Use the table to the right to find when the ball would be at least 78 ft. above the ground.

t (sec)	d (ft)
0	6
0.5	46
1	78
1.5	102
2	118
2.5	126
3	126
3.5	118
4	102
4.5	78
5	46
5.5	6

Instructional Resources	
Tasks	**Additional Resources**
Planes and Wheat (Illustrative Mathematics) Throwing a Ball (Illustrative Mathematics) Buying a Car (Illustrative Mathematics)	

Algebra – Creating Equations

OCS Priority Standard: NC.M1.A-CED.2
Create equations that describe numbers or relationships.
Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities.

Concepts and Skills
Pre-requisite
- Construct a linear function that models the relationship between two quantities (8.F.4)   - Graph linear equations (8.EE.6)   - The graph of a function is the set of ordered pairs consisting of input and a corresponding output (8.F.1)   - Understand that the graph of a two-variable equation represents the set of all solutions to the equation (NC.M1.A-REI.10)
Connections
- Interpret parts of an expression in context (NC.M1.A-SSE.1a,b)   - Creating linear equations for a system (NC.M1.A-CED.3)   - Solving for a variable of interest in a formula (NC.M1.A-CED.4)   - The graph a function f is the graph of the equation $y = f(x)$ (NC.M1.F-IF.1)   - Interpret a function's domain and range in context (NC.M1.F-IF.5)   - Identify key features of linear, exponential and quadratic functions (NC.M1.F-IF.7)   - Building a function through patterns or by combining other functions (NC.M1.F-BF.1a , NC.M1.F-BF.1b)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard. 4 – Model with mathematics 6 – Attend to precision 7 – Look for and make use of structure
Disciplinary Literacy
New Vocabulary: exponential function, quadratic function Students should be able to describe the origins of created equations and demonstrate its relation to the context.

Mastering the Standard	
Clarification	**Checking for Understanding**

This standard is closely related to the building functions standards, as a two-variable equation can be meaningfully reinterpreted as a function. Equations place a focus on equivalent expressions, while functions place a focus on the relationship between the variables.
Students graph equations on coordinate axes with labels and scales clearly labeling the axes defining what the values on the axes represent and the unit of measure. Students also select intervals for the scale that are appropriate for the context and display adequate information about the relationship.
Students interpret the context and choose appropriate minimum and maximum values for a graph.
In Math I, focus on linear, exponential and quadratic **contextual** situations for students to create equations in two variables.
While students will only be asked to rewrite
	Students should be able to create two variable equations from various representations, such as verbal descriptions, and use them to solve problems.
	
	Indicator: The larger leg of a right triangle is 3 cm longer than its smaller leg. The hypotenuse is 6 cm longer than the smaller leg. How many centimeters long is the smaller leg?
	
	Indicator: The floor of a rectangular cage has a length 4 feet greater than its width, w. James will increase both dimensions of the floor by 2 feet. Which equation represents the new area, N, of the floor of the cage?
	  a. $N = w^2 + 4w$
	  b. $N = w^2 + 6w$
	  c. $N = w^2 + 6w + 8$
	  d. $N = w^2 + 8w + 12$
	
	Students should be able to create two variable equations, graph the relationship, and use graph to recognize key feature of the graph.
	Indicator: The FFA had a fundraiser by selling hot dogs for $1.50 and drinks for $2.00. Their total sales were $400.
	  a. Write an equation to calculate the total of $400 based on the hot dog and drink sales.
	  b. Graph the relationship between hot dog sales and drink sales.
	Note: This make a good connection to NC.M1.F-IF.5

expressions with integer exponents, in exponential functions, the domain is not restricted and students should use technology to understand the continuity of exponential functions.

Indicator: In a woman's professional tennis tournament, the money a player wins depends on her finishing place in the standings. The first-place finisher wins half of $1,500,000 in total prize money. The second-place finisher wins half of what is left; then the third-place finisher wins half of that, and so on.
    a.   Write a rule to calculate the actual prize money in dollars won by the player finishing in nth place, for any positive integer n.
    b.   Graph the relationship between the first 10 finishers and the prize money in dollars.
        What pattern is indicated in the graph? What type of relationship exists between the two variables?

Indicator: Misha has a new rabbit that she named "Wascal." She wants to build Wascal a pen, so that the rabbit has space to move around safely. Misha has purchased a 72-foot roll of fencing to build a rectangular pen.
    a.   If Misha uses the whole roll of fencing, what are some of the possible dimensions of the pen?
    b.   Write a model for the area of the rectangular pen in terms of the length of one side. Include both an equation and a graph.
    c.   What are the dimensions of the pen that would allow Wascal the most area to run around? How do you know?

(www.mathematicsvisionproject.org)

Instructional Resources	
Tasks	**Additional Resources**
	Match My Line (DESMOS)
	Build a Bigger Field (DESMOS)

Algebra – Creating Equations

NC.M1.A-CED.3
Create equations that describe numbers or relationships.
Create systems of linear equations and inequalities to model situations in context.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite • Understanding a system of equations (8.EE.8) • Creating linear equations in two variables (NC.M1.A-CED.2)	**Connections** *The following SMPs can be highlighted for this standard.* 4 – Model with mathematics 6 – Attend to precision
Connections • Interpret parts of an expression in context (NC.M1.A-SSE.1a,b) • Use tables, graphs and algebraic methods to solve systems of linear equations (NC.M1.A-REI.6) • Represent the solution to a system of linear inequalities as a region of the plane (NC.M1.A-REI.12)	**Disciplinary Literacy** Students should be able to describe the origins of created equations and demonstrate its relation to the context.

Mastering the Standard

Clarification	Checking for Understanding
Students create a system of linear equations and inequalities that model real world situations. The expectation for this standard is to create a system of linear equations or a system of linear inequalities that model a contextual situation. The system can include inequalities that limit the domain and range, if necessary. Connect this standard to NC.M1.A-REI.11 & 12 for solving the system of linear equations algebraically and graphically and NC.M1.A-REI.12 for representing the solutions to a system of linear inequalities as a region of the plane. *Note: Linear programming and optimization are not the intent of this standard. While it may be an extension of this standard and could be used as an application, it is not the expectation that students be fluent in maximizing or minimizing a quantity based on constraints.*	Students should be able to write inequalities that describe the limitations from a context for a system of inequalities. **Indicator:** A club is selling hats and jackets as a fundraiser. Their budget is $1500 and they want to order at least 250 items. They must buy at least as many hats as they buy jackets. Each hat costs $5 and each jacket costs $8. Write a system of inequalities to represent the situation. Students should be able to write the system of equations based on context. **Indicator:** The only coins that Alexis has are dimes and quarters. • Her coins have a total value of $5.80. • She has a total of 40 coins. Which system of equations can be used to find the number of dimes, d, and the number of quarters, q, Alexis has? (https://www.illustrativemathematics.org/content-standards/HSA/CED/A/3/tasks/220)

Instructional Resources

Tasks	Additional Resources
	Solutions to Systems of Equations (DESMOS) Solving Linear Equations in Two Variables (Mathematics Assessment Project)

Algebra – Creating Equations

NC.M1.A-CED.4
Create equations that describe numbers or relationships.
Solve for a quantity of interest in formulas used in science and mathematics using the same reasoning as in solving equations.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite	**Connections**
• Solve linear equations in one variable (8.EE.7 and NC.M1.A-REI.3) • Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$ where p is a positive rational number (8.EE.2) • Justify a solution method and each step in the solving process (NC.M1.A-REI.1)	*The following SMPs can be highlighted for this standard.* 4 – Model with mathematics 7 – Look for and make use of structure
Connections	**Disciplinary Literacy**
• Create an equation in two variables that represent a relationship between quantities (NC.M1.A-CED.2) • Justify a solving method and each step in the solving process (NC. M1.A-REI.1)	Students should be able to justify the steps in their solving process.

Mastering the Standard

Clarification	Checking for Understanding

Students should be able to solve an equation for a given variable. In Math 1, focus on real mathematical and scientific formulas. This may be a good opportunity to collaborate with science teachers and ask them for formulas that they use often.
 This standard also covers solving for variables in mathematical forms as well as formulas.
 (Students are <u>not</u> expected to write linear equations into "proper" standard form.)
 	
 This standard should be taught in conjunction with NC.M1.A-REI.1 in which students must justify each step of the solving process and justify a particular solving method.
	Students should be able to solve for variables in mathematical forms as well as formulas.
	    Indicator: Solve $(y - y_1) = m(x - x_1)$ for m.
	
	 Students should be able to solve for variable in science and math formula.
	    Indicator: Energy and mass are related by the formula $E = mc^2$, where m is the mass and c is the speed of light. Write an equation that finds m, given E and c.
	
	    Indicator: In each of the equations below, rewrite the equation, solving for the indicated variable.
	      a. If F denotes a temperature in degrees Fahrenheit and C is the same temperature measured in degrees Celsius, then F and C are related by the equation $F = 95C + 32$. Rewrite this expression to solve for C in terms of F.
	      b. The surface area S of a sphere of radius r is given by $S = 4\pi r^2$. Solve for r in terms of S.
	      (https://www.illustrativemathematics.org/content-standards/HSA/CED/A/4/tasks/1828)
	
	    Indicator: The equation for an object that is launched from the ground is given by $h(t) = -16t^2 + v_0t$ where h is the height, t is the time, and v_0 is the initial velocity. What is the initial velocity of an object that is one-hundred feet off the ground four seconds after it is launched?

Instructional Resources

Tasks	Additional Resources
Rewriting Equations (Illustrative Mathematics)	

Algebra – Reasoning with Equations and Inequalities

NC.M1.A-REI.1

Understand solving equations as a process of reasoning and explain the reasoning.
Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning.

Concepts and Skills
Pre-requisite
- Students have been using properties of operations and equality throughout middle school. (6.EE.3, 7.EE.1, 7.EE.4). This is the first time that justification is required by a content standard.   - Solve multi-step equations (8.EE.7)
Connections
- Understand the relationship between factors of a quadratic equation and the solution of the equation (NC.M1.A-APR.3)   - Create and solve one variable linear and quadratic equations (NC.M1.A-CED.1)   - Solve for a quantity of interest in a formula (NC.M1.A-CED.4)   - Solve linear and quadratic equations and systems of linear equations (NC.M1.A-REI.3, NC.M1.A-REI.4, NC.M1.A-REI.5, NC.M1.A-REI.6)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard. 3 – Construct viable arguments and critique the reasoning of others
Disciplinary Literacy
New Vocabulary: quadratic equation Students should be able to defend their method of solving an equation and each step of the solving process.

Mastering the Standard	
Clarification	**Checking for Understanding**
When solving equations, students will use the properties of equality to justify and explain each step obtained from the previous step, assuming the original equation has a solution, and develop an argument that justifies their solution method. Properties of operations can be used to change expressions on either side of the equation to equivalent expressions. In the properties of equality, adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions. While students are not required to memorize the proper names of the various properties, they should be able to describe the property they are using to justify their reasoning. The concepts of the properties are important; however, communication of these concepts is just as important. For this reason, students should be able to use reasoning to identify the	Students should be able to justify a chosen solution method and justify each step in the process. This would be a good opportunity to discuss efficiency. **Indicator:** To the right are two methods to solve the same equation. Justify each step in the solving process. Which method do you prefer? Why? **Method 1:** $5(x + 3) - 3x = 55$ $5x + 15 - 3x = 55$ $2x + 15 = 55$ $2x + 15 - 15 = 55 - 15$ $2x = 40$ $\dfrac{2x}{2} = \dfrac{40}{2}$ $x = 20$ **Method 2:** $5(x + 3) - 3x = 55$ $\dfrac{5(x + 3)}{5} - \dfrac{3x}{5} = \dfrac{55}{5}$ $x + 3 - \dfrac{3}{5}x = 11$ $\dfrac{2}{5}x + 3 = 11$ $\dfrac{2}{5}x + 3 - 3 = 11 - 3$ $\dfrac{2}{5}x = 8$ $\dfrac{5}{2}\left(\dfrac{2}{5}\right)x = \dfrac{5}{2}(8)$ $x = 20$

mathematical meaning of a property from its proper name. For example, students should recognize that the properties of equality tell us how to change the value of expressions while maintaining equality between the expressions.

Indicator: To the right are two methods for solving the equation $5x^2 + 10 = 90$. Select one of the solution methods and construct a viable argument for the use of the method.

$$5x^2 + 10 = 90$$
$$-10 = -10$$
$$5x^2 = 80$$
$$\frac{5x^2}{5} = \frac{80}{5}$$
$$x^2 = 16$$
$$x = \pm\sqrt{16}$$
$$x = 4 \text{ or } x = -4$$

$$5x^2 + 10 = 90$$
$$-90 = -90$$
$$5x^2 - 80 = 0$$
$$5(x^2 - 16) = 0$$
$$5(x + 4)(x - 4) = 0$$
$$x + 4 = 0 \text{ or } x - 4 = 0$$
$$x = 4 \text{ or } x = -4$$

Instructional Resources	
Tasks	**Additional Resources**
Reasoning with Linear Inequalities (Illustrative Mathematics)	

Algebra – Reasoning with Equations and Inequalities

NC.M1.A-REI.3
Solve equations and inequalities in one variable.
Solve linear equations and inequalities in one variable.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite	**Connections**
- Solving multi-step equations (8.EE.7) - Solving two-step inequalities (7.EE.4)	*The following SMPs can be highlighted for this standard.* 1 – Make sense of problems and persevere in problem solving. 5 – Use appropriate tools strategically. 6 – Attend to precision. 7 – Look for and make use of structure. 8 – Look for and express regularity in repeated reasoning.
Connections	**Disciplinary Literacy**
- Create one variable linear equations and inequalities (NC.M1.A-CED.1) - Justify a solution methods and the steps in the solving process (NC.M3.A-REI.1) - Solve systems of linear equations (NC.M1.A-REI.6)	Students should be able to discuss their solution method and the steps in the solving process and should be able to interpret the solutions in context, when applicable.

Mastering the Standard	
Clarification	**Checking for Understanding**
Students solved multi-step equations in 8th grade connecting the process to the order of operations. Additionally, students in 7th graded solved <u>two</u>-step inequalities. This concept will be extended to include solving <u>multi</u>-step inequalities in NC Math 1. These processes should be taught with the mathematical reasoning found in NC.M1.A-REI.1. Students should not be asked to memorize steps to solve a linear equation/inequality. Like many purely procedural practices, such steps are only effective for linear equations. It is more effective for students to be taught the mathematical reasoning for the solving process as these concepts can be applied to all types of equations. Teaching the process of solving linear equations and inequalities in conjunction with NC.M1.A-CED.1 (where students learn how to create linear equations in context) deepens students' knowledge of the purpose for solving equations and inequalities.	Students should be able to solve multistep linear equations and inequalities. **Indicator:** Solve: $\frac{7}{3}y - 8 = 111$ $3x - 2 > 9 + 5x$ $\frac{3+x}{7} = \frac{x-9}{4}$ $\frac{2}{3}x + 9 < 8\left(\frac{1}{3}x - 2\right)$ $\frac{1}{5}(10 - 20x) \leq -14$ **Indicator:** Jackson observed a graph with a y-intercept of 7 that passes through the point $(2, 3)$. What is the slope of the line of Jackson's graph?

Instructional Resources	
Tasks	**Additional Resources**
	Building and Solving Complex Equations (MAP FAL)

Algebra – Reasoning with Equations and Inequalities

NC.M1.A-REI.4
Solve equations and inequalities in one variable.
Solve for the real solutions of quadratic equations in one variable by taking square roots and factoring.

Concepts and Skills
Pre-requisite
- Factor linear expressions with rational coefficients (7.EE.1)   - Use square root to represent solutions to equations of the form $x^2 = p$, where p is a positive rational number; evaluate square roots of perfect squares (8.EE.2)   - Factor a quadratic expression to reveal the solution of a quadratic equation (NC.M1.A-SSE.3)   - Understand the relationship between linear factors and solutions (NC.M1.A-APR.3)
Connections
- Create one variable quadratic equations and inequalities and solve (NC.M1.A-CED.1)   - Justify a solution method and each step in the solution process (NC.M1.A-REI.1)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard. 6 – Attend to precision 7 – Look for a make use of structure
Disciplinary Literacy
New Vocabulary: quadratic equation Students should be able to discuss their solution method and the steps in the solving process and should be able to interpret the solutions in context.

Mastering the Standard	
Clarification	**Checking for Understanding**

Students should focus on quadratics with one or two real solutions that can be solved by factoring or taking the square root.
 	
The focus for this standard is the algebraic reasoning of how to solve a quadratic equation, the "how" to solve a quadratic equation. NC.M1.A-APR.3 places a focus the "why." Therefore, these two standards should be taught together.
 	
Students should be able to use the structure of the quadratic equation to determine whether to solve by using the square root as an inverse operation or by factoring.
 	
When solving using the square root, students are only expected to evaluate perfect squares. All other square root solutions should either be left in square root form or estimated appropriately based on the context. Therefore, solving using the quadratic formula is not expected at this level.
	Students should be able to solve quadratic equations using square root as the inverse operation.
	    Indicator: Solve:
	        a) $x^2 = 49$
	        b) $3x^2 + 9 = 72$
	
	Students should be able to solve quadratic equations using factoring.
	    Indicator: Solve: $6x^2 + 13x = 5$
	
	Students should be able to discuss their chosen solution method.
	    Indicator: Stephen and Brianna are solving the quadratic equation, $(x - 4)^2 - 25 = 0$, in a classroom activity. Stephen believes that the equation can be solving using a square root. Brianna disagrees, saying that it can be solved by using factoring. Who is correct? Be prepared to defend your position.

Algebra – Reasoning with Equations and Inequalities

OCS Priority Standard: NC.M1.A-REI.5
Solve systems of equations.
Explain why replacing one equation in a system of linear equations by the sum of that equation and a multiple of the other produces a system with the same solutions.

Concepts and Skills
Pre-requisite
• Analyze and solve pairs of simultaneous linear equations by graphing and substitution (8.EE.8)   • Operations with polynomials (NC.M1.A-APR.1)   • Justify steps in a solving process (NC.M1.A-REI.1)
Connections
• Solving systems of equations and inequalities (NC.M1.A-REI.6)   • Understand that all points on the graph of an equation is a solution to that equation (NC.M1.A-REI.10)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard.   2 – Reason abstractly and quantitatively   3 – Construct a viable argument and critique the reasoning of others   7 – Look for and make use of structure
Disciplinary Literacy
New Vocabulary: elimination   Students should be able to explain why the process of elimination works.

Mastering the Standard

Clarification	Checking for Understanding
The focus of this standard is to explain a mathematical justification for the addition (elimination) method of solving systems of equations which ultimately transforms a given system of two equations into a simpler equivalent system that has the same solutions as the original system. Students should use the properties of equality to discuss why the process of elimination maintains the same solutions. • When an equation is multiplied by a constant the set of solutions remains the same. Graphically it is the same line. • When two linear equations are added together, a third linear equation is formed that shares a common solution as the original equations. Graphically this means the three linear equations all intersect at the same point. • The goal for process of elimination is to obtain the value for one of the coordinates of intersection. Graphically, it is to get either a horizontal or vertical line that goes through the point of intersection.	Students should be able to understand the process of elimination through simple intuitive problems. **Indicator:** Given that the sum of two numbers is 10 and their difference is 4, what are the numbers? Explain how your answer can be deduced from the fact that the two numbers, x and y, satisfy the equations $x + y = 10$ and $x - y = 4$. Students should be able to identify systems composed of equivalent equations. **Indicator:** Four students were assigned to write an equivalent system of equations. Determine which of the following students created a system that is equivalent to $\begin{cases} x - 2y = 4 \\ 3x + y = 9 \end{cases}$ and explain how you know. Brenda $\begin{cases} x - 2y = 4 \\ 6x + 2y = 9 \end{cases}$ Evan $\begin{cases} x - 2y = 4 \\ 6x - 2y = 18 \end{cases}$ Kevon $\begin{cases} -3x + 6y = 4 \\ 3x + y = 9 \end{cases}$ Brie $\begin{cases} \frac{1}{2}x - y = 2 \\ 3x + y = 9 \end{cases}$

Instructional Resources

Tasks	Additional Resources

Algebra – Reasoning with Equations and Inequalities

NC.M1.A-REI.6
Solve systems of equations.
Use tables, graphs, or algebraic methods (substitution and elimination) to find approximate or exact solutions to systems of linear equations and interpret solutions in terms of a context.

Concepts and Skills
Pre-requisite
- Analyze and solve pairs of simultaneous linear equations by graphing (8.EE.8)   - Create equations for systems of equations (NC.M1.A-CED.3)   - Justify the steps in a solving process (NC.M1.A-REI.1)   - Solve linear equations in one variable (NC.M1.A-REI.3)   - Understand the mathematical reasoning behind the process of elimination (NC.M1.A-REI.5)   - Understand every point on a graph is a solution to its associated equation (NC.M1.A-REI.10)
Connections
- Understand the mathematical reasoning behind the methods of graphing, using tables and technology to solve systems and equations (NC.M1.A-REI.11)   - Analyze linear functions (NC.M1.F-IF.7)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard. 3 – Construct a viable argument and critique the reasoning of others 6 – Attend to precision
Disciplinary Literacy
New Vocabulary: elimination Students should be able to discuss their solution method and the steps in the solving process and should be able to interpret the solutions in context.

Mastering the Standard	
Clarification	**Checking for Understanding**
Students solve a system of equations and then interpret its solution for the given context. Students should be able to create and solve a system from a contextual situation. Therefore, this standard should be taught in conjunction to NC.M1.A-CED.3 Students should not be required to use one method over another when solving a system of equations but should be allowed to choose the best option for the given scenario and justify their chosen solution method. The focus of this standards should also not be limited to the algebraic methods. This is a capstone standard supported by several standards in this course. To have a complete understanding of this standard, the following standards must be incorporated:	Students should be able to create equations for system (NC.M1.A-CED.3), select an appropriate solution method, solve that system, and interpret the solution in context. **Indicator:** José had 4 times as many trading cards as Philippe. After José gave away 50 cards to his little brother and Philippe gave 5 cards to his friend for his birthday, they each had an equal number of cards. Write a system to describe the situation and solve the system. **Indicator:** A restaurant serves a vegetarian and a chicken lunch special each day. Each vegetarian special is the same price. Each chicken special is the same price. However, the price of the vegetarian special is different from the price of the chicken special. - On Thursday, the restaurant collected \$467 selling 21 vegetarian specials and 40 chicken specials. - On Friday, the restaurant collected \$484 selling 28 vegetarian specials and 36 chicken specials. What is the cost of each lunch special? **Indicator:** The math club sells candy bars and drinks during football games.

- The ability to create equations for a system from a contextual situation is addressed in **NC.M1.A-CED.3**.
- The understanding of the elimination method is addressed **NC.M1.A-REI.5**.
- The understanding of solving a system by graphing and how to recognize a solution to a system in tables is taught in **NC.M1.A-REI.11**.

Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to NC.M1.G-GPE.5, which requires students to prove the slope criteria for parallel lines.

- 60 candy bars and 110 drinks will sell for $265.
- 120 candy bars and 90 drinks will sell for $270.

How much does each candy bar sell for?

Indicator: Two times Antonio's age plus three times Sarah's age equals 34. Sarah's age is also five times Antonio's age. How old is Sarah?

Indicator: Lucy and Barbara began saving money the same week. The table below shows the models for the amount of money Lucy and Barbara had saved after x weeks.

Lucy's Savings	$f(x) = 10x + 5$
Barbara's Savings	$g(x) = 7.5x + 25$

After how many weeks will Lucy and Barbara have the same amount of money saved?

Indicator: A streaming movie service has three monthly plans to rent movies online. Graph the equation of each plan and analyze the change as the number of rentals increase. When is it beneficial to enroll in each of the plans?
- Basic Plan: $3 per movie rental
- Watchers Plan: $7 fee + $2 per movie with the first two movies included with the fee
- Home Theater Plan: $12 fee + $1 per movie with the first four movies included with the fee

Instructional Resources	
Tasks	**Additional Resources**
	Card Sort: Linear Systems (DESMOS)

Algebra – Reasoning with Equations and Inequalities

OCS Priority Standard: NC.M1.A-REI.10
Represent and solve equations and inequalities graphically
Understand that the graph of a two-variable equation represents the set of all solutions to the equation.

Concepts and Skills
Pre-requisite
- Use substitution to determine if a number if a solution (6.EE.5)   - Graphing lines (8.EE.5, 8.EE.6, 8.F.3)   - Analyze and solve pairs of simultaneous linear equations by graphing and substitution (8.EE.8)   - Understanding functions as a rule that assigns each input with exactly one output (8.F.1)
Connections
- Creating and graphing two-variable equations (NC.M1.A-CED.2)   - Solutions to systems of equations (NC.M1.A-REI.5, NC.M1.A-REI.6)   - Understanding that the relationship between the solution of system of equations and the associated equation (NC.M1.A-REI.11)   - Representing the solutions to linear inequalities (NC.M1.A-REI.12)   - Relating a function to its graph, domain and range of a function (NC.M1.F-IF.1, NC.M1.F-IF.2, NC.M1.F-IF.5)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard. 3 – Construct a viable argument and critique the reasoning of others
Disciplinary Literacy
Students should be able to discuss the solutions to a two-variable equation and the link to a function.

| Mastering the Standard	
Clarification	**Checking for Understanding**

Students understand that the graph of an equation is the set of all ordered pairs that make that equation a true statement.
 	
This standard contains no limitation and so applies to all function types, including those functions that a student cannot yet algebraically manipulate.
 	
Students can explain and verify that every point (x, y) on the graph of an equation represents all values for x and y that make the equation true.
 	
In connection with NC.M1.F-IF.1, NC.M1.F-IF.2, NC.M1.F-IF.5, students can begin to understand how the context of a problem determines whether the solutions should be graphed discretely or continuously.
	Students should be able to assess if a point is a solution to an equation.
	Indicator: Consider three points in the plane, $P = (-4, 0), Q = (-1, 12)$ *and* $R = (4, 32)$.
	  a. Find the equation of the line through P and Q.
	  b. Use your equation in (a) to show that R is on the same line as P and Q.
	    (https://www.illustrativemathematics.org/content-standards/HSA/REI/D/10/tasks/1066)
	
	Indicator: Which of the following points are on the graph of the equation $-5x + 2y = 20$? Which of the following points are of the graph of the equation? How do you know?
	  a. (4, 0)
	  b. (0, 10)
	  c. (-1, 7.5)
	  d. (2.3, 5)
	
	Indicator: Verify that $(-1, 60)$ is a solution to the equation $y = 15\left(\frac{1}{4}\right)^x$. Explain what this means for the graph of the function.

It is common to refer to the zeros of functions as solutions. However, this is inaccurate language that can cause misconceptions and confusion among students. Every point that is graphed, including the zeros, are solutions to the function.

The zeros of a function do correspond to the solution of the functions related equation. See NC.M1.APR.3

Indicator: Without graphing, determine if the ordered pair $(2, -15)$ is on the graph of $y = 3x^2 + 2x - 1$. Explain.

**Indicator:** The graph below shows the height of a hot air balloon as a function of time.
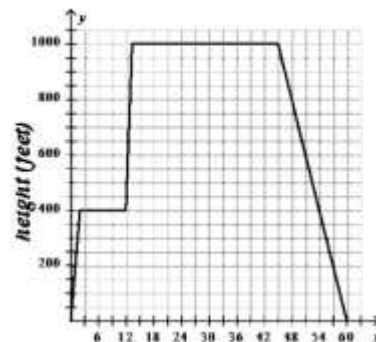
Use the graph to answer the following:

a. What is the height of the hot air balloon 10 minutes after it has left the ground?
b. Approximately, when will the hot air balloon reach a height of 600 feet?
c. Explain what the point $(48, 800)$ on this graph represents.

Indicator: Given the function to the right, determine if the following points are solutions and explain each answer.

a. (2, 1)
b. (3, 8)
c. (-1, -4)

Note: ($y < x^2 - x - 1$ is the inequality of the graph)
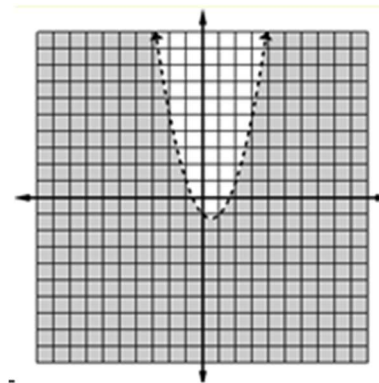
Instructional Resources	
Tasks	**Additional Resources**
Collinear Points (Illustrative Mathematics)	

Algebra – Reasoning with Equations and Inequalities

NC.M1.A-REI.11
Represent and solve equations and inequalities graphically
Build an understanding of why the x-coordinates of the points where the graphs of two linear, exponential, or quadratic equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ and approximate solutions using a graphing technology or successive approximations with a table of values.

Concepts and Skills
Pre-requisite
- Solving multi-step linear equations (8.EE.7)   - Analyze and solve pairs of simultaneous linear equations by graphing and substitution (8.EE.8)   - Understand every point on a graph is a solution to its associated equation (NC.M1.A-REI.10)
Connections
- Creating and solving one variable equations and systems of equations (NC.M1.A-CED.1, NC.M1.A-CED.3)   - Solving systems of equations (NC.M1.A-REI.6)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard. 4 – Model with mathematics 6 – Attend to precision
Disciplinary Literacy
New Vocabulary: exponential function, quadratic function

Mastering the Standard

Clarification

For a complete understanding, students will need exposure to both parts of this standard. First, students should be able to see the connection between graphs and tables of two functions, the points they have in common and the truthfulness of the equation.
For example:

x	$f(x) = 2x - 4$
0	-4
1	-2
2	0
3	2
4	4

x	$g(x) = \frac{1}{2}x + \frac{1}{2}$
0	.5
1	1
2	1.5
3	2
4	2.5

Because $f(x) = g(x)$ when $x = 3$ the solution to the equation $2x - 4 = \frac{1}{2}x + \frac{1}{2}$ is 3.
(As an extension, students could write an inequality to describe the relationship between the functions when $x < 3$ and when $x > 3$.)

In Math 1, students are expected to solve linear systems of equations algebraically. All other systems should be solved with technology, tables, and graphs.

Second, students should be able to use a system of equations to solve systems of equations.
For example:
Solve: $3x^2 - 2x + 1 = \frac{1}{2}x + 5$

Checking for Understanding

Indicator: The functions $f(m) = 18 + 0.4m$ and $g(m) = 11.2 + 0.54m$ give the lengths of two different springs in centimeters, as mass is added in grams, m, to each separately.
  a. Graph each equation on the same set of axes.
  b. What mass makes the springs the same length?
  c. What is the length at that mass?
  d. Write a sentence comparing the two springs.

Indicator: The population of a country is initially 2 million people and is increasing at 4% per year. The country's annual food supply is initially adequate for 4 million people and is increasing at a constant rate adequate for an additional 0.5 million people per year.
  a. Based on these assumptions, in approximately what year will this country first experience shortages of food?
  b. If the country doubled its initial food supply and maintained a constant rate of increase in the supply adequate for an additional 0.5 million people per year, would shortages still occur? In approximately which year?
  c. If the country doubled the rate at which its food supply increases, in addition to doubling its initial food supply, would shortages still occur?
(https://www.illustrativemathematics.org/content-standards/HSA/REI/D/11/tasks/645)

Rewrite the equations as a system of equations

$$\{f(x) = 3x^2 - 2x + 1 \quad g(x) = \frac{1}{2}x + 5$$

Using technology, graph the equations and look for points of intersection, where the same x produces $f(x) = g(x)$.

In Math 1, students are expected to solve linear equations using inverse operations and quadratic equations with square roots and factoring. In all other equations, such as exponential equations, solutions should be approximated with technology, tables and graphs.

Example: Solve the following equations by graphing. Give your answer to the nearest tenth.

a. $3(2^x) = 6x - 7$

b. $10x + 5 = -x + 8$

Instructional Resources	
Tasks	**Additional Resources**
Population and Food Supply (Illustrative Mathematics)	

Algebra – Reasoning with Equations and Inequalities

NC.M1.A-REI.12
Represent and solve equations and inequalities graphically
Represent the solutions of a linear inequality or a system of linear inequalities graphically as a region of the plane.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite	**Connections**
- Solve two-step linear inequalities (7.EE.4b) - Solve linear inequalities in one variable (NC.M1.A-REI.3) - Understand every point on a graph is a solution to its associated equation (NC.M1.A-REI.10)	*The following SMPs can be highlighted for this standard.* 5 – Use appropriate tools strategically 6 – Attend to precision
Connections	**Disciplinary Literacy**
- Create one variable linear inequalities and use the inequality to solve problems (NC.M1.A-CED.1) - Create a system of linear inequalities to model a situation in context (NC.M1.A-CED.3)	Students should be able to explain the reasoning behind their graphical representation of an inequality or system of inequalities.

Mastering the Standard

| Comprehending the Standard	Assessing for Understanding
Students should understand that since there is no way to list every solution to a linear inequality in two variables, the solutions must be represented graphically. Similarly, we recognize linear inequalities to have infinitely many solutions. It is an American tradition to shade the region that represent the solutions of the inequality. In other countries, they shade regions of the plane that do <u>not</u> contain solutions, marking that region out. This results in an unmarked solution region making it easier to identify and work with points in the solution region. This means that it is important for students to understand what the shaded region represents according to the context of the problem.	Students should be able to represent solutions to linear inequalities and systems of linear inequalities as a region of a plane. **Indicator:** Graph the solution set for the following system of inequalities: $3x + 5y \leq 10$ $y > -4$ **Indicator:** Graph the system of linear inequalities below and determine if (3,2) is a solution to the system. $x - 3y > 0$ $x + y \leq 2$ $x + 3y > -3$ **Indicator:** Graph the following inequalities: $3x - 4y \leq 7$ $y > -2x + 6$ $-9x + 4y \geq 1$

Indicator: What scenario could be modeled by the graph below?
a. The number of pounds of apples, y, minus two times the number of pounds of oranges, x, is at most 5.
b. The number of pounds of apples, y, minus half the number of pounds of oranges, x, is at most 5.
c. The number of pounds of apples, y, plus two times the number of pounds of oranges, x, is at most 5.
d. The number of pounds of apples, y, plus half the number of pounds of oranges, x, is at most 5.
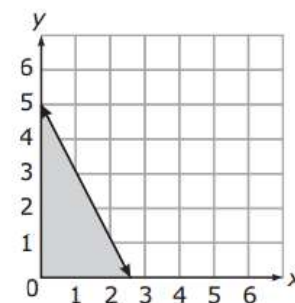
Indicator: Given below are the graphs of two lines, $y = -0.5x + 5$ and $y = -1.25x + 8$, and several regions and points are shown. Note that C is the region that appears completely white in the graph.
a. For each region and each point, write a system of equations or inequalities, using the given two lines, that has the region or point as its solution set and explain the choice of $\leq, \geq, or\ =$ in each case. (You may assume that the line is part of each region.)
b. The coordinates of a point within a region have to satisfy the corresponding system of inequalities. Verify this by picking a specific point in each region and showing that the coordinates of this point satisfy the corresponding system of inequalities for that region.
c. In the previous part, we checked that specific coordinate points satisfied our inequalities for each region. Without picking any specific numbers, use the same idea to explain how you know that all points in the 3rd quadrant must satisfy the inequalities for region A.
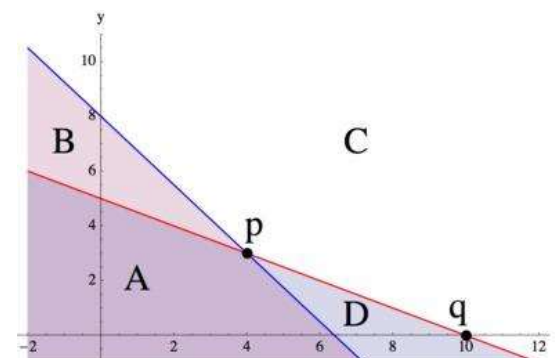
(https://www.illustrativemathematics.org/content-standards/HSA/REI/D/12/tasks/1205)

Indicator: Fishing Adventures rents small fishing boats to tourists for day-long fishing trips. Each boat can hold at most eight people. Additionally, each boat can only carry 900 pounds of weight for safety reasons.
a. Let p represent the total number of people. Write an inequality to describe the number of people that a boat can hold. Draw a number line diagram that shows all possible solutions.
b. Let w represent the total weight of a group of people wishing to rent a boat. Write an inequality that describes all total weights allowed in a boat. Draw a number line diagram that shows all possible solutions.
(https://www.illustrativemathematics.org/content-standards/tasks/642)

Instructional Resources	
Tasks	**Additional Resources**
Solution Sets (Illustrative Mathematics)	

Algebra, Functions & Function Families

NC Math 1	NC Math 2	NC Math 3
Functions represented as graphs, tables or verbal descriptions in context		
Focus on comparing properties of linear function to *specific* non-linear functions and rate of change. • Linear • Exponential • Quadratic	**Focus on properties of quadratic functions and an introduction to inverse functions through the inverse relationship between quadratic and square root functions.** • Quadratic • Square Root • Inverse Variation	**A focus on more complex functions** • Exponential • Logarithm • Rational functions w/ linear denominator • Polynomial w/ degree $\leq$ three • Absolute Value and Piecewise • Intro to Trigonometric Functions
A Progression of Learning of Functions through Algebraic Reasoning		

The conceptual categories of Algebra and Functions are inter-related. Functions describe situations in which one quantity varies with another. The difference between the Function standards and the Algebra standards is that the Function standards focus more on the characteristics of functions (e.g. domain/range or max/min points), function definition, etc. whereas the Algebra standards provide the computational tools and understandings that students need to explore specific instances of functions. As students progress through high school, the coursework with specific families of functions and algebraic manipulation evolve. Rewriting algebraic expressions to create equivalent expressions relates to how the symbolic representation can be manipulated to reveal features of the graphical representation of a function.

Note: The Numbers conceptual category also relates to the Algebra and Functions conceptual categories. As students become more fluent with their work within particular function families, they explore more of the number system. For example, as students continue the study of quadratic equations and functions in Math 2, they begin to explore the complex solutions. Additionally, algebraic manipulation within the real number system is an important skill to creating equivalent expressions from existing functions.

Functions – Interpreting Functions

NC.M1.F-IF.1
Understand the concept of a function and use function notation.
Build an understanding that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range by recognizing that:
- if f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x.
- the graph of f is the graph of the equation $y = f(x)$.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite	**Connections**
• Understand that a function is a rule that assigns to each input exactly one output (8.F.1) • Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)	*The following SMPs can be highlighted for this standard.* 1 – Make sense of problems and persevere in solving them
Connections	**Disciplinary Literacy**
• Create and graph two variable equations (NC.M1.A-CED.2) • All other function standards	*New Vocabulary: function notation* Students should be able to accurate describe a function in their own terms.

Mastering the Standard

Clarification	Checking for Understanding
Students should understand the definition of a function. It is deeper than just "x" cannot repeat or the vertical line test. Students should understand what it takes to be a function in categorical, numerical, and graphical scenarios. In 8th grade, students defined a function. Function notation is introduced in NC Math 1. While this standard focuses on the correspondence definition of a function, that is the input and output of values, a function can also be defined by how one variable changes in relation to another variable. This is known as the covariation definition of a function. This view of a function is highlighted in other standards throughout NC Math 1 when students are asked to identify, interpret, and use the rate of change.	Students should be able to understand functions in categorical scenarios. **Indicator:** A certain business keeps a database of information about its customers. a. Let C be the rule which assigns to each customer shown in the table his or her home phone number. Is C a function? Explain your reasoning. b. Let P be the rule which assigns to each phone number in the table above, the customer's name(s) associated with it. Is P a function? Explain your reasoning. c. Explain why a business would want to use a person's social security number as a way to identify a particular customer instead of their phone number. (https://www.illustrativemathematics.org/content-standards/HSF/IF/A/1/tasks/624) Students should be able to understand functions as a correspondence between inputs and outputs. **Indicator:** A pack of pencils cost \$0.75. If n number of packs are purchased, then the total purchase price is represented by the function $t(n) = 0.75n$. a. Explain why t is a function. b. What is a reasonable domain and range for the function t? **Indicator:** Suppose f is a function. a. If $10 = f(-4)$, give the coordinates of a point on the graph of f. b. If 6 is a solution of the equation $f(w) = 1$, give a point on the graph of f. (https://www.illustrativemathematics.org/content-standards/HSF/IF/A/1/tasks/630)

Table (right side of Checking for Understanding):

Customer Name	Home Phone Number
Heather Baker	3105100091
Mike London	3105200256
Sue Green	3234132598
Bruce Swift	3234132598
Michelle Metz	2138061124

Instructional Resources

Tasks	Additional Resources
The Customers (Illustrative Mathematics) Points on a Graph (Illustrative Mathematics)	Card Sort: Functions (DESMOS) Understanding Range (DESMOS)

Functions – Interpreting Functions

NC.M1.F-IF.2
Understand the concept of a function and use function notation.
Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Concepts and Skills
Pre-requisite
- Use substitution to determine if a number if a solution (6.EE.5)   - Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)   - Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)   - Define a function and use functions notation (NC.M1.F-IF.1)
Connections
- Creating and solving one variable equations (NC.M1.A-CED.1)   - Creating and graphing two variable equations (NC.M1.A-CED.2)   - Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)   - Function standards that relate domain and range (NC.M1.F-IF.3, NC.M1.F-IF.4, NC.M1.F-IF.5, NC.M1.F-IF.7)   - Comparing the end behavior of functions (NC.M1.F-LE.3)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard. 1 – Make sense of problems and persevere in solving them
Disciplinary Literacy
New Vocabulary: exponential function, quadratic function Students should be able to discuss the domain, range, input, output and the relationship between the variables of a function in context.

Mastering the Standard

Clarification	Checking for Understanding

Students should be fluent in using function notation to evaluate a linear, quadratic, and exponential function.
 	
Students should be able to interpret statements in function notation in contextual situations. This standard should be linked to the algebra standard NC.M1.A-SSE.1 where students are expected to interpret expressions in context.
	Students should be able to use evaluate functions written in function notation.
	Indicator: Evaluation $f(2)$ for the following functions:
	   a) $f(x) = 5(x - 3) + 17$.
	   b) Evaluate $f(2)$ for the function $f(x) = 1200(1 + .04)^x$.
	   c) Evaluate $f(2)$ for the function $f(x) = 3x^2 + 2x - 5$.
	
	Students should be able to evaluate functions and interpret the result in a context.
	Indicator: You placed a yam in the oven and, after 45 minutes, you take it out. Let f be the function that assigns to each minute after you placed the yam in the oven, its temperature in degrees Fahrenheit. Write a sentence for each of the following to explain what it means in everyday language.
	   a. $f(0) = 65$ b. $f(5) < f(10)$
	   c. $f(40) = f(45)$ d. $f(45) > f(60)$
	      (https://www.illustrativemathematics.org/content-standards/HSF/IF/A/2/tasks/625)
	
	Indicator: The rule $f(x) = 50(0.85)^x$ represents the amount of a drug in milligrams, $f(x)$, which remains in the bloodstream after x hours. Evaluate and interpret each of the following:
	   a. $f(0)$
	   b. $f(2) = k \cdot f(1)$. What is the value of k?
	   c. $f(x) < 6$

Indicator: Suppose that the function $f(x) = 2x + 12$ represents the cost to rent x movies a month from an internet movie club. Makayla now has $10. How many more dollars does Makayla need to rent 7 movies next month?

(NCDPI Math 1 released EOC #12)

Indicator: Let $f(t)$ be the number of people, in millions, who own cell phones t years after 1990. Explain the meaning of the following statements.

   a. $f(10) = 100.3$
   b. $f(a) = 20$
   c. $f(20) = b$
   d. $n = f(t)$

(https://www.illustrativemathematics.org/content-standards/HSF/IF/A/2/tasks/634)

Indicator: Suppose Matthew throws a baseball into the air. The height of the baseball at any given time, t, can be modeled by the function $h(t) = -16t^2 + 65t + 5$.

   a. What is the height of the baseball after 2 seconds?
   b. If $h(1) = 54$, what does this mean in context of the baseball scenario?

Functions – Interpreting Functions

NC.M1.F-IF.3
Understand the concept of a function and use function notation.
Recognize that recursively and explicitly defined sequences are functions whose domain is a subset of the integers, the terms of an arithmetic sequence are a subset of the range of a linear function, and the terms of a geometric sequence are a subset of the range of an exponential function.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite	**Connections**
• Interpret the equation $y = mx + b$ as being from a linear function and compare to nonlinear functions (8.F.3) • Define a function and use functions notation (NC.M1.F-IF.1) • Evaluating functions (NC.M1.F-IF.2)	*The following SMPs can be highlighted for this standard.* 8 – Look for and express regularity in repeated reasoning
Connections	**Disciplinary Literacy**
• Relating the domain and range to a context (NC.M1.F-IF.5) • Analyzing linear and exponential functions (NC.M1.F-IF.7) • Build linear and exponential functions (NC.M1.F-BF.1) • Translate between explicit and recursive forms (NC.M1.F-BF.2) • Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)	*New Vocabulary: arithmetic sequence, geometric sequence, explicit form, recursive form, exponential function* Students should be able to explain a function written in recursive form using subset notation.

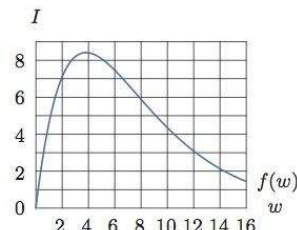| Mastering the Standard	
Clarification	**Checking for Understanding**

Students should recognize that sequences are functions. A sequence can be described as a function, with the domain consisting of a subset of the integers, and the range being the terms of the sequence.
 	
This standard connects to arithmetic and geometric sequences and should be taught with NC.M1.F-BF.2.
 	
Students should be able to examine the pattern of a sequence and connect it to the rate of change of the associated function.
 	
It is also important to note that sequences are not limited to arithmetic and geometric. It is expected that recursive form should be written in subset notation. Students should be familiar with writing and interpreting subset notation.
Now-Next can be used as a tool to introduce the concepts of recursive form, but the expectation is that students will move to the more formal representations of recursive form.
	Indicator: A theater has 60 seats in the first row, 68 seats in the second row, 76 seats in the third row, and so on in the same increasing pattern.
	   a. If the theater has 20 rows of seats, how many seats are in the twentieth row?
	   b. Explain why the sequence is considered a function.
	   c. What is the domain of the sequence? Explain what the domain represents in context.
	   d. What is the range of the sequence? Explain what the range represents in context.
	
	Indicator: A geometric sequence can be represented by the exponential function $f(x) = 400 \left(\frac{1}{2}\right)^x$. In terms of the geometric sequence, explain what $f(3) = 50$ represents.
	
	Indicator: Represent the following sequence in explicit form: $1, 4, 9, 16, 25$
	
	Indicator: The Fibonacci numbers are sequence that are often found in nature. This sequence is defined by $a_n = a_{n-1} + a_{n-2}$ where $a_0 = 0$ and $a_1 = 1$. What are the first 10 terms of the Fibonacci sequence? Could you easily represent this pattern in explicit form?

Functions – Interpreting Functions

OCS Priority Standard: NC.M1.F-IF.4

*Interpret functions that arise in applications in terms of the context.*
Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.

Concepts and Skills
Pre-requisite
- Describe quantitatively the functional relationship between two quantities by analyzing a graph (8.F.5)   - Define a function and use functions notation (NC.M1.F-IF.1)   - Evaluating functions (NC.M1.F-IF.2)
Connections
- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)   - Relate domain and range of a function to its graph (NC.M1.F-IF.5)   - Calculate the average rate of change (NC.M1.F-IF.6)   - Use equivalent forms of quadratic and exponential function to reveal key features (NC.M1.F-IF.8a, NC.M1.F-IF.8b)   - Compare key features of two functions in different representations (NC.M1.F-IF.9)   - Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard. 1 – Make sense of problems and persevere in solving them 4 – Model with mathematics
Disciplinary Literacy
New Vocabulary: maximum, minimum Students should be able to justify their identification of key features and interpret those key features in context.

Mastering the Standard	
Clarification	**Checking for Understanding**

Students should understand the key features of any contextual situation. For example, plots over time represent functions as do some scatterplots. These are often functions that "tell a story". Students should have experience with a wide variety of these types of functions and be flexible in thinking about functions and key features using tables, graphs, and verbal descriptions.
 	
Students should understand the concept behind the key features (intercepts, increasing/decreasing, positive/negative, and maximum/minimum) for any given graph, not just "function families". This means that students should be asked to work with graphical and tabular representations of functions that the student could not solve or manipulate algebraically.
 	
Given a problem that asks students to identify a region, students are expected to write answers using inequality notation. Students in Math 1 are not responsible for using interval notation to represent a solution.
	Students should be able to identify and interpret key features of functions.
	    Indicator: An epidemic of influenza spreads through a city. The figure below is the graph of $I = f(w)$, where I is the number of individuals (in thousands) infected w weeks after the epidemic begins.
	    a. Estimate $f(2)$ and explain its meaning in terms of the epidemic.
	    b. Approximately how many people were infected at the height of the epidemic? When did that occur? Write your answer in the form $f(a) = b$.
	    c. For approximately which w is $f(w) = 4.5$; explain what the estimates mean in terms of the epidemic.
	
	(https://www.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/637)
	
	Example: The figure shows the graph of T, the temperature (in degrees Fahrenheit) over one 20-hour period in Santa Elena as a function of time t.

It is important for students to begin developing an understanding of end behavior and interpreting mathematical notation (such as $x \to \infty$). As students study intervals of increasing and decreasing, connect their mathematical thinking from "as we keep going out" or "as x gets really big" to "as x goes to infinity".


$y = T(t)$
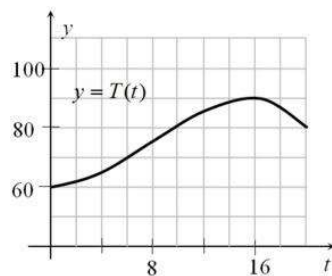
a. Estimate $T(14)$.
b. If $t = 0$ corresponds to midnight, interpret what we mean by $T(14)$ in words.
c. Estimate the highest temperature during this period from the graph.
d. When was the temperature decreasing?
e. If Anya wants to go for a two-hour hike and return before the temperature gets over 80 degrees, when should she leave?

(https://www.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/639)

Indicator: Eliana observed her dog, Lola, running around the yard and recorded the time and distance that Lola was away from her doghouse in the table to the right.
    a.   Sketch a graph of Lola's play time away from her doghouse.
   b.   Describe what is happening between minutes 2 & 3.

Time (minutes)	Distance (feet)
0	0
1	5
2	30
3	15
4	25
5	50
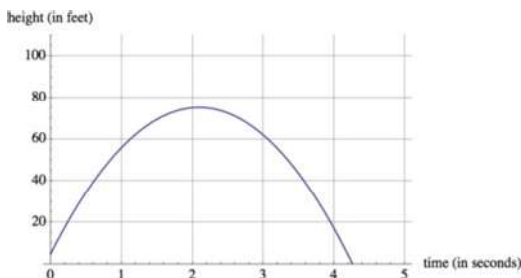
Indicator: Suppose Brett and Andre each throw a baseball into the air. The height of Brett's baseball is given by $h(t) = -16t^2 + 79t + 6$, where h is in feet and t is in seconds. The height of Andre's baseball is given by the graph. Interpret the x-intercept, y-intercept, and maximum in context of the baseball scenario.
Adapted from Illustrative Math

Indicator: Identify the key features of the grade below.
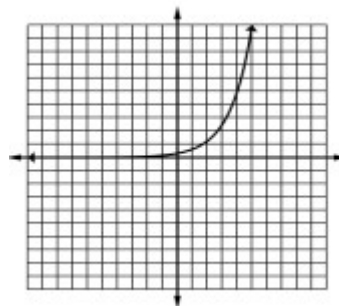
There are several videos on this site http://graphingstories.com Some are aligned to Math I while others are more appropriate for Math 2 or 3. The following are suggested videos for Math I: (1) Water Volume, (2) Weight, (3) Bum Height Off Ground, (4) Air Pressure, (5) Height of Stack.

Function Carnival (DESMOS)

Function Carnival, Part 2 (DESMOS)

Representing Functions of Everyday Situations (Mathematics Assessment Project)
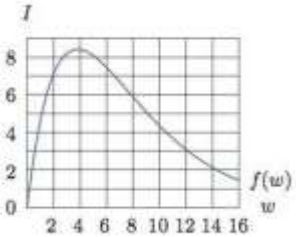
Functions – Interpreting Functions

NC.M1.F-IF.5
Interpret functions that arise in applications in terms of the context.
Interpret a function in terms of the context by relating its domain and range to its graph and, where applicable, to the quantitative relationship it describes.

Concepts and Skills
Pre-requisite
• In middle school, students only informally considered restrictions to the domain and range based on context, such as understanding that measurements cannot be negative.
• Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
• Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)
• Formally define a function (NC.M1.F-IF.1)
• Evaluating functions and interpret in context (NC.M1.F-IF.2)
Connections
• Recognize the domain of sequences (NC.M1.F-IF.3)
• Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)
• Analyze linear, quadratic, and exponential functions to identify key features (NC.M1.F-IF.7)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard. 4 – Model with mathematics
Disciplinary Literacy

Mastering the Standard	
Clarification	**Checking for Understanding**
Students should be able to associate a reasonable domain and range to a graph as well as to a contextual situation. The domain of a graph should be taught in the context of the situation it represents. Graphs represented should be both discrete and continuous forms.	Students should be able to identify a reasonable domain and range to its graph as well as to a contextual situation. **Indicator:** An epidemic of influenza spreads through a city. The figure below is the graph of $I = f(w)$, where I is the number of individuals (in thousands) infected w weeks after the epidemic begins. An equation for the function used to plot the image above is $f(w) = 6w(1.3)^{-w}$. Use the graph to estimate the solution of the inequality $6w(1.3)^{-w} \geq 6$. Explain what the solution means in terms of the epidemic. **Indicator:** Maggie tosses a coin off of a bridge into a stream below. The distance the coin is above the water is modeled by the equation $y = -16x^2 + 96x + 112$, where x represents time in seconds. What is a reasonable domain for the function?

Indicator: SoFi Stadium in Los Angeles is capable of seating 70,000 spectators for a football game. For each game, the amount of money that is brought in as revenue is a function of the number of people, n, in attendance.

    a) The average of the cheapest ticket prices of the Los Angeles Ram in 2022 is $94. Find the domain of the function of the revenue to the number of people in attendance, using the average cheapest ticket.

    b) At a game, the Rams have decided to honor fans who served in the military. For this event, the Rams will be giving away 1,500 tickets to military families. How does this effect the domain and range of the function? What does this mean for the Raiders and their fans?
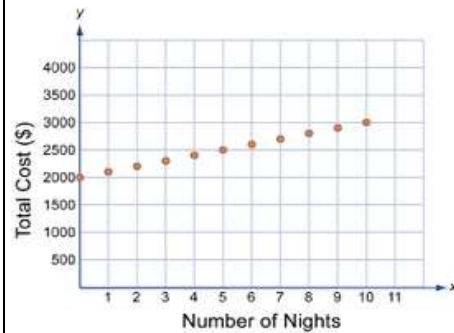
Indicator: An all-inclusive resort in Los Cabos, Mexico provides everything for their customers during their stay including food, lodging, and transportation. Use the graph at the left to describe the domain of the total cost function.

Indicator: Jacob is observing bacterial growth in a lab. He noted that the bacteria double in number every hour. There are 50 bacteria at the beginning of his experiment. Determine the appropriate domain and range of the function if Jacob runs the experiment for 8 hours.

Functions – Interpreting Functions

NC.M1.F-IF.6
Interpret functions that arise in applications in terms of the context.
Calculate and interpret the average rate of change over a specified interval for a function presented numerically, graphically, and/or symbolically.

Concepts and Skills
Pre-requisite
- Determine and interpret the rate of change of a linear function (8.F.4)   - Describe qualitatively the functional relationship between two quantities and sketch a graph from a verbal description (8.F.5)
Connections
- Interpret key features of graphs and tables (NC.M1.F-IF.4)   - Analyze linear, quadratic and exponential functions by generating different representations (NC.M1.F-IF.7)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard. 4 – Model with mathematics
Disciplinary Literacy
New Vocabulary: average rate of change

Mastering the Standard

| Clarification	Checking for Understanding

Students calculate the average rate of change of a function given a graph, table, and/or equation.
 	
The average rate of change of a function $y = f(x)$ over an interval $a \leq x \leq b$ is $\frac{chang\ in\ y}{change\ in\ x} = \frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$.
 	
This standard is more than just slope. It is asking students to find the average rate of change of any function over any given interval. Be sure to include multiple representations (numerically, graphically, or symbolically) of functions for students to work with.
 	
It is an important connection for later courses that students recognize that linear functions have consistent average rate of change over any interval, while functions like quadratics and exponentials do not have constant rates of change due to their curvature.
	Indicator: Find the average rate of change of each of the following functions over the interval $1 \leq x \leq 5$.
	
	    a) $f(x) = 3x - 7$
	    b) $g(x) = x^2 + 2x - 5$
	    c) $h(x) = 3(2)^x$
	
	Indicator: The table below shows the average weight of a type of plankton after several weeks. What is the average rate of change in weight of the plankton from week 8 to 12?
	
	|Time(weeks)|Weight (ounces)|
	|---|---|
	|8|0.04|
	|9|0.07|
	|10|0.14|
	|11|0.25|
	|12|0.49|
	
	Indicator: The table below shows the temperature, T, in Tucson, Arizona t hours after midnight. When does the temperature decrease the fastest: between midnight and 3 a.m. or between 3 a.m. and 4 a.m.?
	
	| t(hours after midnight) | 0 | 3 | 4 |
	|---|---|---|---|
	| T(temp. in °F) | 85 | 76 | 70 |
	
	(https://www.illustrativemathematics.org/content-standards/HSF/IF/B/6/tasks/1500)
	
	Indicator: You are a marine biologist working for the Environmental Protection Agency (EPA). You are concerned that the rare coral mathemafish population is being threatened by an invasive species

Time(weeks)	Weight (ounces)
8	0.04
9	0.07
10	0.14
11	0.25
12	0.49

t(hours after midnight)	0	3	4
T(temp. in °F)	85	76	70

known as the fluted dropout shark. The fluted dropout shark is known for decimating whole schools of fish. Using a catch-tag-release method, you collected the following population data over the last year.

# months since 1st measurement	0	1	2	3	4	5	6	7	8	9	10	11	12
Mathemafish population	480	472	417	318	240	152	103	84	47	32	24	29	46

Mathemafish Population
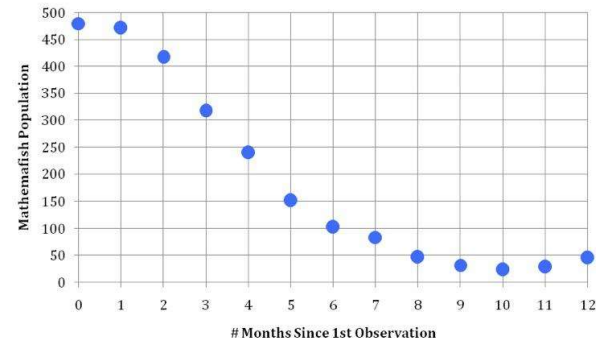
Through intervention, the EPA was able to reduce the dropout population and slow the decimation of the mathemafish population. Your boss asks you to summarize the effects of the EPA's intervention plan in order to validate funding for your project.

Item to include in your summary report:

a) Calculate the average rate of change of the mathemafish population over specific intervals. Indicate how and why you chose the intervals you chose.

b) When was the population decreasing the fastest?

c) During what month did you notice the largest effects of the EPA intervention?

d) Explain the overall effects of the intervention.

Remember to justify all your conclusions using supporting evidence.

(https://www.illustrativemathematics.org/content-standards/HSF/IF/B/6/tasks/686)

Functions – Interpreting Functions

OCS Priority Standard: NC.M1.F-IF.7
Analyze functions using different representations.
Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite	**Connections**
- Interpret $y = mx + b$ as being linear (8.F.3) - Determine rate of change and initial value of linear functions from tables and graphs (8.F.4) - Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b) - Formally define a function (NC.M1.F-IF.1) - Evaluating functions and interpret in context (NC.M1.F-IF.2) - Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)	*The following SMPs can be highlighted for this standard.* 4 – Model with mathematics
Connections	**Disciplinary Literacy**
- Creating and graphing two variable equations (NC.M1.A-CED.2) - Solving systems of equations (NC.M1.A-REI.6) - Recognize the domain of sequences as integers (NC.M1.F-IF.3) - Relate domain and range of a function to its graph (NC.M1.F-IF.5) - Calculate the average rate of change (NC.M1.F-IF.6) - Use equivalent forms of quadratic and exponential function to reveal key features (NC.M1.F-IF.8a, NC.M1.F-IF.8b) - Compare key features of two functions in different representations (NC.M1.F-IF.9) - Build functions that describe a relationship between two quantities (NC.M1.F-BF.1a, NC.M1.F-BF.1b) - Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1) - Interpret the parameters of a linear and exponential function in context (NC.M1.F-LE.5)	*New Vocabulary: exponential function, quadratic function* Students should be able to justify their use of a representation.

Mastering the Standard	
Clarification	**Checking for Understanding**
Students should identify the key features of the three function families covered in Math 1: linear, quadratic, and exponential. Students should be aware of the key functions typically associated with each function type. **<u>Linear functions</u>** – domain & range, rate of change, intercepts, increasing/decreasing	Students should be able to identify key feature of linear, quadratic and exponential functions from the symbolic representation. **Indicator:** Describe the key features of the graph $f(x) = \frac{-2}{3}x + 8$ and use the key features to create a sketch of the function. **Indicator:** Without using the graphing capabilities of a calculator, sketch the graph of

Quadratic functions – domain & range, y-intercept, x-intercepts (zeros), intervals of increasing and decreasing, intervals of positive and negative values, maximums and minimums, and end behavior

Exponential functions – domain & range, rate of change, increasing or decreasing (growth and decay), intervals of positive and negative values, and end behavior

It is important for students to begin developing an understanding of end behavior and interpreting mathematical notation (such as $x \to \infty$). As students study end behavior of these function families, connect their mathematical thinking from "as we keep going out" or "as x gets really big" to "as x goes to infinity".

At the Math 1 level, students should **not** be exposed to finding the line of symmetry of a quadratic function using the formula $x = \frac{-b}{2a}$, unless it is developed conceptually. This concept should be developed with a study of the quadratic formula, which will be done in Math 2.
If the students need to find the line of symmetry (not a requirement of Math 1), they can find the midpoint of the zeros of the function.

$f(x) = x^2 + 7x + 10$ and identify the x-intercepts, y-intercept, and the maximum or minimum point.

Indicator: The function $f(x) = 300(0.70)^x - 25$ models the amount of aspirin left in the bloodstream after x hours. Graph the function showing the key features of the graph. Interpret the key features in context of the problem.

Instructional Resources	
Tasks	**Additional Resources**
Which Function? (Illustrative Mathematics)	Polygraph: Lines (DESMOS) Polygraph: Lines, Part 2 (DESMOS) Polygraph: Quadratics (DESMOS)

Functions – Interpreting Functions

NC.M1.F-IF.8a
Analyze functions using different representations.
Use equivalent expressions to reveal and explain different properties of a function.
    a.   Rewrite a quadratic function to reveal and explain different key features of the function

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite	**Connections**
- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b) - Factor to reveal key features (NC.M1.A-SSE.3) - Operations with polynomials (NC.M1.A-APR.1) - Understand the relationship between linear factors and zeros (NC.M1.A-APR.3) - Formally define a function (NC.M1.F-IF.1)	*The following SMPs can be highlighted for this standard.* 4 – Model with mathematics 5 – Use appropriate tools strategically
Connections	**Disciplinary Literacy**
- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4) - Identify and interpret key features of functions from different representations (NC.M1.F-IF.7) - Compare key features of two functions in different representations (NC.M1.F-IF.9)	*New Vocabulary: quadratic function*

Mastering the Standard

| Clarification	Checking for Understanding

This set of standards requires that students rewrite expressions of quadratic and exponential functions to reveal key features of their graphs.
This is the "why" behind rewriting an expression where NC.M1.A-SSE.1 is the "how". Therefore, these two standards should be taught together.
This standard should also tie to the key features of graphs in NC.M1.F.IF.7
At the Math 1 level, students only know two forms of quadratics; standard and factored. Students are not expected to complete the square or write a quadratic in vertex form. Therefore, other methods for finding the vertex should be used, such as calculating the midpoint between two zeros to find the x-value of the vertex and using function notation to determine the y-value of the vertex. Using a graphing utility to analyze key features of a quadratic function may be necessary.
 	
Students should be able to examine key features of a function given different forms of the equation. For example, they should know that standard form of a quadratic equation reveals the y-intercept, and the coefficient of the quadratic terms indicates the direction the graph opens while the factored form of a quadratic function reveals the x-intercepts or zeros of the function.
 	
In the Math 1 level, students are not expected to find the line of symmetry of a quadratic function using the formula $x = \frac{-b}{2a}$. This concept should be developed with a study of the quadratic formula in Math 2. If the students need to find the line of
	Students should be able to factor quadratic expressions to find key features of the quadratic function.
	  Indicator: Suppose $h(t) = -5t^2 + 10t + 15$ is the height of a diver above the water (in meters), t seconds after the diver leaves the springboard. When does the diver hit the water?
	  (https://www.illustrativemathematics.org/content-standards/HSF/IF/C/8/tasks/375)
	
	Indicator: Which of the following is the function graphed below?
	A) $f(x) = 4x^2 - 8x + 7$
	B) $f(x) = x^2 + 7x + 3$
	C) $f(x) = 7x^2 - 4x + 3$
	D) $f(x) = 3x^2 + x + 7$
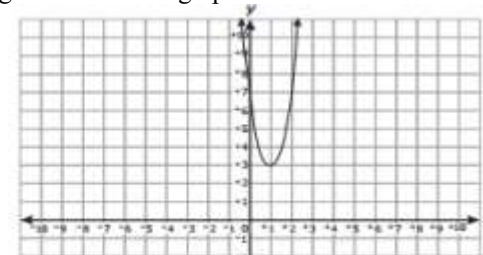
symmetry (not a requirement of Math 1) using the midpoint of the zeros of the function.

The typical key features of a quadratic functions are: domain and range, y-intercept, x-intercepts (zeros), intervals of increasing and decreasing, intervals of positive and negative values, maximums and minimums, and end behavior.

Indicator: The function $f(t) = -5t^2 + 20t + 60$ models the approximate height of an object t seconds after it is launched. Rewrite the function to show how many seconds does it take the object to hit the ground.

Indicator: Suppose that the equation $V = 20.8x^2 - 458.3x + 3500$ represents the value of a car from 1964 to 2002. Rewrite the function to show what year did the car have the lease value. ($x = 0$ in 1964)

Indicator: Which of the following could be the function of a real variable x whose graph is shown below? Explain.
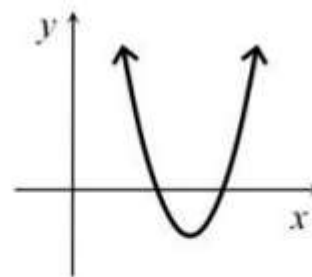
$f_1(x) = (x + 12)^2 + 4$	$f_5(x) = -4(x + 2)(x + 3)$
$f_2(x) = -(x - 2)^2 - 1$	$f_6(x) = (x + 4)(x - 6)$
$f_3(x) = (x + 18)^2 - 40$	$f_7(x) = (x - 12)(-x + 18)$
$f_4(x) = (x - 12)^2 - 9$	$f_8(x) = (24 - x)(40 - x)$

(https://www.illustrativemathematics.org/content-standards/HSF/IF/C/8/tasks/640)

Functions – Interpreting Functions

NC.M1.F-IF.8b

Analyze functions using different representations.

Use equivalent expressions to reveal and explain different properties of a function.

    b.   Interpret and explain growth and decay rates for an exponential function.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite	**Connections**
- Identify and interpret parts of expression (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b) - Formally define a function (NC.M1.F-IF.1)	*The following SMPs can be highlighted for this standard.* 4 – Model with mathematics
Connections	**Disciplinary Literacy**
- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4) - Identify and interpret key features of functions from different representations (NC.M1.F-IF.7) - Compare key features of two functions in different representations (NC.M1.F-IF.9)	*New Vocabulary: exponential function, growth rate, decay rate, growth factor*

Mastering the Standard

Clarification	Checking for Understanding

This set of standards requires that students rewrite expressions of quadratic and exponential functions to reveal key features of their graphs.
 	
This is the "why" behind rewriting an expression where NC.M1.A-SSE.1 interprets the rate in context. Therefore, these two standards should be taught together.
 	
This standard should also tie to the key features of graphs in NC.M1.F.IF.7
Students should know the key features of an exponential function and how they relate to a contextual situation.
 	
Understanding the difference between the ***growth factor*** and the ***growth rate*** is an important part of understanding this standard. Students should be able to find the initial value, growth/decay rate and the growth factor for the interval based on the given context. Both the growth and decay ***rate*** are the distance from one. This concept is directly connected to percent increase/decrease from middle school mathematics.
   - The *growth factor* is the value being repeatedly multiplied in the function.   - The *growth/decay rate* is the value added to or subtracted from one to create the *growth factor*.   For example, in the exponential function, $f(x) = 5(.75)^x$, **.75** is the growth **factor** as it is repeatedly multiplied in the function. Whereas the **growth rate** would be $-.25$ indicating decay because the *growth factor* is less than one.
	Students should know the key features of an exponential function and how they relate to a contextual situation.
	Indicator: The expression $50(0.85)^x$ represents the amount of a drug in milligrams that remains in the bloodstream after x hours.
	   a. Describe how the amount of drug in milligrams changes over time.
	   b. What as the initial value of the drug in the bloodstream?
	   c. What would the expression $50(0.80)^x$ represent?
	   d. What new or different information is revealed by the changed expression?
	
	Indicator: City Bank pays a simple interest rate of 3% per year, meaning that each year the balance increases by 3% of the initial deposit. National Bank pays an compound interest rate of 2.6% per year, compounded monthly.
	  a. Which bank will provide the largest balance if you plan to invest $10,000 for 10 years? For 15 years?
	  b. Write an expression for $C(y)$, the City Bank balance, y years after a deposit is left in the account. Write an expression for $N(m)$, the National Bank balance, m months after a deposit is left in the account.
	  c. For which years is City Bank a better investment, and for which years is National Bank a better investment?

Functions – Interpreting Functions

OCS Priority Standard: NC.M1.F-IF.9
Analyze functions using different representations.
Compare key features of two functions (linear, quadratic, or exponential) each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

Concepts and Skills
Pre-requisite
- Compare properties of two functions each represented in different ways (8.F.2)   - Formally define a function (NC.M1.F-IF.1)   - Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)   - Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)   - Rewrite quadratic functions to identify key features (NC.M1.F-IF.8a)   - Interpret and explain growth and decay rates for an exponential function (NC.M1.F-IF.8b)
Connections

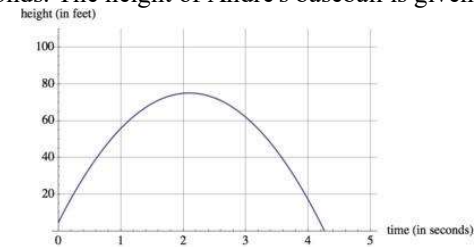| The Standards for Mathematical Practices |
| --- |
| **Connections** |
| *The following SMPs can be highlighted for this standard.*
4 – Model with mathematics
5 – Use appropriate tools strategically |
| **Disciplinary Literacy** |
| *New Vocabulary: exponential function, quadratic function*
Students should be able to justify their use of a representation to make the comparison. |

Mastering the Standard

Clarification	Checking for Understanding
Students should compare two functions in two different forms. The function types may be the same (linear & linear) or different (linear & exponential), but the representations should be different (e.g. numerical & graphical). It is important to note that the point of this standard is not to have students simply translate one function into the same form as the other function when given in different forms. Students should be able to use appropriate tools to compare the key features of functions.	**Indicator:** Suppose Brett and Andre each throws a baseball into the air. The height of Brett's baseball is given by the function $h(t) = -16t^2 + 79t + 6$, where h is in feet and t is in seconds. The height of Andre's baseball is given by the graph below: Brett claims that his baseball went higher than Andre's, and Andre says that his baseball went higher. - Who is right? - How long is each baseball airborne? - Construct a graph of the height of Brett's throw as a function of time on the same set of axes as the graph of Andre's throw (if not done already), and explain how this can confirm your claims to parts (a) and (b).

**Indicator:** Joe is trying to decide which job would allow him to earn the most money after a few years.

- His first job offer agrees to pay him $500 per week. If he does a good job, they will give him a 2% raise each year.
- His other job offer agrees to pay him according to the following equation
  $f(x) = 20,800(1.03)^x$, where x represents the number of years and $f(x)$ his salary.

Which job would you suggest Joe take? Justify your reasoning.

Indicator: Mario compared the slope of the function graphed below to the slope of the linear function that has an x-intercept of $\frac{4}{3}$ and a y-intercept of -2.

What is the slope of the function with the smaller slope?
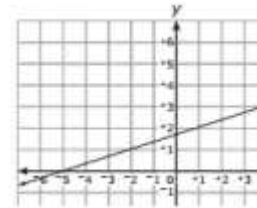
Indicator: Kevin compared the y-intercept of the graph of the function $f(x) = 3x^2 + 5$ to the y-intercept of the graph of the linear function that includes the points in the table to the right. What is the difference when the y-intercept of $f(x)$ is subtracted from the y-intercept of $g(x)$?

x	g(x)
-7	2
-5	3
-3	4
-1	5

Instructional Resources	
Tasks	**Additional Resources**
	Card Sort: Linear Functions (DESMOS)

Functions – Building Functions

NC.M1.F-BF.1a
Build a function that models a relationship between two quantities.
Write a function that describes a relationship between two quantities.
   a. Build linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two ordered pairs (include reading these from a table).

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite	**Connections**
- Construct a function to model a linear relationship (8.F.4) - Formally define a function (NC.M1.F-IF.1) - Recognize arithmetic and geometric sequences as linear and exponential functions (NC.M1.F-IF.3) - Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)	*The following SMPs can be highlighted for this standard.* 4 – Model with mathematics
Connections	**Disciplinary Literacy**
- Create and graph two variable equations (NC.M1.A-CED.2) - Identify key feature of graphs and tables of functions (NC.M1.F-IF.4) - Identify and interpret key features of functions from different representations (NC.M1.F-IF.7) - Translate between explicit and recursive forms (NC.M1.F-BF.2)	*New Vocabulary: arithmetic sequence, geometric sequence, exponential function* Students should be able to justify claims that a sequence defines a linear or exponential relationship.

| Mastering the Standard	
Clarification	**Checking for Understanding**
This standard is about building a function from different representations. In this part of the standard, the different representations include sequences, graphs, verbal descriptions, tables, and ordered pairs. This standard pairs well with Interpreting Functions standards, in that the purpose behind building a function is to then use that function to solve a problem. These functions can be written in function notation (linear or exponential) or as a sequence in explicit or recursive form. Students should recognize explicit form of an arithmetic sequence as an equivalent structure to slope-intercept form of a linear function and explicit form of a geometric sequence as an equivalent structure to standard	Students should write functions from verbal descriptions as well as a table of values **Indicator:** Suppose a single bacterium lands on one of your teeth and starts reproducing by a factor of 2 every hour. If nothing is done to stop the growth of the bacteria, write a function for the number of bacteria as a function of the number of days. **Indicator:** The table below shows the cost of a pizza based on the number of toppings. <table><tr><th>Number of Toppings (n)</th><th>Cost (C)</th></tr><tr><td>1</td><td>$12.00</td></tr><tr><td>2</td><td>$13.50</td></tr><tr><td>3</td><td>$15.00</td></tr><tr><td>4</td><td>$16.50</td></tr></table> Write a function that represents the cost of a pizza with n toppings?

form of an exponential function. Using the concepts of rate of change, students should recognize that the forms of these sequences are one iteration forward from the y-intercept, which gives meaning to the $n-1$ notation.

Students should be familiar with both function and sequence notation for defining a sequence recursively.

	Function Notation	Sequence Notation
Term	$f(n)$	a_n
Previous term	$f(n-1)$	a_{n-1}
Next term	$f(n+1)$	a_{n+1}

Indicator: The height of a stack of cups is a function of the number of cups in the stack. If a 7.5" cup with a 1.5" lip is stacked vertically, determine a function that would provide you with the height based on any number of cups.
   Hint: Start with height of one cup and create a table, list, graph, or description that describes the pattern of the stack as an additional cup is   added.
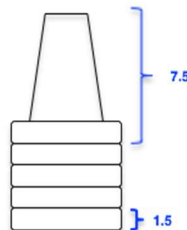
Indicator: There were originally 4 trees in an orchard. Each year the owner planted the same number of trees. In the 29th year, there were 178 trees in the orchard. Which function, $t(n)$, can be used to determine the number of trees in the orchard in any year, n?

a. $t(n) = \frac{178}{29}n + 4$

b. $t(n) = \frac{178}{29}n - 4$

c. $t(n) = 6n + 4$

d. $t(n) = 29n - 4$

Students should write linear or exponential relationships as a sequence in explicit or recursive form.
   Indicator: The price of a new computer decreases with age. Examine the table by analyzing the outputs.
   a.   Describe the recursive relationship.
   b.   Analyze the input and the output pairs to determine an explicit function that represents the value of the computer when the age is known.

Age	Value
1	$1575
2	$1200
3	$900
4	$650
5	$500
6	$400
7	$300

Indicator: Investigate the following sequence:

a. Create a recursive formula for the number of stars in the next pattern.
b. Build an explicit formula for the number of stars in the nth pattern.
c. How many stars are in the 43rd pattern?

(www.visualpatterns.org)

Instructional Resources	
Tasks	**Additional Resources**
	[Put the Point on the Line](#) (DESMOS)
	[Modeling Population Growth: Having Kittens](#) (Mathematics Assessment Project)

Functions – Building Functions

NC.M1.F-BF.1b
Build a function that models a relationship between two quantities.
Write a function that describes a relationship between two quantities.
  b. Build a function that models a relationship between two quantities by combining linear, exponential, or quadratic functions with addition and subtraction or two linear functions with multiplication.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite	**Connections**
- Construct a function to model a linear relationship (8.F.4) - Operations with polynomials (NC.M1.A-APR.1) - Formally define a function (NC.M1.F-IF.1)	*The following SMPs can be highlighted for this standard.* 4 – Model with mathematics
Connections	**Disciplinary Literacy**
- Create and graph two variable equations (NC.M1.A-CED.2) - Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)	*New Vocabulary: exponential function, quadratic function* Students should be able to justify their process of building a new function.

Mastering the Standard

Clarification	Checking for Understanding

This standard is about building functions. In this part of the standard students should combine functions to represent a contextual situation.
 	
This standard pairs well with Interpreting Functions standards, in that the purpose behind building a function is to then use that function to solve a problem.
 	
The algebraic skills behind this standard occur in NC.M1.A-APR.1. This standard should be taught throughout the year as each new function family is added to the course.
	Students should combine functions to represent a contextual situation.
	    Indicator: Cell phone Company Y charges a $10 start-up fee plus $0.10 per minute, x. Cell phone Company Z charges $0.20 per minute, x, with no start-up fee. Write a function that represents the difference in cost between Company Y and Company Z?
	
	    Indicator: A retail store has two options for discounting items to go on clearance.
	        ● Option 1: Decrease the price of the item by 15% each week.
	        ● Option 2: Decrease the price of the item by $5 each week.
	    If the cost of an item is $45, write a function rule for the difference in price between the two options.
	
	    Indicator: Blake has a monthly car payment of $225. He has estimated an average cost of $0.32 per mile for gas and maintenance. He plans to budget for the car payment the minimal he needs with an additional 3% of his total budget for incidentals that may occur. Build a function that gives the amount Blake needs to budget as a function of the number of miles driven.
	
	    Indicator: The floor of a rectangular cage has a length 4 feet greater than its width, w. James will increase both dimensions of the floor by 2 feet. Write an equation that represents the new area, N, of the floor of the cage?

Instructional Resources

Tasks	Additional Resources
	[Will it Hit the Hoop?](#) (DESMOS: Quadratic specifically)

Functions – Building Functions

NC.M1.F-BF.2

Build a function that models a relationship between two quantities.
Translate between explicit and recursive forms of arithmetic and geometric sequences and use both to model situations.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite	**Connections**
- Construct a function to model a linear relationship (8.F.4) - Formally define a function (NC.M1.F-IF.1) - Recognize sequences as function and link arithmetic sequences to linear functions and geometric sequences to exponential functions (NC.M1.F-IF.3) - Build functions from arithmetic and geometric sequences (NC.M1.F-BF.1a)	*The following SMPs can be highlighted for this standard.* 4 – Model with mathematics
Connections	**Disciplinary Literacy**
	New Vocabulary: arithmetic sequence, geometric sequence, explicit form, recursive form Students should be able to explain their model in context.

Mastering the Standard	
Clarification	**Checking for Understanding**

Students should be able to use both the explicit and recursive forms of arithmetic and geometric sequences where the explicit form is a linear or exponential function, respectively.
 	
Students are expected to use formal notation (function or sequence notation, see NC.M1.F-BF.1a). Use of the informal notation, NEXT-NOW, can be used to build conceptual understanding of recursive functions, however the expectation is for students to know and use formal notation.
 	
Students should recognize explicit form of an arithmetic sequence as an equivalent structure to slope-intercept form of a linear function and explicit form of a geometric sequence as an equivalent structure to standard form of an exponential function. Using the concepts of rate of change, students should recognize that the forms of these sequences are one iteration forward from the y-intercept, which gives meaning to the $n-1$ notation.
	Students should be able to build explicit and recursive forms of arithmetic and geometric sequences.
	
	Indicator: The following sequence shows the number of trees that a nursery plants each year. $2, 8, 32, 128 \ldots$
	Let a_n represent the current term in the sequence and a_{n-1} represent the previous term in the sequence. Write a formula that could be used to determine the number of trees the nursery will plant in year n?
	
	Indicator: A single bacterium is placed in a test tube and splits in two after one minute. After two minutes, the resulting two bacteria split in two, creating four bacteria. This process continues.
	   a) How many bacteria are in the test tube after 5 minutes? 15 minutes?
	   b) Write a recursive rule to find the number of bacteria in the test tube from one minute to the next.
	   c) Convert this rule into explicit form. How many bacteria are in the test tube after one hour?
	
	Indicator: A concert hall has 58 seats in Row 1, 62 seats in Row 2, 66 seats in Row 3, and so on. The concert hall has 34 rows of seats.
	   a) Write a recursive formula to find the number of seats in each row. How many seats are in row 5?
	   b) Write the explicit formula to determine which row has 94 seats?
	
	Indicator: Given the sequence defined by the function $a_{n+1} = a_n + 12$ with $a_1 = 4$. Write an explicit function rule.
	Note: Students may interpret 4 as the y-intercept since it is the first value; however, attending to the notation when $x = 1, y = 4$. Thus, the y-intercept for the explicit form is -8.
	
	Indicator: Given the sequence defined by the function $a_{n+1} = \frac{3}{4} a_n$ with $a_1 = 424$. Write an explicit function rule.

Functions – Linear, Quadratic, and Exponential Models

NC.M1.F-LE.1
Construct and compare linear and exponential models and solve problems.
Identify situations that can be modeled with linear and exponential functions and justify the most appropriate model for a situation based on the rate of change over equal intervals.

Concepts and Skills
Pre-requisite
- Construct a function to model a linear relationship (8.F.4)   - Describe qualitatively the functional relationship between two quantities by analyzing a graph (8.F.5)   - Formally define a function (NC.M1.F-IF.1)   - Recognize sequences as function and link arithmetic sequences to linear functions and geometric sequences to exponential functions (NC.M1.F-IF.3)
Connections
- Build explicit and recursive forms of arithmetic and geometric sequences (NC.M1.F-BF.1a)   - Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)   - Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard. 3 – Construct a viable argument and critique the reasoning of others 4 – Model with mathematics 7 – Look for and make use of structure
Disciplinary Literacy
New Vocabulary: exponential function

Mastering the Standard	
Clarification	**Checking for Understanding**

Students should differentiate whether a situation (contextual, graphical, or numerical) can be represented best by a linear or exponential model.
 	
Students should be able to identify whether a situation is linear or exponential based on the context in relation to the rate of change.
 	
This standard can be taught with NC.MI.F-IF.3 and NC.MI.F-BF.2.
	Students should be able to identify whether a situation is linear or exponential based on the context of the scenario and justify their decision.
	Indicator: Town A adds 10 people per year to its population, and town B grows by 10% each year. In 2006, each town has 145 residents. For each town, determine whether the population growth is linear or exponential. Explain.
	
	Indicator: In (a)–(e), say whether the quantity is changing in a linear or exponential fashion.
	    a. A savings account, which earns no interest, receives a deposit of $723 per month.
	    b. The value of a machine depreciates by 17% per year.
	    c. Every week, 9/10 of a radioactive substance remains from the beginning of the week.
	    d. A liter of water evaporates from a swimming pool every day.
	    e. Every 124 minutes, ½ of a drug dosage remains in the body.
	                    (https://www.illustrativemathematics.org/content-standards/HSF/LE/A/1/tasks/629)

Indicator: Monica did an experiment to compare two methods of warming an object. The results are shown in the table

Time (hours)	Temperature (°F) Method 1	Temperature (°F) Method 2
0	0	1.5
1	5	3
2	11	6
3	15	12
4	19	24
5	25	48

below.
Which statement best describes her results?
  a. The temperature using both methods changed at a constant rate.
  b. The temperature using both methods changed exponentially.
  c. The temperature using Method 2 changed at a constant rate.
  d. The temperature using Method 2 changed exponentially.

Indicator: For each of the scenarios below, decide whether the situation can be modeled by a linear function, an exponential function, or neither. For those with a linear or exponential model, create a function which accurately describes the situation.
  a. From 1910 until 2010 the growth rate of the United States has been steady at about 1.5% per year. The population in 1910 was about 92,000,000.
  b. The circumference of a circle as a function of the radius.
  c. According to an old legend, an Indian King played a game of chess with a traveling sage on a beautiful, hand-made chessboard. The sage requested, as reward for winning the game, one grain of rice for the first square, two grains for the second, four grains for the third, and so on for the whole chess board. How many grains of rice would the sage win for the nth square?
  d. The volume of a cube as a function of its side length.
                    (https://www.illustrativemathematics.org/content-standards/HSF/LE/A/1/tasks/1910)

Instructional Resources	
Tasks	**Additional Resources**
Basketball Rebound (Illustrative Mathematics) Linear or Exponential? (Illustrative Mathematics) Finding Linear and Exponential Models (Illustrative Mathematics)	Penny Circle (DESMOS)

Functions – Linear, Quadratic, and Exponential Models

NC.M1.F-LE.3
Construct and compare linear and exponential models and solve problems.
Compare the end behavior of linear, exponential, and quadratic functions using graphs and tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.

Concepts and Skills
Pre-requisite
- Construct a function to model a linear relationship and interpret rate of change (8.F.4)   - Formally define a function (NC.M1.F-IF.1)   - Evaluate functions (NC.M1.F-IF.2)
Connections
- Calculate the average rate of change of an interval (NC.M1.F-IF.6)   - Identify and interpret key features, like rate of change, of functions from different representations (NC.M1.F-IF.7)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard. 4 – Model with mathematics
Disciplinary Literacy
New Vocabulary: exponential function, quadratic function

Mastering the Standard

Clarification	Checking for Understanding
Students experiment with the function types to build an understanding that the average rate of change over an interval for an exponential function will eventually surpass the rate of change of a linear or quadratic function over the same interval. Students should be able to demonstrate this using various representations. It is important for students to begin developing an understanding of end behavior and interpreting mathematical notation (such as $x \to \infty$). As students study end behavior of these function families, connect their mathematical thinking from "as we keep going out" or "as x gets really big" to "as x goes to infinity".	Students should realize that an exponential function is eventually always bigger than a linear or quadratic function. **Indicator:** Kevin and Joseph each decide to invest $100. Kevin decides to invest in an account that will earn $5 every month. Joseph decided to invest in an account that will earn 3% interest every month. a. Whose account will have more money in it after two years? b. After how many months will the accounts have the same amount of money in them? c. Describe what happens as the money is left in the accounts for longer periods of time. **Indicator:** Using technology, determine the average rate of change of the following functions for intervals of their domains in the table.

Functions	Average rate of change $0 \leq x \leq 10$	Average rate of change $10 \leq x \leq 20$	Average rate of change $20 \leq x \leq 30$	Average rate of change $30 \leq x \leq 40$	Average rate of change $40 \leq x \leq 50$
$f(x) = x^2$					
$f(x) = 1.17^x$					

a. When does the average rate of change of the exponential function exceed the average rate of change of the quadratic function?

b. Using a graphing technology, graph both functions. How do the average rates of change in your table relate to what you see on the graph?

Note: You can use the information in your table to determine how to change the setting to see where the functions intersect.

c. In your graphing technology, change the first function to $f(x) = 10x^2$ and adjust the settings to see where the functions intersect. What do you notice about the rates of change interpreted from the graph?

d. Make a hypothesis about the rates of change about polynomial and exponential function. Try other values for the coefficient of the quadratic function to support your hypothesis.

Functions – Linear, Quadratic, and Exponential Models

NC.M1.F-LE.5
Interpret expressions for functions in terms of the situation they model.
Interpret the parameters a and b in a linear function $f(x) = ax + b$ or an exponential function $g(x) = ab^x$ in terms of a context.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite	**Connections**
- Construct a function to model a linear relationship and interpret rate of change and initial value (8.F.4) - Compare the coefficients and constants of linear equations in similar form (8.EEb) - Identify and interpret parts of expression (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)	*The following SMPs can be highlighted for this standard.* 4 – Model with mathematics
Connections	**Disciplinary Literacy**
Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)	*New Vocabulary: exponential function, growth factor, decay rate, growth rate*

Mastering the Standard

Clarification	Checking for Understanding
Students should know the meaning of the parameters in both linear and exponential functions in the context of the situation. Use real-world situations to help students understand how the parameters of linear and exponential functions depend on the context. In a linear function $y = ax + b$ the value of "a" represents the slope (constant rate of change) while "b" represents the y intercept (initial value). In an exponential function $y = a(b)^x$ the value of "a" represents the y intercept (initial value) and "b" represents the growth or decay factor. When $b > 1$ the function models growth. When $0 < b < 1$ the function model decays. Be cautious when interpreting the growth or decay rate. If the factor is 0.85 this means that it is decreasing by 15%. If the factor is 1.05, this means that is increasing by 5%.	Students should be able to describe the effects of changes to the parameters of a linear and exponential functions. **Indicator:** A plumber who charges $50 for a house call and $85 per hour can be expressed as the function $y = 85x + 50$. If the rate were raised to $90 per hour, how would the function change? **Indicator:** The equation $y = 8,000(1.04)^x$ models the rising population of a city with 8,000 residents when the annual growth rate is 4%. a. What would be the effect on the equation if the city's population were 12,000 instead of 8,000? b. What would happen to the population over 25 years if the growth rate were 6% instead of 4%? Students should be able to interpret the parameters of a linear and exponential function. **Indicator:** A function of the form $f(n) = P(1 + r)^n$ is used to model the amount of money in a savings account that earns 8% interest, compounded annually, where n is the number of years since the initial deposit. a. What is the value of r? Interpret what r means in terms of the savings account? b. What is the meaning of the constant P in terms of the savings account? Explain your reasoning. c. Will n or $f(n)$ ever take on the value 0? Why or why not? **Indicator:** Lauren keeps records of the distances she travels in a taxi and what it costs: a. If you graph the ordered pairs (d, f) from the table, they lie on a line. How can this be determined without graphing them? b. Show that the linear function in part a. has equation $f = 2.25d + 1.5$. c. What do the 2.25 and the 1.5 in the equation represent?

Distance d in miles	Fare f in dollars
3	8.25
5	12.75
11	26.25

Instructional Resources

Tasks	Additional Resources
	[Representing Linear and Exponential Growth](#) (Mathematics Assessment Project)

Geometry

NC Math 1	NC Math 2	NC Math 3
Analytic & Euclidean		
Focus on coordinate geometry • Distance on the coordinate plane • Midpoint of line segments • Slopes of parallel and perpendicular lines • Prove geometric theorems algebraically	**Focus on triangles** • Congruence • Similarity • Right triangle trigonometry ○ Special right triangles	**Focus on circles and continuing the work with triangles** • Introduce the concept of radian • Angles and segments in circles • Centers of triangles • Parallelograms
A Progression of Learning		
Integration of Algebra and Geometry • Building off what students know from $5^{th} – 8^{th}$ grade with work in the coordinate plane, the Pythagorean theorem and functions. • Students will integrate the work of algebra and functions to prove geometric theorems algebraically. • Algebraic reasoning as a means of proof will help students to build a foundation to prepare them for further work with geometric proofs.	**Geometric proof and SMP3** • An extension of transformational geometry concepts, lines, angles, and triangles from 7th and 8th grade mathematics. • Connecting proportional reasoning from 7th grade to work with right triangle trigonometry. • Students should use geometric reasoning to prove theorems related to lines, angles, and triangles. *It is important to note that proofs here are not limited to the traditional two-column proof. Paragraph, flow proofs and other forms of argumentation should be encouraged.*	**Geometric Modeling** • Connecting analytic geometry, algebra, functions, and geometric measurement to modeling. • Building from the study of triangles in Math 2, students will verify the properties of the centers of triangles and parallelograms.

Expressing Geometric Properties with Equations

OCS Priority Standard: NC.M1.G-GPE.4

Use coordinates to prove simple geometric theorems algebraically.

Use coordinates to solve geometric problems involving polygons algebraically

- Use coordinates to compute perimeters of polygons and areas of triangles and rectangles.
- Use coordinates to verify algebraically that a given set of points produces a particular type of triangle or quadrilateral.

Concepts and Skills
Pre-requisite
• Finding the distance between points in the coordinate plane (8.G.8)   • Calculating rate of change from two points (8.F.4)   • Using slope to determine parallelism and perpendicularity (NC.M1.G-GPE.5)   • Finding midpoint/endpoint of a line segment, given either (NC.M1.G-GPE.6)
Connections
• Experiment with transformations in the plane (NC.M2.G-CO.2, NC.M2.G-CO.3, NC.M2.G-CO.3, NC.M2.G-CO.4)   • Geometric transformations as functions (NC.M2.F-IF.1)

The Standards for Mathematical Practices
Connections

The following SMPs can be highlighted for this standard.
 3 – Construct viable arguments and critique the reasoning of others.
    • Students must use algebraic reasoning as they solve geometric problems.
 8 – Look for and express regularity in repeated reasoning
    • The <u>distance formula</u> is a generalization where students notice general methods and/or shortcuts for performing mathematical calculations.

Disciplinary Literacy
Students should be able to justify their claim that a set of points forms a particular shape using mathematical reasoning.

Mastering the Standard	
Clarification	**Checking for Understanding**

In upper elementary and middle grades, students calculated the area of triangles and special quadrilaterals using all four quadrants of the coordinate plane. Students also applied geometric measurement to real-world and mathematical problems and made use of properties of two-dimensional figures in order to calculate or estimate their lengths and areas.
 	
 This standard emphasizes the use of coordinates to solve geometric problems algebraically and continues with geometric measurement. Students will begin to demonstrate and analyze properties of geometric shapes using equations and graphs. This includes:
 • Using previously learned formulas to find the perimeter of polygons and the area of triangles and rectangles.
	Given coordinates of a polygon in the coordinate plane, students should be able to compute the lengths of segments and side lengths of polygons by finding the distance between points in the coordinate plane to calculate the perimeter of polygons and calculate the area of triangles and rectangles.
	    Indicator: Find the perimeter and area of a polygon with vertices at C $(-1,1)$, $D(3,4)$, $E(6,0)$, F $(2,-3)$ and G $(-4,-4)$. Round your answer to the nearest hundredth.
	
	 Given coordinates of a polygon in the coordinate plane, students should be able to verify the properties of any triangle or quadrilateral using the slopes of lines and lengths of segments that comprise the figure.
	    Indicator: Given $\triangle ABC$ with altitude$\overline{CD}$, given A $(-4,-2)$, $B(8,7)$, $C(1,8)$ and $D(4,4)$.
	    a. Calculate the area of $\triangle ABC$.
	    b. The altitude of a triangle is defined as is a line that extends from one vertex of a **triangle** perpendicular to the opposite side. Verify that $\overline{CD}$ is an altitude of $\triangle ABC$.
	
	    Indicator: The coordinates for the vertices of quadrilateral $MNPQ$ are M $(3,0)$, N $(1,3)$, P $(-2,1)$, and Q $(0,-2)$.
	    a. Classify quadrilateral $MNPQ$.
	    b. Identify the properties used to determine your classification.

- Applying the slope to determine right angles in triangles and rectangles (perpendicular lines), to verify parallel sides in geometric figures; and to determine intersecting lines.

- Finding the perimeter of figures by computing the distance between points on the coordinate plane.

The distance formula ($d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$) is an appropriate generalization and should be ***developed*** through SMP 8 where students notice general methods and/or shortcuts for performing mathematical calculations. This is based on what students know about finding the length of line segments in the coordinate plane (Pythagorean Theorem) from MS mathematics.

Given the properties of a rectangle or triangle, students can determine the missing coordinate(s).

Indicator*:* If quadrilateral $ABCD$ is a rectangle, where A $(1,2)$, B $(6,0)$, C $(10,10)$ and D (x, y) is unknown.

    a.   Find the coordinates of the fourth vertex Point D.

    b.   Verify that $ABCD$ is a rectangle providing evidence related to the sides and angles.

Instructional Resources	
Tasks	**Additional Resources**
Squares on a coordinate grid (Illustrative Mathematics) Is this a rectangle? (Illustrative Mathematics) Mathematics Diagnostic Testing Project Area Problem (Regents of University of CA)	Classifying Equations of Parallel and Perpendicular Lines (Mathematics Assessment Project)

Expressing Geometric Properties with Equations

OCS Priority Standard: NC.M1.G-GPE.5

Use coordinates to prove simple geometric theorems algebraically.

Use coordinates to prove the slope criteria for parallel and perpendicular lines and use them to solve problems.

- Determine if two lines are parallel, perpendicular, or neither.
- Find the equation of a line parallel or perpendicular to a given line that passes through a given point.

Concepts and Skills
Pre-requisite
• Calculating rate of change given two points, a table or a graph (8.F.4)   • Derive the equation for a line in the coordinate plane (8.EE.6)
Connections
• Calculating and interpreting rate of change for a function (NC.M1.F-IF.6)   • Using coordinates to solve geometric problems algebraically (NC.M1.G-GPE.4)   • Analyze functions using different representations (NC.M1.F-IF.7, NC.M1.F-IF.9)   • Using concepts of points lines and planes to develop definitions of rigid motions in the plane (NC.M2.G-CO.2, NC.M2.G-CO.3, NC.M2.G-CO.4)   • Prove theorems about lines (NC.M2.G-CO.9)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard.   3 – Construct viable arguments and critique the reasoning of others.   8 – Look for and express regularity in repeated reasoning.   • The <u>slope formula</u> is a generalization where students notice general methods and/or shortcuts for performing mathematical calculations.
Disciplinary Literacy
Compare and contrast the equations of parallel and perpendicular lines. What similarities/differences must be present for parallel lines? Perpendicular lines? Intersecting lines?

| Mastering the Standard	
Clarification	**Checking for Understanding**
Students in 8th grade determine the slope and write the equation of non-vertical lines given two points, a table or graph. This standard is an extension and an application of this work as it asks students to compare two or more lines based on the characteristics of the lines presented. • Parallelism – same slope $m_1 = m_2$, where $m = \frac{\Delta y}{\Delta x}$ • Perpendicularity – slopes are opposite reciprocals OR slopes have a product of (-1). $m_1 \cdot m_2 = -1$, where $m = \frac{\Delta y}{\Delta x}$	Given coordinates, students can compare the characteristics, slopes and intercepts, of two or more lines. Student should be able to determine if two lines are parallel, perpendicular, or intersecting based on the slopes of the two lines. **Indicator:** Investigate the slopes of each of the sides of the rectangle ABCD (pictured on the right). What do you notice about the slopes of the sides that meet at a right angle? What do you notice about the slopes of the opposite sides that are parallel? Can you generalize what happens when you multiply the slopes of perpendicular lines? Students should be able to find the slope and/or endpoint(s) of a line given the graph or coordinates of a line parallel or perpendicular to the given line.
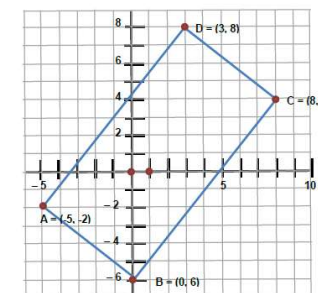
- Intersecting – have different rates of change. It is useful to note that perpendicular lines are a subset of intersecting lines on the coordinate plane.

$m_1 \neq m_2$, where $m = \frac{\Delta y}{\Delta x}$

The <u>slope formula</u> ($m = \frac{y_2 - y_1}{x_2 - x_1}$) is an appropriate generalization and should be **developed** through SMP 8 where students notice general methods and/or shortcuts for performing mathematical calculations. This is based on what students know about rate of change (slope) from MS mathematics.

Likewise, the properties for parallel and perpendicular lines should be **developed conceptually**.

Indicator: Suppose a line k in a coordinate plane has slope $\frac{c}{d}$.

a. What is the slope of a line parallel to k? Why must this be the case?
b. What is the slope of a line perpendicular to k? Why does this seem reasonable?

Students should be able to write the equation of line parallel or perpendicular to a given line.

Indicator: Two points $A(0, -4)$, $B(2, -1)$ determines a line, $\overleftrightarrow{AB}$.

a. **FC:** What is the equation of the line AB?
b. What is the equation of the line perpendicular to $\overleftrightarrow{AB}$ passing through the point $(2, -1)$?

Instructional Resources	
Tasks	**Additional Resources**
Midpoint Miracle (Illustrative Mathematics)	Classifying Equations of Parallel and Perpendicular Lines (MAP FAL)
Slope Criterion for Parallel and Perpendicular Lines (Illustrative Mathematics)	Graphing resource: http://www.geogebra.org/graphing

Expressing Geometric Properties with Equations

OCS Priority Standard: NC.M1.G-GPE.6

Use coordinates to prove simple geometric theorems algebraically.

Use coordinates to find the midpoint or endpoint of a line segment.

Concepts and Skills
Pre-requisite
• Finding the distance between points in the coordinate plane (8.G.8)   • (7.RP.2d)
Connections
• Use coordinates to solve geometric problems involving polygons (NC.M1.G-GPE.4)   • Prove theorems about lines (NC.M2.G-CO.9)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard.   3 – Construct viable arguments and critique the reasoning of others.   8 – Look for and express regularity in repeated reasoning.   • The <u>midpoint formula</u> is a generalization where students notice general methods and/or shortcuts for performing mathematical calculations.
Vocabulary

Mastering the Standard

Clarification	Checking for Understanding

The midpoint partitions the ratio of two distinct points on the same line segment into 1:1; thus, from either direction the point is the same.
 	
 The midpoint is always halfway between the two endpoints. The x-coordinate of the midpoint will be the mean of the x-coordinates of the endpoints and the y-coordinate will be the mean of the y-coordinates of the endpoints as indicated through the use of the midpoint formula.
 This should be derived from what students understand about distance.
 	
 The <u>midpoint formula</u> $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ is an appropriate generalization and should be ***developed*** through SMP 8 where students notice general methods and/or shortcuts for performing mathematical calculations.
	Given two points on a line, students can find the point that divides the segment into an equal number of parts.
	
	     Indicator*:* Jennifer and Jane are best friends. They placed a map of their town on a coordinate grid and found the point at which each of their house lies. If Jennifer's house lies at (9, 7) and Jane's house is at (15, 9) and they wanted to meet in the middle, what are the coordinates of the place they should meet?
	
	 Given the midpoint and an endpoint, students can use what they know about the midpoint to locate the other endpoint.
	
	     Indicator*:* If you are given the midpoint of a segment and one endpoint. Find the other endpoint.
	         a. midpoint: $(6, 2)$ endpoint: $(1, 3)$
	         b. midpoint: $(-1, -2)$ endpoint: $(3.5, -7)$

Instructional Resources

Tasks	Additional Resources
[Midpoint Miracle](#) (Illustrative Mathematics)	

Statistics & Probability

A statistical process is a problem-solving process consisting of four steps:
1. Formulating a statistical question that anticipates variability and can be answered by data.
2. Designing and implementing a plan that collects appropriate data.
3. Analyzing the data by graphical and/or numerical methods.
4. Interpreting the analysis in the context of the original question.

NC Math 1	NC Math 2	NC Math 3
Focus on analysis of univariate and bivariate data • Use of technology to represent, analyze and interpret data • Shape, center and spread of univariate numerical data • Scatter plots of bivariate data • Linear and exponential regression • Interpreting linear models in context.	**Focus on probability** • Categorical data and two-way tables • Understanding and application of the Addition and Multiplication Rules of Probability • Conditional Probabilities • Independent Events • Experimental vs. theoretical probability	**Focus on the use of sample data to represent a population** • Random sampling • Simulation as it relates to sampling and randomization • Sample statistics • Introduction to inference

A Progression of Learning		
• A continuation of the work from middle grades mathematics on summarizing and describing quantitative data distributions of univariate (6th grade) and bivariate (8th grade) data.	• A continuation of the work from 7th grade where students are introduced to the concept of probability models, chance processes and sample space; and 8th grade where students create and interpret relative frequency tables. • The work of MS probability is extended to develop understanding of conditional probability, independence and rules of probability to determine probabilities of compound events.	• Bringing it all back together • Sampling and variability • Collecting unbiased samples • Decision making based on analysis of data

Interpreting Categorical and Quantitative Data

NC.M1.S-ID.1
Summarize, represent, and interpret data on a single count or measurement variable.
Use technology to represent data with plots on the real number line (histograms and box plots).

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite	**Connections**
• Displaying numerical data on line plots, dot plots, histograms and dot plots (6.SP.4)	*The following SMPs can be highlighted for this standard.* 4 – Model with mathematics
Connections	**Vocabulary**
• Comparing two or more data distributions using shape and summary statistics (NC.M1.S-ID.2) • Examining the effects of outliers on the shape, center, and/or spread (NC.M1.S-ID.3)	*New Vocabulary: outlier, standard deviation*

Mastering the Standard

Clarification

This standard is an extension of 6th grade where students display numerical data using dot plots, histograms and box plots.
The standard involves representing data from contextual situations with histograms and box plots ***using technology***. Students should now be able to see that dot plots (line plots) are no longer appropriate for larger data sets. They should see that technology can quickly perform calculations and create graphs so that more emphasis can be placed on interpretation of the data. Students should be able to extract information about the data set from each type of graph (6.SP.4).

- **Histograms** represent the counts and order of the data set and easily display the *shape* of the data.
- *Modified* **Box plots** show a picture of the data set and the *range* of the data values broken up in quartiles. It doesn't show every value as does the histogram, but it is a good visual image of *spread*. The summary statistics needed to build a modified box plot are the 5-Number summary, which includes the minimum value (minX), maximum value (maxX), median (Med), lower quartile (Q_1), upper quartile (Q_3) and outlier(s) (NC.M1.S-ID.3).

Checking for Understanding

Students can use appropriate technology to calculate summary statistics and graph a given set of data. Appropriate technology includes graphing calculators, software, or online applications (e.g. http://technology.cpm.org/general/stats/).

Indicator: The table shows the length of a class period for each of the schools listed in a NC school district. Choose and create an appropriate plot to represent the data. Explain your choice of plot.

School	Class period (minutes)	School	Class period (minutes)
Lincoln Middle	45	New Hope Middle	55
Central Middle	65	Sunnyside Middle	50
Oak Grove Middle	70	Pine Grove Middle	60
Fairview Middle	55	Green Middle	65
Jefferson Middle	60	Hope Middle	55
Roosevelt Middle	60		

Indicator: The following data set shows the number of songs downloaded in one week by each student in Mrs. Jones class: 10, 20, 12, 14, 12, 27, 88, 2, 7, 30, 16, 16, 32, 25, 15, 4, 0, 15, 6, 1, 0, 15, 12, 10, and 7.
   a) Construct a histogram and boxplot of the data.
   b) What are the advantages to each data display? Explain.

Instructional Resources

Tasks	Additional Resources
Speed Trap (Illustrative Mathematics) S-ID Haircut Costs (Illustrative Mathematics)	Statistics on Basketball Team (Smarter Balanced CAT Sample Questions) Interactive Box Plot Activity (Shodor) Representing Data with Boxplots (Mathematics Assessment Project) Representing Data with Frequency Graphs (Mathematics Assessment Project)

Interpreting Categorical and Quantitative Data

NC.M1.S-ID.2

Summarize, represent, and interpret data on a single count or measurement variable.

Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Interpret differences in shape, center, and spread in the context of the data sets.

Concepts and Skills
Pre-requisite
- Relating the choice of center and variability to shape of data (6.SP.5d)   - Informally compare graphical displays of two distributions to make inferences about two populations (7.SP.3)   - Informally compare numerical summaries of two distributions to make inferences about two populations (7.SP.4)   - Use technology to represent data (NC.M1.S-ID.1)
Connections
Effects of outliers on shape, center, and/or spread (NC.M1.S-ID.3)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard. 4 – Model with mathematics 5 – Use appropriate tools strategically 6 – Attend to precision
Vocabulary
New Vocabulary: standard deviation, outlier

Mastering the Standard

Clarification	Checking for Understanding

In middle school, students related the measure of center and variability to the shape and context of the data. Students know that symmetrical displays are more appropriate for the mean as a measure of center and mean absolute deviation (M.A.D) as a measure of variability. Likewise, they understand that skewed distributions or distributions with outliers are better described using median as a measure of center due to the fact that it is a *resistant* measure of center; and the interquartile range (IQR) as a measure of variability.
 	
The standard deviation is a new summary statistic for students. The development of the sample standard deviation is based on the M.A.D (Mean Absolute Deviation) learned in the middle grades. Essentially, students need to understand that SD like M.A.D is a measure of variability in the data. The larger SD, the more variable the data. Students should also know that standard deviation allows comparison of variability in multiple data sets regardless of the unit of measurement for the data sets.
 	
An understanding of how the standard deviation is calculated can help students to conceptualize the value and why it's primarily used in association with mean as a measure of center. Using a relatively smaller data set and the list feature in the
	Given two or more sets of data, students compare datasets and identify similarities and differences in shape, center and spread within the context of the data.
	    Indicator: Ms. Williams wants to analyze the scores for the first unit test of her 1st period and 4th period NC Math 1 classes. The scores for each class are below.
	
	    1st Period: **4th Period:**
	    82, 100, 94, 68, 34, 72, 70, 96, 99, 92, 100, 95, 72, 80, 97, 78, 89, 100, 93, 95, 66,
	    90, 85, 70, 46, 71, 71, 77, 78, 95, 82, 87, 85, 98, 89, 86, 80, 79, 94, 90, 92, 87, 88
	    80, 100, 99, 72, 69, 74, 84, 87 81, 82
	
	a) Calculate the mean, median, standard deviation, and interquartile range for each class.
	    *Note: In this example, the entire class is being used, therefore the **population** mean (μ)and standard deviation (σ) should be used when comparing the data sets.*
	b) Construct an appropriate graph to compare the two classes.
	c) Write several sentences to compare the class grades in context.
	
	Given two or more graphs, students compare datasets and identify similarities and differences in shape, center and spread within the context of the data.

graphing calculator can make the calculations easier during development of the concept.

There are two formulas for SD for which students should become familiar:

$$\text{Sample (S): } S_x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - x)^2}$$

$$\text{Population (σ): } \sigma_x = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2}$$

Context is important for determining when each measure should be used. If the whole population is used, then students should use the population mean (μ) and SD (σ_x). For situations where samples are stated or implied, students should use the sample mean (x) and SD (S_x).

Indicator: Delia wanted to find the best type of fertilizer for her tomato plants. She purchased three types of fertilizer and used each on a set of seedlings. After 15 days, she measured the heights (in cm) of each set of seedlings. The data she collected and plotted are shown below. Write a brief description comparing the three types of fertilizer. Which fertilizer do you recommend that Delia use? Explain your answer.

Fertilizer A			Fertilizer B			Fertilizer C		
7.1	6.3	1.0	11.0	9.2	5.6	10.5	11.8	15.5
5.0	4.5	5.2	8.4	7.2	12.1	14.7	11.0	10.8
3.2	4.6	2.4	10.5	14.0	15.3	13.9	12.7	9.9
5.5	3.8	1.5	6.3	8.7	11.3	10.3	10.1	15.8
6.2	6.9	2.6	17.0	13.5	14.2	9.5	13.2	9.7
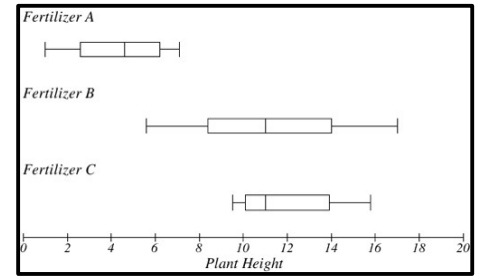
Instructional Resources	
Tasks	**Additional Resources**
Measuring Variability in a Data Set (Illustrative Mathematics) Understanding the Standard Deviation (Illustrative Mathematics) Speed Trap (Illustrative Mathematics) Haircut Costs (Illustrative Mathematics)	Airline Arrival Times (Smarter Balanced CAT Sample Questions) Boxplot Grapher: http://www.imathas.com/stattools/boxplot.html

Interpreting Categorical and Quantitative Data

OCS Priority Standard: NC.M1.S-ID.3
Summarize, represent, and interpret data on a single count or measurement variable.

Examine the effects of extreme data points (outliers) on shape, center, and/or spread.

Concepts and Skills
Pre-requisite
• Describing striking deviations from the overall pattern of a distribution (6.SP.5c) • Use technology to create boxplots and histograms (NC.M1.S-ID.1)
Connections
• Comparing two or more data distributions using shape and summary statistics (NC.M1.S-ID.2)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard. 3 – Construct a viable argument and critique the reasoning of others 4 – Model with mathematics 5 – Use appropriate tools strategically 6 – Attend to precision
Vocabulary
New Vocabulary: outlier, standard deviation

Mastering the Standard

Clarification	Checking for Understanding
An important part of data analysis includes examining data for values that represent abnormalities in the data. In MS, students *informally* addressed "striking deviations from the overall pattern" of a data distribution. The identification of <u>outliers</u> is formalized in this standard. A value is mathematically determined to be an outlier if the value falls 1.5 IQRs below the 1st quartile or above the third quartile in a data set. • Lower outlier(s) $< 1.5 \cdot IQR$ • Upper outlier(s) $> 1.5 \cdot IQR$ The mean and standard deviation are most commonly used to describe sets of data. However, if the distribution is extremely skewed and/or has outliers, it is best to use the median and the interquartile range to describe the distribution since these measures are not sensitive to outliers. It is important to detect outliers within a distribution because they can alter the results of the data analysis. The mean is more sensitive to the existence of outliers than other measures of center.	Students understand and use the context of the data to explain why its distribution takes on a particular shape (e.g., Why is the data skewed? Are there outliers?) **Indicator:** Why does the shape of the distribution of incomes for professional athletes tend to be skewed to the right? Why does the shape of the distribution of test scores on a really easy test tend to be skewed to the left? Why does the shape of the distribution of heights of the students at your school tend to be symmetrical? Students should identify outliers of the data set and determine the effect outliers will have on the shape, center, and spread of a data set. **Indicator:** The heights of players on the Washington High School's Girls basketball team are recorded below: <table><tr><td>5' 10"</td><td>5' 4"</td><td>5' 7"</td><td>5' 6"</td><td>5' 5"</td><td>5' 3"</td><td>5' 7"</td><td>5' 7"</td><td>5' 8"</td></tr></table> A student transfers to Washington High and joins the basketball team. Her height is 6' 2" a) What affect does her height have on the team's height distribution and stats (center and spread)? b) Which measure of center most accurately describes the team's average height? Explain.

Indicator: The table shows the length of a class period for each of the schools listed. If Cherry Lane Middle School's class period length of 100 minutes is added to the data above, what effect will it have on the mean, median, interquartile range, standard deviation and on the graph of the data?

School	Length of class period (minutes)
Lincoln Middle	45
Central Middle	65
Oak Grove Middle	70
Fairview Middle	55
Jefferson Middle	60
Roosevelt Middle	60
New Hope Middle	55
Sunnyside Middle	50
Pine Grove Middle	60
Green Middle	65
Hope Middle	55

Instructional Resources	
Tasks	**Additional Resources**
Identifying Outliers (Illustrative Mathematics) Describing Data Sets with Outliers (Illustrative Mathematics) Student Heights (PISA Sample) Test Scores (PISA Sample)	

Interpreting Categorical and Quantitative Data

OCS Priority Standard: NC.M1.S-ID.6a
Summarize, represent, and interpret data on two categorical and quantitative variables.
Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
   a.   Fit a least squares regression line to linear data using technology. Use the fitted function to solve problems.

Concepts and Skills
Pre-requisite
- Construct and interpret scatterplots for two-variable data and describe patterns of association (8.SP.1)   - Informally fit a straight line assess the model fit judging the closeness of the data to line (8.SP.2)   - Analyze patterns and describe relationships between variables in context. (NC.M1.S-ID.8)
Connections
- Assess linearity by analyzing residuals (NC.M1.S-ID.6b)   - Fit a function to exponential data using technology and use the model to solve problems (NC.M1.S-ID.6c)   - Use technology to analyze patterns and describe relationships between two variables in context. (NC.M1.S-ID.7)   - Distinguish between association and causation (NC.M1.S-ID.9)   - Write a function that describes a relationship between two quantities (NC.M1.F-BF.1)   - Identify situations that can be modeled with linear and exponential functions and justify the appropriate model (NC.M1.F-LE.1)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard. 4 – Model with mathematics 5 – Use appropriate tools strategically 6 – Attend to precision
Vocabulary

Mastering the Standard

Clarification	Checking for Understanding
In 8th grade, students created scatter plots and described patterns of association between two quantities. They also informally fit a straight line to data based on how closely the data points resembled a line. That knowledge is extended to fitting a linear regression equation to a set of data using technology. Technology includes graphing calculators, computer software/programs and web-based applets and tools. The initial exploration with technology should include a discussion of domain and range and their relationship to the graphing window. Most technology tools include an automatic feature that graphs data within a window representative of the data, however understanding of the graphing window can lead to further discussions about domain, range, interpolation, and extrapolation.	Students can represent data on a scatter plot using an appropriate scale and describe the relationship between two quantitative variables. **Indicator:** Determine if and what the relationship is between the population of each high school and the number of active band members. **Indicator:** The data gives the number of miles driven and advertised price for 11 used models of a particular car.

HS Population	# of active band members
1200	150
1450	155
900	100
1500	125
1400	125
1005	120

Miles (thousands)	Price($)
22	17,998
29	16,450
35	14,998
39	13,998
45	14,599
49	14,988
55	13,599
56	14,599
69	11,998
70	14,450
86	10,998

a) FC: Use a calculator or graphing technology to make a scatter plot of the data.

b) FC: Find the correlation coefficient for the data above. Describe what the correlation coefficient means about the data.

c) Fit a linear function to model the relationship of miles driven and the price of these cars.

d) How do you know that this is the best-fit model?

e) If a used car is driven 98,000 miles, what will the price be to the nearest dollar?

f) If the price of the car is $12,540, how many miles, to the nearest thousand, could have been driven?

Instructional Resources	
Tasks	**Additional Resources**
Olympic Men's 100-meter dash (Illustrative Mathematics) Laptop Battery Charge 2 (Illustrative Mathematics)	Lego Prices (DESMOS)

Interpreting Categorical and Quantitative Data

OCS Priority Standard: NC.M1.S-ID.6b
Summarize, represent, and interpret data on two categorical and quantitative variables.
Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
    b.   Assess the fit of a linear function by analyzing residuals.

Concepts and Skills
Pre-requisite
• Fit a regression line to linear data using technology (NC.M1.S-ID.6a)
Connections
• Use technology to analyze patterns and describe relationships between two variables in context. (NC.M1.S-ID.7)   • Analyze patterns and describe relationships between variables in context. (NC.M1.S-ID.8)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard.   3 – Construct a viable argument and critique the reasoning of others
Vocabulary
New Vocabulary: residual

Mastering the Standard

Clarification

A ***residual***, a measure of the error in prediction, is the difference between the actual y-value (y) and the predicted y-value ($\hat{y}$). Residuals are represented on the graph by the vertical distance between a data point and the graph of the function.
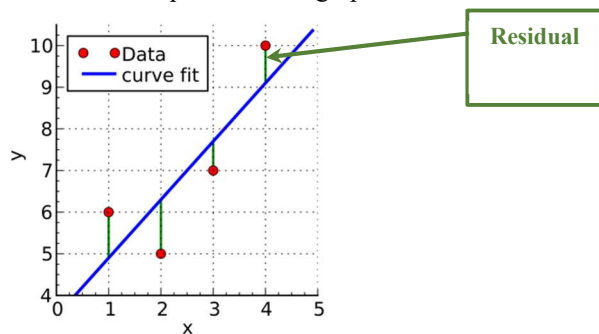
Checking for Understanding

Students can create a residual plot from a given set of data and interpret the appropriateness of a linear model for the data set.

Students can determine the residual for any value in a data set.

Indicator: The table to the right displays the annual tuition rates of a state college in the U.S. between 1990 and 2000, inclusively. The linear function $R(t) = 326x + 6440$ has been suggested as a good fit for the data.

  a) Extend the table to find the predicted rates based on the model and the residual values for each year.

  b) Create the residual plot for the tuition rates.

  c) Use the residual plot to determine the goodness of fit of the function for the data provided in the table.

Year (0 = 1990)	Tuition Rate	Predicted Rate	Residuals
0	6546		
1	6996		
2	6996		
3	7350		
4	7500		
5	7978		
6	8377		
7	8710		
8	9110		
9	9411		
10	9800		

Students can use a residual plot to determine the appropriateness of a linear model for a set of data.

A *residual plot* is a graph that shows the residuals on the vertical axis and the independent variable on the horizontal axis. If the points in a residual plot are randomly dispersed around the horizontal axis, a linear regression model is appropriate for the data; otherwise, a non-linear model is more appropriate.

(a)
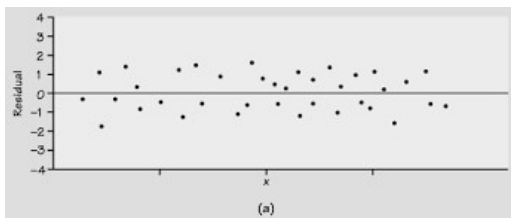
Indicator: What do the following residual plots tell you about the appropriateness of a linear model for the functions they represent? Explain your responses.
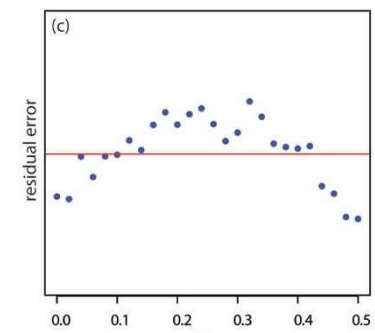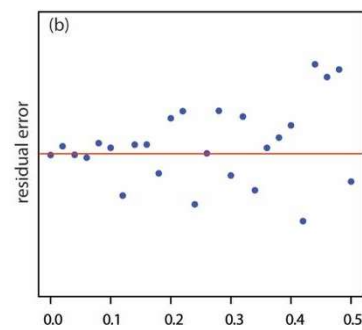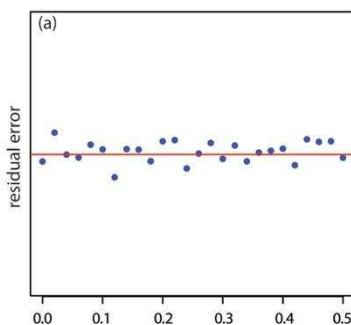
Instructional Resources	
Tasks	**Additional Resources**
Restaurant Bill and Party Size (Illustrative Mathematics)	

Interpreting Categorical and Quantitative Data

OCS Priority Standard: NC.M1.S-ID.6c
Summarize, represent, and interpret data on two categorical and quantitative variables.
Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
    c.   Fit a function to exponential data using technology. Use the fitted function to solve problems.

Concepts and Skills
Pre-requisite
● Fit a regression line to linear data using technology (NC.M1.S-ID.6a)
Connections
● Create and graph equations that represent exponential relationships (NC.M1.A-CED.1)
● Recognize a geometric sequence as a subset of the range of an exponential function (NC.M1.F-IF.3)
● Exponential growth and decay (NC.M1.F-IF.8b)
● Use technology to analyze patterns and describe relationships between two variables in context. (NC.M1.S-ID.7)
● Identify situations that can be modeled with linear and exponential functions, and justify the most appropriate model (NC.M1.F-LE.1)
● Interpret the parameters in linear or exponential functions in terms of a context (NC.M1.F-LE.5)
● Interpret key features in context to describe functions relating two quantities (NC.M1.F-IF.4)
● Interpret a function in terms of its domain and range in context (NC.M1.F-IF.5)
● Calculate and interpret the avg. rate of change for a function (NC.M1.F-IF.6)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard.
4 – Model with mathematics
5 – Use appropriate tools strategically
6 – Attend to precision
Vocabulary

Mastering the Standard

Clarification	Checking for Understanding
Work with exponential functions is new to students. In 8th grade, students focused on identifying characteristics of linear functions and distinguishing them from non-linear functions. Students will use the same tools to explore exponential functions specifically. This standard should be explored in context to help students make meaning of the behavior of exponential models. Technology can be used as a tool to make connections between symbolic, tabular and graphical representations of exponential functions. This will also help to build conceptual understanding of exponential growth and decay.	Students can use graphing technology or a graphing calculator to determine the exponential model for a given data set or scatter plot. **Indicator:** What is the exponential function that best models the number of gnats the scientists have gathered after the number of hours listed? How many hours will it take for 200 gnats to gather? Students can make connections between the graph, table, and symbolic representations of an exponential function.

Table within Checking for Understanding:

Hours	0	1	2	3	4
Number of gnats	12	20	35	60	80

At this level, students should be able to support the use of an exponential model based on the graphical display and the understanding of the constant ratio between consecutive terms; a concept supported by the study of geometric sequences.

Students should be presented with exponential data and asked to fit the function to the data using technology. They should **NOT** have to *verify* the appropriateness of an exponential model; analysis at that level requires transformations for linearity, which is an advanced statistical concept.

Indicator: In an experiment, 300 pennies were shaken in a cup and poured onto a table. Any penny 'heads up' was removed. The remaining pennies were returned to the cup and the process was repeated. The results of the experiment are shown below. Write a function rule suggested by the context. Use the context to explain all values of the function. How are those values reflected in the table?

# of Rolls	0	1	2	3	4	5
# of Pennies	300	164	100	46	20	8

Instructional Resources	
Tasks	**Additional Resources**
Income vs Literacy (Smarter Balanced CAT Sample Question) Stopping Distance vs. Speed (UCLA Curtis Center)	

Interpreting Categorical and Quantitative Data

NC.M1.S-ID.7
Interpret linear models.
Interpret in context the rate of change and the intercept of a linear model. Use the linear model to interpolate and extrapolate predicted values. Assess the validity of a predicted value.

Concepts and Skills
Pre-requisite
• Interpret the slope and y-intercept of a linear model in context (8.SP.3)
Connections
• Fit a regression line to linear data using technology (NC.M1.S-ID.6a)
• Interpret the parameters in linear or exponential functions in terms of a context (NC.M1.F-LE.5)
• Interpret key features in context to describe functions relating two quantities (NC.M1.F-IF.4)
• Calculate and interpret the avg. rate of change for a function (NC.M1.F-IF.6)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard.
3 – Construct a viable argument and critique the reasoning of others
4 – Model with mathematics
5 – Use appropriate tools strategically
6 – Attend to precision
Vocabulary

Mastering the Standard	
Clarification	**Checking for Understanding**

Students have interpreted the slope and y-intercept of a linear model in 8th grade. This standard expands upon this notion to using the model to make **predictions**.
 	
Interpolation is using the function to predict the value of the dependent variable for an independent variable that is in the midst of the data.
 	
Extrapolation is using the function to predict the value of the dependent variable for an independent variable that is outside the range of our data.
	Students can interpret the meaning of the rate of change and y-intercept in context.
	Students can interpolate and/or extrapolate predicted values using the linear model.
	    Indicator: Data was collected of the weight of a male white laboratory rat for the first 25 weeks after its birth. A scatterplot of the rat's weight (in grams) and the time since birth (in weeks) indicates a fairly strong, positive linear relationship. The linear regression equation $W = 100 + 40t$ (where W = weight in grams and t = number of weeks since birth) models the data fairly well.
	    a) Explain the meaning of the slope in context.
	    b) Explain the meaning of the y-intercept in context.
	    c) Based on the linear regression model, what will be the weight of the rat 10 weeks after birth?
	    d) Based on the linear regression model, at how many weeks will the rat be 760 grams?

Instructional Resources	
Tasks	**Additional Resources**
Texting and Grades II (Illustrative Mathematics) Used Subaru Foresters II (Illustrative Mathematics)	Charge! (DESMOS)

Interpreting Categorical and Quantitative Data

NC.M1.S-ID.8
Interpret linear models.
Analyze patterns and describe relationships between two variables in context. Using technology, determine the correlation coefficient of bivariate data and interpret it as a measure of the strength and direction of a linear relationship. Use a scatter plot, correlation coefficient, and a residual plot to determine the appropriateness of using a linear function to model a relationship between two variables.

Concepts and Skills
Pre-requisite
• Construct and interpret scatterplots for two-variable data and describe patterns of association (8.SP.1) • Fit a regression line to linear data using technology (NC.M1.S-ID.6a) • Assess linearity by analyzing residuals (NC.M1.S-ID.6b)
Connections
• Identify situations that can be modeled with linear and exponential functions, and justify the most appropriate model (NC.M1.F-LE.1)

The Standards for Mathematical Practices
Connections
The following SMPs can be highlighted for this standard. 3 – Construct viable arguments and critique the reasoning of others 4 – Model with mathematics 5 – Use appropriate tools strategically 6 – Attend to precision
Vocabulary
New Vocabulary: correlation, correlation coefficient

Mastering the Standard

Clarification

In working with bivariate data in MS, students have previously investigated patterns of association between two quantities (specifically, positive and negative associations and linear and non-linear associations).

The correlation coefficient, *r*, is a measure of the strength and direction of a linear relationship between two quantities in a set of data.

The magnitude (absolute value) of *r* indicates how closely the data points fit a linear pattern.

If $r = \pm 1$, all points fall exactly on a line. The sign of *r* indicates the direction of the relationship. The closer $|r|$ is to 1, the stronger the correlation and the closer $|r|$ is to zero, the weaker the correlation.

Checking for Understanding

Students can interpret the correlation coefficient.
  Indicator: The correlation coefficient of a given data set is 0.97. List three specific things this tells you about the data.
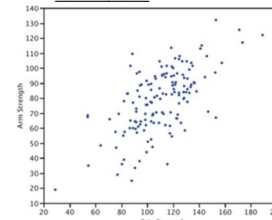
Students recognize the strength of the association of two quantities based on the scatter plot.
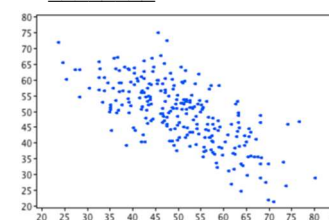 **Indicator:** Which correlation coefficient best matches each graph? Explain.
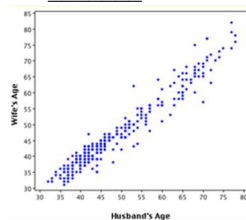
A. *r* = _____   B. *r* = _____   C. *r* = _____

$r = -.48$	$r = .98$	$r = .88$	$r = -.17$	$r = 1$	$r = .31$	$r = -1$

Students will be able to analyze patterns in context between two variables and use graphing technology to determine whether a linear model is appropriate for the data.

Indicator: The following data set indicates the average weekly temperature and the number of sno-cones sold by Sno-Show Sno-cones each week in May for the temperatures noted.

a) Determine the correlation coefficient.
b) Determine the strength and direction of the linear relationship.
c) Create a residual plot.
   Is a linear model appropriate for the data shown? Explain.

NOTE: Remind students to turn the Diagnostics on in the graphing calculator so that the correlation coefficient (r) appears when the regression equation is calculated.

Average weekly temperature	# of Sno-cones sold
68	500
74	600
74	700
80	800
82	1200

Instructional Resources	
Tasks	**Additional Resources**
Used Subaru Foresters I (Illustrative Mathematics)	

Interpreting Categorical and Quantitative Data

NC.M1.S-ID.9
Interpret linear models.
Distinguish between association and causation.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite	**Connections**
• Construct and interpret scatterplots for two-variable data and describe patterns of association (8.SP.1) • Fit a regression line to linear data using technology (NC.M1.S-ID.6a) • Assess linearity by analyzing residuals (NC.M1.S-ID.6b)	*The following SMPs can be highlighted for this standard.* 3 – Construct viable arguments and critique the reasoning of others
Connections	**Vocabulary**
• Fit a function to exponential data using technology (NC.M1.S-ID.6c)	*New Vocabulary: correlation, causation, association*

Mastering the Standard

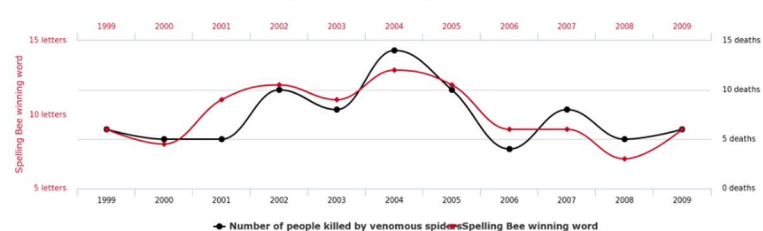| Clarification	Checking for Understanding
In working with bivariate data in MS, students have previously investigated patterns of association between two quantities (specifically, positive and negative associations and linear and non-linear associations). This standard addresses an often-made misconception regarding association, correlation and causation. Association indicates a relationship between two or more variables and correlation indicates the degree of association between two quantities. Causation, on the other hand, implies a cause-and-effect relationship when a strong relationship is observed. Determining causation goes beyond the idea of mere association or a high degree of correlation and requires the design and analysis of a randomized experimental process.	Students will recognize that association does not imply causation. **Indicator:** The following graph shows the correlation between *Letters in Winning Word of Scripps National Spelling Bee* and *Number of people killed by venomous spiders*. How does the graph support the phrase: <u>association does not imply causation</u>? For more examples, explore the site http://tylervigen.com/. Students will determine if statements of causation are reasonable or not and justify their opinion. **Indicator:** A study found a strong, positive correlation between the number of cars owned and the length of one's life. Larry concludes that owning more cars means you will live longer. Does this seem reasonable? Explain your answer. **Indicator:** Choose two variables that could be correlated because one is the cause of the other; defend and justify the selection of variables.

Instructional Resources

Tasks	Additional Resources
Coffee vs. Crime (Illustrative Mathematics) Golf and Divorce (Illustrative Mathematics)	

HESS COGNITIVE RIGOR MATRIX (MATH-SCIENCE CRM):
Applying Webb's Depth-of-Knowledge Levels to Bloom's Cognitive Process Dimensions

TOOL 2

Revised Bloom's Taxonomy	Webb's DOK Level 1 Recall & Reproduction	Webb's DOK Level 2 Skills & Concepts	Webb's DOK Level 3 Strategic Thinking/Reasoning	Webb's DOK Level 4 Extended Thinking
Remember Retrieve knowledge from long-term memory, recognize, recall, locate, identify	o Recall, observe, & recognize facts, principles, properties o Recall/ identify conversions among representations or numbers (e.g., customary and metric measures)	Use these Hess CRM curricular examples with most mathematics or science assignments or assessments.		
Understand Construct meaning, clarify, paraphrase, represent, translate, illustrate, give examples, classify, categorize, summarize, generalize, infer a logical conclusion), predict, compare/contrast, match like ideas, explain, construct models	o Evaluate an expression o Locate points on a grid or number on number line o Solve a one-step problem o Represent math relationships in words, pictures, or symbols o Read, write, compare decimals in scientific notation	o Specify and explain relationships (e.g., non-examples/examples; cause-effect) o Make and record observations o Explain steps followed o Summarize results or concepts o Make basic inferences or logical predictions from data/observations o Use models /diagrams to represent or explain mathematical concepts o Make and explain estimates	o Use concepts to solve non-routine problems o Explain, generalize, or connect ideas using supporting evidence o Make and justify conjectures o Explain thinking/reasoning when more than one solution or approach is possible o Explain phenomena in terms of concepts	o Relate mathematical or scientific concepts to other content areas, other domains, or other concepts o Develop generalizations of the results obtained and the strategies used (from investigation or readings) and apply them to new problem situations
Apply Carry out or use a procedure in a given situation; carry out (apply to a familiar task), or use (apply) to an unfamiliar task	o Follow simple procedures (recipe-type directions) o Calculate, measure, apply a rule (e.g., rounding) o Apply algorithm or formula (e.g., area, perimeter) o Solve linear equations o Make conversions among representations or numbers, or within and between customary and metric measures	o Select a procedure according to criteria and perform it o Solve routine problem applying multiple concepts or decision points o Retrieve information from a table, graph, or figure and use it solve a problem requiring multiple steps o Translate between tables, graphs, words, and symbolic notations (e.g., graph data from a table) o Construct models given criteria	o Design investigation for a specific purpose or research question o Conduct a designed investigation o Use concepts to solve non-routine problems o Use & show reasoning, planning, and evidence o Translate between problem & symbolic notation when not a direct translation	o Select or devise approach among many alternatives to solve a problem o Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze Break into constituent parts, determine how parts relate, differentiate between relevant-irrelevant, distinguish, focus, select, organize, outline, find coherence, deconstruct	o Retrieve information from a table or graph to answer a question o Identify whether specific information is contained in graphic representations (e.g., table, graph, T-chart, diagram) o Identify a pattern/trend	o Categorize, classify materials, data, figures based on characteristics o Organize or order data o Compare/ contrast figures or data o Select appropriate graph and organize & display data o Interpret data from a simple graph o Extend a pattern	o Compare information within or across data sets or texts o Analyze and draw conclusions from data, citing evidence o Generalize a pattern o Interpret data from complex graph o Analyze similarities/differences between procedures or solutions	o Analyze multiple sources of evidence o Analyze complex/abstract themes o Gather, analyze, and evaluate information
Evaluate Make judgments based on criteria, check, detect inconsistencies or fallacies, judge, critique	"UG"—unsubstantiated generalizations = stating an opinion without providing any support for it!		o Cite evidence and develop a logical argument for concepts or solutions o Describe, compare, and contrast solution methods o Verify reasonableness of results	o Gather, analyze, & evaluate information to draw conclusions o Apply understanding in a novel way, provide argument or justification for the application
Create Reorganize elements into new patterns/structures, generate, hypothesize, design, plan, produce	o Brainstorm ideas, concepts, or perspectives related to a topic	o Generate conjectures or hypotheses based on observations or prior knowledge and experience	o Synthesize information within one data set, source, or text o Formulate an original problem given a situation o Develop a scientific/mathematical model for a complex situation	o Synthesize information across multiple sources or texts o Design a mathematical model to inform and solve a practical or abstract situation