

**NCDPI Unpacked Content  
with  
OCS Priority Standards Identified**

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**Eighth Grade  
i-Ready Classroom Mathematics**

**2022 Alignment**



# Introduction

## Purpose:

- Ensure educators understand the expectations of the standards
- Facilitate discussion among teachers
- Encourage coherence in the sequence, pacing and units of study for grade-level curricula
- Used to understand and teach the NCSCOS

## Standards:

<p><b>OCS Priority Standards:</b> the most important standards within a domain that are deemed the highest priority or most important for students to learn based on:</p> <ul style="list-style-type: none"><li>➤ Endurance</li><li>➤ Leverage</li><li>➤ Readiness</li><li>➤ Assessment</li></ul>	<p><b>Supporting Standards:</b></p> <ul style="list-style-type: none"><li>➤ Taught in context of the priority standards but do not receive the same emphasis or degree of instruction</li><li>➤ Support, enhance or connect to the priority standards</li></ul>
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## Lesson 0 – First 5 days of Math Instruction:

- **Try-Discuss-Connect Routine** - engages students in mathematical practices and supports student-to-student discourse.
- **Language Routines** - *Three Reads* supports use of Tier III academic vocabulary
- **Teacher Moves** – instructional strategies that help facilitate discussions:
  - Turn and Talk
  - Individual Think Time
  - Four Rs: Repeat, Rephrase, Reword, Record

## Standards for Mathematical Practice:

- Teaching approach that guides effective instruction
- Develops a mathematical mindset
- Creates real-world problem-solvers
- Builds mathematical communication

# North Carolina 8<sup>th</sup> Grade Standards

The Number System	Expressions & Equations	Functions	Geometry	Statistics & Probability
<p><i>Know that there are numbers that are not rational and approximate them by rational numbers.</i></p> <p><a href="#"><u>NC.8.NS.1</u></a> <a href="#"><u>NC.8.NS.2</u></a></p>	<p><i>Work with radicals and integer exponents.</i></p> <p><a href="#"><u>NC.8.EE.1</u></a> <a href="#"><u>NC.8.EE.2</u></a> <a href="#"><u>NC.8.EE.3</u></a> <a href="#"><u>NC.8.EE.4</u></a></p> <p><i>Analyze and solve linear equations and inequalities.</i></p> <p><a href="#"><u>NC.8.EE.7</u></a></p> <p><i>Analyze and solve pairs of simultaneous linear equations.</i></p> <p><a href="#"><u>NC.8.EE.8</u></a></p>	<p><i>Define, evaluate, and compare functions.</i></p> <p><a href="#"><u>NC.8.F.1</u></a> <a href="#"><u>NC.8.F.2</u></a> <a href="#"><u>NC.8.F.3</u></a></p> <p><i>Use functions to model relationships between quantities.</i></p> <p><a href="#"><u>NC.8.F.4</u></a> <a href="#"><u>NC.8.F.5</u></a></p>	<p><i>Understand congruence and similarity using physical models, transparencies, or geometry software.</i> <a href="#"><u>NC.8.G.2</u></a></p> <p><a href="#"><u>NC.8.G.3</u></a> <a href="#"><u>NC.8.G.4</u></a></p> <p><i>Analyze angle relationships.</i></p> <p><a href="#"><u>NC.8.G.5</u></a></p> <p><i>Understand and apply the Pythagorean Theorem.</i></p> <p><a href="#"><u>NC.8.G.6</u></a> <a href="#"><u>NC.8.G.7</u></a> <a href="#"><u>NC.8.G.8</u></a></p> <p><i>Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.</i></p> <p><a href="#"><u>NC.8.G.9</u></a></p>	<p><i>Investigate patterns of association in bivariate data.</i></p> <p><a href="#"><u>NC.8.SP.1</u></a> <a href="#"><u>NC.8.SP.2</u></a> <a href="#"><u>NC.8.SP.3</u></a> <a href="#"><u>NC.8.SP.4</u></a></p>

**8<sup>th</sup> Grade OCS Priority Standards**  
**Standards for Mathematical Practice**

<b>Practice</b>	<b>Explanation and Example</b>
1. Make sense of problems and persevere in solving them.	In grade 8, students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”
2. Reason abstractly and quantitatively.	In grade 8, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree to which the pattern models a line. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3. Construct viable arguments and critique the reasoning of others.	In grade 8, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like “How did you get that?”, “Why is that true?” “Does that always work?” They explain their thinking to others and respond to others’ thinking.
4. Model with mathematics.	In grade 8, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students solve systems of linear equations and compare properties of functions provided in different forms. Students use scatterplots to represent data and describe associations between variables. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.

Practice	Explanation and Example
5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 8 may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal.
6. Attend to precision.	In grade 8, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays.
7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. In grade 8, students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.
8. Look for and express regularity in repeated reasoning.	In grade 8, students use repeated reasoning to understand algorithms and make generalizations about patterns. Students use iterative processes to determine more precise rational approximations for irrational numbers. They analyze patterns of repeating decimals to identify the corresponding fraction. Students flexibly make connections between covariance, rates, and representations showing the relationships between quantities.

### 8<sup>th</sup> Grade Instructional Blueprint

Unit	Concept	Duration	OCS Priority Standards	Supporting Standards	Document Pages
<b>Unit 1</b> <b>Lessons 0-3</b>	Geometric Figures: Rigid Transformations and Congruence	20 days		NC.8.G.2	8-12
				NC.8.G.4	
<b>Unit 2</b> <b>Lessons 4-7</b>	Geometric Figures: Transformations, Similarity, and Angle Relationships	18 days	NC.8.G.3	NC.8.G.2	13-22
			NC.8.G.5	NC.8.G.4	
<b>Unit 3</b> <b>Lessons 8-14</b>	Linear Relationships: Slope, Linear Equations, and Systems	32 days	NC.8.EE.7		23-29
			NC.8.EE.8		
			NC.8.F.4		
<b>Unit 4</b> <b>Lessons 15-18</b>	Functions: Linear and Non-Linear Relationships	19 days	NC.8.F.2	NC.8.F.1	30-37
			NC.8.F.4	NC.8.F.3	
				NC.8.F.5	
<b>Unit 5</b> <b>Lessons 19-22</b>	Integer Exponents: Properties and Scientific Notation	20 days	NC.8.EE.1	NC.8.EE.3	38-41
			NC.8.EE.4		
<b>Unit 6</b> <b>Lessons 23-28</b>	Real Numbers: Rational Numbers, Irrational Numbers, and the Pythagorean Theorem	26 days	NC.8.NS.2	NC.8.NS.1	42-49
			NC.8.G.7	NC.8.EE.2	
			NC.8.G.9	NC.8.G.6	
				NC.8.G.8	
<b>Unit 7</b> <b>Lessons 29-32</b>	Statistics: Two-Variable Data and Fitting a Linear Model	19 days	NC.8.SP.2	NC.8.SP.1	50-59
			NC.8.SP.3	NC.8.SP.4	
			NC.8.F.4		

# Online Resources

- <https://login.i-ready.com/> Digital teacher resource designed to provide teachers access to the *i-Ready Classroom*, *Ready Math NC*, and *Thinkup! NC* lessons and additional resources, which can be used for whole class or small groups to help differentiate instruction.
- <https://readycentral.com/> From how-to tips to planning tools, find everything you need for successfully implementing *Ready Math*.
- <https://i-readycentral.com/> From videos to tips and planning tools, find everything you need to be successful with *i-Ready*
- <https://www.dpi.nc.gov/districts-schools/classroom-resources/academic-standards/standard-course-study/mathematics> NCDPI K-12 Mathematics site.
- <http://www.tools4ncteachers.com/> (Tools for Teachers Project-Created by North Carolina educators in conjunction with NCDPI consultants) - grade level (K-8) material access which includes NC Standards, Unpacking Documents and Instructional Frameworks.
- <https://www.nc2ml.org/> (North Carolina Collaborative for Mathematics Learning, i.e. NC<sup>2</sup>ML) - NC network of support for teachers. Provides resources, the ability to share best practices, and develop mathematical mindsets.



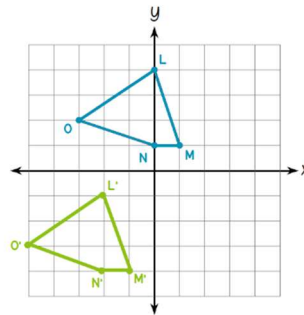
Domain									Conceptual Category
K	1	2	3	4	5	6	7	8	HS
Counting and Cardinality						Ratio and Proportions	Functions		Algebra
Operations and Algebraic Thinking						Expressions and Equations			Functions
Number and Operations Base Ten						The Number System			Number and Quantity
		Number and Operations Fractions							
Measurement and Data						Statistics and Probability			Statistics and Probability
Geometry									Geometry

# Unit 1

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## Geometric Figures: Rigid Transformations and Congruence

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## Unit 1: Geometric Figures: Rigid Transformations and Congruence

Source: NCSCOS 6-8 Mathematics. Retrieved from: <https://files.nc.gov/dpi/documents/curriculum/mathematics/scos/current/6-8.pdf>

<i>OCS Priority Standard(s):</i>	<i>Supporting Standard(s):</i>
	<p><b>NC.8.G.2</b> Use transformations to define congruence:</p> <ul style="list-style-type: none"><li>• Verify experimentally the properties of rotations, reflections, and translations that create congruent figures.</li><li>• Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations.</li><li>• Given two congruent figures, describe a sequence that exhibits the congruence between them.</li></ul> <p><b>NC.8.G.4</b> Use transformations to define similarity.</p> <ul style="list-style-type: none"><li>• Verify experimentally the properties of dilations that create similar figures.</li><li>• Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations.</li><li>• Given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.</li></ul>

# Unit 1 Unpacking

Source: NC DPI 8<sup>th</sup> Grade Math Unpacking Document Revised June 2022. Retrieved from <https://www.dpi.nc.gov/nc-8th-grade-math-unpacking-rev-june-2022>

## Supporting Standard: NC.8.G.2

**Understand congruence and similarity using physical models, transparencies, or geometry software.**

**NC.8.G.2** Use transformations to define congruence:

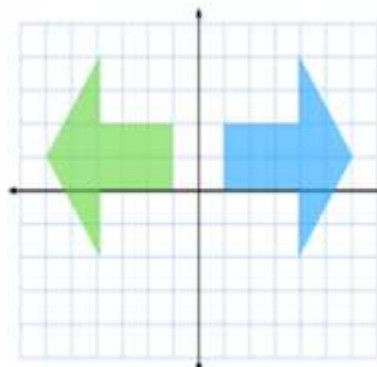
- Verify experimentally the properties of rotations, reflections, and translations that create congruent figures.
- Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations.
- Given two congruent figures, describe a sequence that exhibits the congruence between them.

### Clarification

The focus on this standard is the conceptual development of the idea of **congruent figures**. Congruent figures have the same shape and size. Two figures in the plane are said to be congruent if there is a sequence of rigid motions that takes one figure onto the other. (Progressions for CCSSM Geometry, Grade 7-8, HS, 2016).

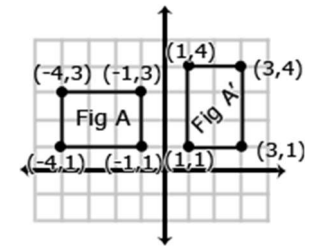
Given two congruent figures, students explore characteristics of the figures, such as lengths of line segments, angle measures and parallel lines as they develop a definition for congruent figures. The coordinate plane can be used as a tool to develop understanding of this concept because it gives a visual image of the correspondence between the two figures.

**In the following example**, students should be able to compare the side lengths and angles created by adjacent sides for each figure and the correspondence of the sides and angles between the figures. They should also be able to determine what type of rigid transformation will map one figure onto the other. In this case, a reflection across the y-axis will map the green arrow onto the blue arrow and vice-versa.

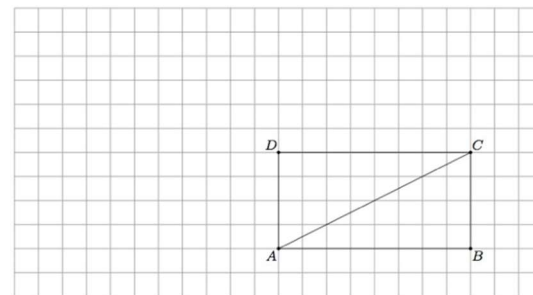


### Checking for Understanding

Figure A and Figure A' are congruent. Identify at least three characteristics of the figures that supports this fact.



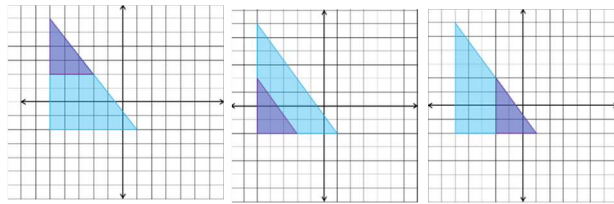
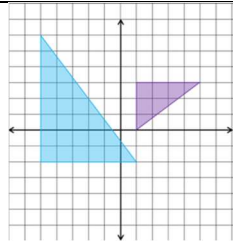
Below is a picture of rectangle ABCD with diagonal AC.



- Draw the image of triangle  $ACD$  when it is rotated  $180^\circ$  about vertex  $D$ . Call  $A'$  the image of point  $A$  under the rotation and  $C'$  the image of point  $C$ .
- Explain why  $\overrightarrow{DA'} \cong \overrightarrow{DA}$  and why  $\overrightarrow{DC'}$  is parallel to  $\overrightarrow{AB}$ .

<p>Students use mathematical language to distinguish the figures, noting that the figure prior to the transformation is called the <b>pre-image</b> (e.g. Figure A) and the post-transformation figure is called the <b>image</b> (e.g. Figure A').</p> <p>Students also examine two figures to identify the rigid transformation(s) that produced the image from the preimage. Students recognize the symbol for congruency (<math>\cong</math>) and write statements of congruence.</p>	<p>c. Show that <math>\triangle A'C'D'</math> can be translated to <math>\triangle CAB</math>. Conclude that <math>\triangle ACD</math> is congruent to <math>\triangle CAB</math>.</p> <p>d. Show that <math>\triangle ACD</math> is congruent to <math>\triangle CAB</math> with a sequence of translations, rotations, and/or reflections different from those chosen in parts (a) and (c).</p>
<p><b>Supporting Standard: NC.8.G.4</b></p>	
<p><b>Understand congruence and similarity using physical models, transparencies, or geometry software.</b></p>	
<p><b>NC.8.G.4</b> Use transformations to define similarity.</p> <ul style="list-style-type: none"> <li>• Verify experimentally the properties of dilations that create similar figures.</li> <li>• Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations.</li> <li>• Given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.</li> </ul>	
<p><b>Clarification</b></p>	<p><b>Checking for Understanding</b></p>
<p>The focus of this standard is on the conceptual development of <b>similar figures</b>. Similar figures have congruent angles and proportional corresponding side lengths. Two figures are <i>similar</i> if there is a sequence of dilations and rigid motions that places one figure directly on top of another (Progressions for CCSSM Geometry, Grade 7-8, HS, 2016).</p> <p>Given two similar figures, students explore the proportional relationship between corresponding characteristics of the figures, such as lengths of line segments, and angle measures as they develop a definition for similar figures. In 8<sup>th</sup> grade, dilations are restricted from the origin. The coordinate plane can be used as a tool to develop understanding of this concept because it gives a visual image of the correspondence between the two figures. Additionally, transparencies or tracing paper can be used to show the congruency of the angles, while measurement tools can be used to examine the proportional relationships between edge lengths.</p>	<p>Triangle ABC undergoes a series of some of the following transformations to become triangle DEF:</p> <ul style="list-style-type: none"> <li>• Rotation</li> <li>• Reflection</li> <li>• Translation</li> <li>• Dilation</li> </ul> <p>Is triangle DEF always, sometimes or never similar to triangle ABC? Justify your response. (Adapted from SBAC)</p> <p>Given the dilation <math>\triangle ABC \rightarrow \triangle A'B'C'</math>, what is the scale factor?</p> <p>a. Verify proportionality using both the dimensions and the coordinates of the figures.</p>

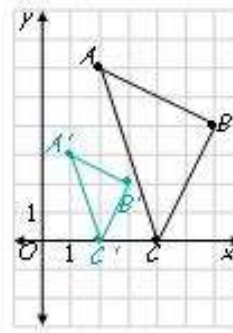
**In the following example**, the blue triangle and the purple triangle are similar. Students should explore the angle and side length relationships by using transformations to map one triangle onto another. Mapping each angle in the purple triangle to the corresponding angle in the blue triangle can help students see the proportionality of the sides and congruence of the angles.



Students use mathematical language to distinguish the figures, noting that the figure prior to the transformation is called the **pre-image** (e.g. Figure A) and the post-transformation figure is called the **image** (e.g. Figure A').

Students also examine two figures to identify the rigid transformation(s) that produced the image from the preimage. Students recognize the symbol for similarity ( $\sim$ ) and write statements of similarity.

b. Verify that the center of dilation is at the origin.

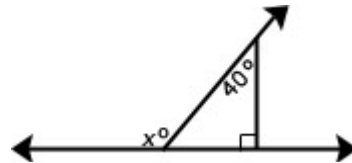


# Unit 2

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## Geometric Figures: Transformations, Similarity, and Angle Relationships

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## Unit 2: Geometric Figures: Transformations, Similarity, and Angle Relationships

Source: NCSCOS 6-8 Mathematics. Retrieved from: <https://files.nc.gov/dpi/documents/curriculum/mathematics/scos/current/6-8.pdf>

<b>OCS Priority Standard(s):</b>	<b>Supporting Standard(s):</b>
<p><b>NC.8.G.3</b> Describe the effect of dilations about the origin, translations, rotations about the origin in 90-degree increments, and reflections across the <math>x</math>-axis and <math>y</math>-axis on two-dimensional figures using coordinates.</p> <p><b>NC.8.G.5</b> Use informal arguments to analyze angle relationships.</p> <ul style="list-style-type: none"><li>• Recognize relationships between interior and exterior angles of a triangle.</li><li>• Recognize the relationships between the angles created when parallel lines are cut by a transversal.</li><li>• Recognize the angle-angle criterion for similarity of triangles.</li><li>• Solve real-world and mathematical problems involving angles.</li></ul>	<p><b>NC.8.G.2</b> Use transformations to define congruence:</p> <ul style="list-style-type: none"><li>• Verify experimentally the properties of rotations, reflections, and translations that create congruent figures.</li><li>• Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations.</li><li>• Given two congruent figures, describe a sequence that exhibits the congruence between them.</li></ul> <p><b>NC.8.G.4</b> Use transformations to define similarity.</p> <ul style="list-style-type: none"><li>• Verify experimentally the properties of dilations that create similar figures.</li><li>• Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations.</li></ul> <p>Given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.</p>



## Unit 2 Unpacking

Source: NC DPI 8<sup>th</sup> Grade Math Unpacking Document Revised June 2022. Retrieved from <https://www.dpi.nc.gov/nc-8th-grade-math-unpacking-rev-june-2022>

### Supporting Standard: NC.8.G.2

**Understand congruence and similarity using physical models, transparencies, or geometry software.**

**NC.8.G.2** Use transformations to define congruence:

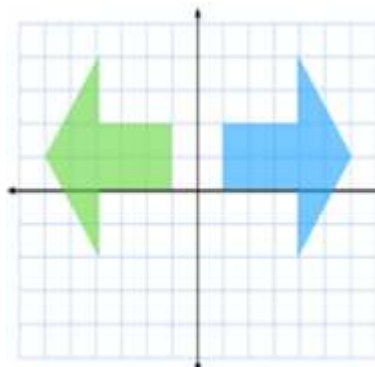
- Verify experimentally the properties of rotations, reflections, and translations that create congruent figures.
- Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations.
- Given two congruent figures, describe a sequence that exhibits the congruence between them.

#### Clarification

The focus on this standard is the conceptual development of the idea of **congruent figures**. Congruent figures have the same shape and size. Two figures in the plane are said to be congruent if there is a sequence of rigid motions that takes one figure onto the other. (Progressions for CCSSM Geometry, Grade 7-8, HS, 2016).

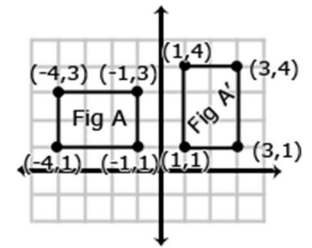
Given two congruent figures, students explore characteristics of the figures, such as lengths of line segments, angle measures and parallel lines as they develop a definition for congruent figures. The coordinate plane can be used as a tool to develop understanding of this concept because it gives a visual image of the correspondence between the two figures.

**In the following example**, students should be able to compare the side lengths and angles created by adjacent sides for each figure and the correspondence of the sides and angles between the figures. They should also be able to determine what type of rigid transformation will map one figure onto the other. In this case, a reflection across the y-axis will map the green arrow onto the blue arrow and vice-versa.

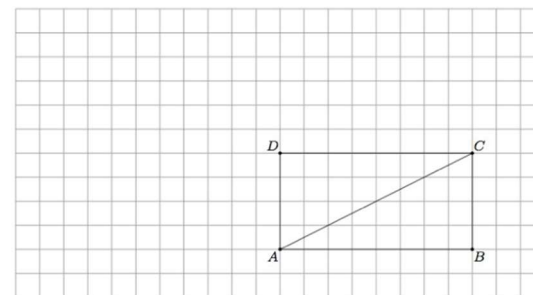


#### Checking for Understanding

Figure A and Figure A' are congruent. Identify at least three characteristics of the figures that supports this fact.



Below is a picture of rectangle ABCD with diagonal AC.



- e. Draw the image of triangle  $ACD$  when it is rotated  $180^\circ$  about vertex  $D$ . Call  $A'$  the image of point  $A$  under the rotation and  $C'$  the image of point  $C$ .

Students use mathematical language to distinguish the figures, noting that the figure prior to the transformation is called the **pre-image** (e.g. Figure A) and the post-transformation figure is called the **image** (e.g. Figure A').

Students also examine two figures to identify the rigid transformation(s) that produced the image from the preimage. Students recognize the symbol for congruency ( $\cong$ ) and write statements of congruence.

- f. Explain why  $\overline{DA'} \cong \overline{DA}$  and why  $\overline{DC'}$  is parallel to  $\overline{AB}$ .
- g. Show that  $\Delta A'C'D'$  can be translated to  $\Delta CAB$ . Conclude that  $\Delta ACD$  is congruent to  $\Delta CAB$ .
- h. Show that  $\Delta ACD$  is congruent to  $\Delta CAB$  with a sequence of translations, rotations, and/or reflections different from those chosen in parts (a) and (c).

**OCS Priority Standard: NC.8.G.3**

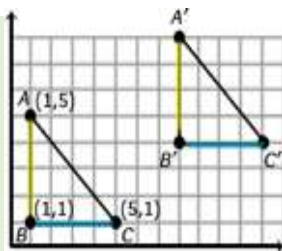
**Understand congruence and similarity using physical models, transparencies, or geometry software.**

**NC.8.G.3** Describe the effect of dilations about the origin, translations, rotations about the origin in 90 degree increments, and reflections across the  $x$ -axis and  $y$ -axis on two-dimensional figures using coordinates.

**Clarification**

The focus of this standard is on developing understanding of transformations using visualization, spatial reasoning, and geometric modeling. The coordinate plane is used as a tool to develop student understanding of transformations unifying the ideas of shape and location. Students study distance-preserving transformations (isometries) and dilations to aid in the development of the concepts of congruence and similarity, respectively.

**For example,** notice that the distance between the respective points in each figure is the same (i.e. there are 4 units between A and B, likewise between A' and B').



**Isometries**  
Isometries are also called rigid

transformations because they preserve size and shape of a geometric figure. Translations, reflections and rotations are **rigid** transformations.

**Translations**

Translations move the object so that every point of the object moves in the same direction as well as the same distance. In a translation, the translated object is *congruent* to its pre-image.

**Checking for Understanding**

Complete the table of transformation rules:

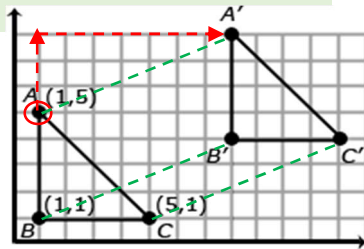
Type of Transformation	Coordinate Change
Vertical Translation ( $\uparrow$ )	$(x, y) \rightarrow (x, y + a)$
Vertical Translation ( $\downarrow$ )	$(x, y) \rightarrow ( \quad , \quad )$
Horizontal Translation ( $\rightarrow$ )	$(x, y) \rightarrow ( \quad , \quad )$
Horizontal Translation ( $\leftarrow$ )	$(x, y) \rightarrow ( \quad , \quad )$
Reflection over $x$ -axis	$(x, y) \rightarrow ( \quad , \quad )$
Reflection over $y$ -axis	$(x, y) \rightarrow ( \quad , \quad )$
Rotation $90^\circ$ (clockwise)	$(x, y) \rightarrow ( \quad , \quad )$
Rotation $90^\circ$ (counter-clockwise)	$(x, y) \rightarrow ( \quad , \quad )$
Rotation $180^\circ$ (clockwise)	$(x, y) \rightarrow ( \quad , \quad )$
Rotation $180^\circ$ (counter-clockwise)	$(x, y) \rightarrow ( \quad , \quad )$
Rotation $270^\circ$ (clockwise)	$(x, y) \rightarrow ( \quad , \quad )$
Rotation $270^\circ$ (counter-clockwise)	$(x, y) \rightarrow ( \quad , \quad )$
Dilation (Scale up)	$(x, y) \rightarrow ( \quad , \quad )$
Dilation (Scale down)	$(x, y) \rightarrow ( \quad , \quad )$

**For example,**

$\Delta ABC$  has been translated 7 units  $\rightarrow$  and 3 units  $\uparrow$ .

To get from  $A (1, 5)$  to  $A' (8, 8)$ , move point  $A$  7 units  $\rightarrow$  (from  $x = 1$  to  $x = 8$ ) and 3 units  $\uparrow$  (from  $y = 5$  to  $y = 8$ ).

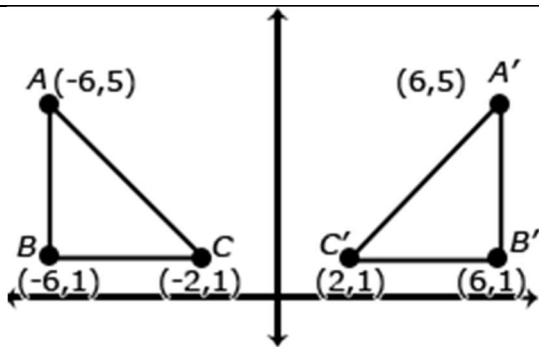
Points  $B$  and  $C$  also move in the same direction (7 units  $\rightarrow$  and 3 units  $\uparrow$ ), resulting in the same changes to each coordinate.



**Reflections**

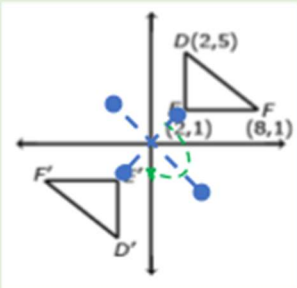
A reflection is the “flipping” of an object over a line, known as the “line of reflection”. In the 8<sup>th</sup> grade, students will only reflect over the  $x$ - and  $y$ -axis. Students recognize that when an object is reflected across the  $y$ -axis, the reflected  $x$ -coordinate is the opposite of the pre-image  $x$ -coordinate. Students can then infer what happens when reflected across the  $x$ -axis. Likewise, a reflection across the  $x$ -axis would change a pre-image coordinate  $A (-6, 5)$  to the image coordinate of  $A' (-6, -5)$ .

The figure below is reflected across the  $y$ -axis. Compare point  $A (-6, 5)$  to point  $A' (6, 5)$ ,  $B$  to  $B'$ , and  $C$  to  $C'$ .



### Rotations

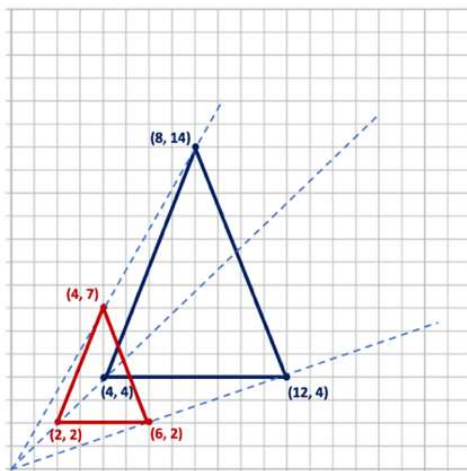
A rotation is a transformation performed by “spinning” the figure around a fixed point known as the center of rotation. The figure may be rotated clockwise or counterclockwise up to  $360^\circ$  (at 8<sup>th</sup> grade, rotations will be around the origin and a multiple of  $90^\circ$ ). Students know that lines passing through the point of rotation, map onto themselves when rotated  $180^\circ$ , however lines that are rotated  $180^\circ$  that do not pass through the point of rotation create parallel lines.



**For example**, consider when triangle  $DEF$  is rotated  $180^\circ$  clockwise about the origin. The coordinates of triangle  $DEF$  are  $(2,5)$ ,  $(2,1)$ , and  $(8,1)$ . When rotated  $180^\circ$  about the origin, the new coordinates are  $(-2,-5)$ ,  $(-2,-1)$ , and  $(-8,-1)$ . In this case, each coordinate is the opposite of its pre-image (see figure). Notice that corresponding sides of the image and preimage are parallel.

## Dilations

Building on understanding of scale factor in earlier grades, students learn that a dilation is a transformation that moves each point along a ray which starts from a fixed center and multiplies distances from this center by a common scale factor. Dilations enlarge (scale factors greater than one) or reduce (scale factors less than one) the size of a figure by the specified factor.



Students recognize the relationship between the coordinates of the pre-image, the image and the scale factor for a dilation from the origin. In 8<sup>th</sup> grade, dilations are limited from the origin.

### Supporting Standard: **NC.8.G.4**

**Understand congruence and similarity using physical models, transparencies, or geometry software.**

**NC.8.G.4** Use transformations to define similarity.

- Verify experimentally the properties of dilations that create similar figures.
- Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations.
- Given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

#### Clarification

The focus of this standard is on the conceptual development of **similar figures**. Similar figures have congruent angles and proportional corresponding side lengths. Two figures are *similar* if there is a sequence of dilations and rigid motions that places one figure directly on top of another (Progressions for CCSSM Geometry, Grade 7-8, HS, 2016).

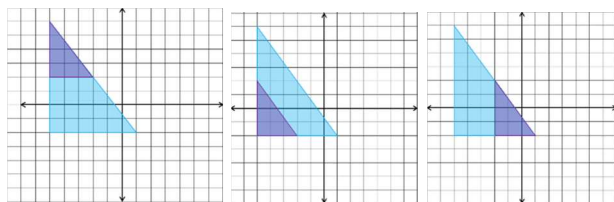
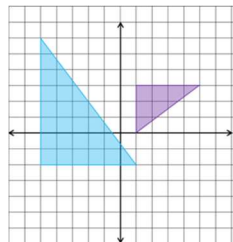
#### Checking for Understanding

Triangle ABC undergoes a series of some of the following transformations to become triangle DEF:

- Rotation
- Reflection
- Translation

Given two similar figures, students explore the proportional relationship between corresponding characteristics of the figures, such as lengths of line segments, and angle measures as they develop a definition for similar figures. In 8<sup>th</sup> grade, dilations are restricted from the origin. The coordinate plane can be used as a tool to develop understanding of this concept because it gives a visual image of the correspondence between the two figures. Additionally, transparencies or tracing paper can be used to show the congruency of the angles, while measurement tools can be used to examine the proportional relationships between edge lengths.

**In the following example**, the blue triangle and the purple triangle are similar. Students should explore the angle and side length relationships by using transformations to map one triangle onto another. Mapping each angle in the purple triangle to the corresponding angle in the blue triangle can help students see the proportionality of the sides and congruence of the angles.



Students use mathematical language to distinguish the figures, noting that the figure prior to the transformation is called the **pre-image** (e.g. Figure A) and the post-transformation figure is called the **image** (e.g. Figure A').

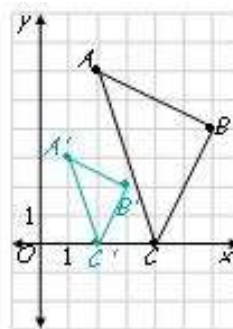
Students also examine two figures to identify the rigid transformation(s) that produced the image from the preimage. Students recognize the symbol for similarity ( $\sim$ ) and write statements of similarity.

- Dilation

Is triangle DEF always, sometimes or never similar to triangle ABC? Justify your response. (Adapted from SBAC)

Given the dilation  $\Delta ABC \rightarrow \Delta A'B'C'$ , what is the scale factor?

- Verify proportionality using both the dimensions and the coordinates of the figures.
- Verify that the center of dilation is at the origin.



**OCS Priority Standard: NC.8.G.5**

**Analyze angle relationships.**

**NC.8.G.5** Use informal arguments to analyze angle relationships.

- Recognize relationships between interior and exterior angles of a triangle.
- Recognize the relationships between the angles created when parallel lines are cut by a transversal.
- Recognize the angle-angle criterion for similarity of triangles.
- Solve real-world and mathematical problems involving angles.

**Clarification**

This standard focuses on an **informal development** of the understanding of angle relationships in triangles and parallel lines. Students build on understandings of angle relationships in 7<sup>th</sup> grade and transformations in 8<sup>th</sup> grade.

➤ **Interior and exterior angles in triangles**

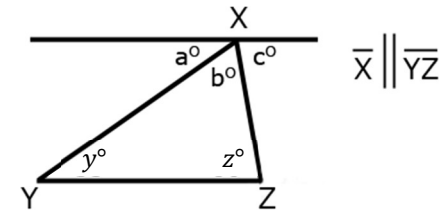
- Students *informally discover* that the interior angles of a triangle have a sum of  $180^\circ$ .
- Students understand the relationships between interior and exterior angles of a triangle. Students know that:
  - every exterior angle is supplementary to its adjacent interior angle.
  - the measure of an interior angle is equivalent to the sum of the remote interior angles.

➤ **Angles created when parallel lines are cut by a transversal**

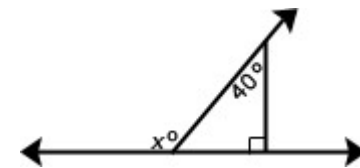
- Building on understanding from transformations, students construct parallel lines and a transversal to examine the relationships between the created angles.
- Students recognize vertical angles, adjacent angles and supplementary angles from 7<sup>th</sup> grade and build on these relationships to identify other pairs of congruent angles created by parallel lines and a transversal.

**Checking for Understanding**

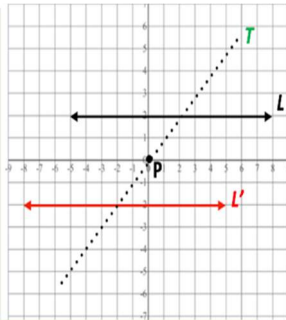
In the figure below line  $X$  is parallel to line  $YZ$ . How could you use this fact to show that the sum of the angles of  $\triangle XYZ$  is  $180^\circ$ ?



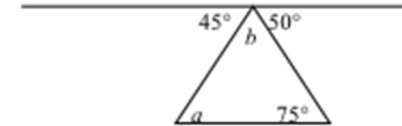
Write and solve an equation to find the measure of angle  $x$ .



**For example,** since line  $L'$  is the image of line  $L$  after a  $180^\circ$  rotation about point  $P$  and line  $T$ , which passes through the point of rotation, maps onto itself after the same  $180^\circ$  rotation, then  $L$  and  $L'$  are parallel lines. This means that the angle relationships between lines  $T$  and  $L$  are the same angle



Find the measures of angles  $a$  and  $b$ .



➤ **Angle-angle criterion for similarity of triangle.**

Students notice through exploration and conjecturing that there are an infinite number of triangles that can be created that have the same exact angle measurements and those triangles are therefore similar to each other and not necessarily congruent.

Josh drew  $\triangle ABC$  with  $m\angle A = 40^\circ$ ,  $m\angle B = 60^\circ$  and  $m\angle C = 80^\circ$ . Howard drew  $\triangle XYZ$  with  $m\angle Y = 60^\circ$  and  $m\angle Z = 80^\circ$ . What can you say about the two triangles? Justify your response.

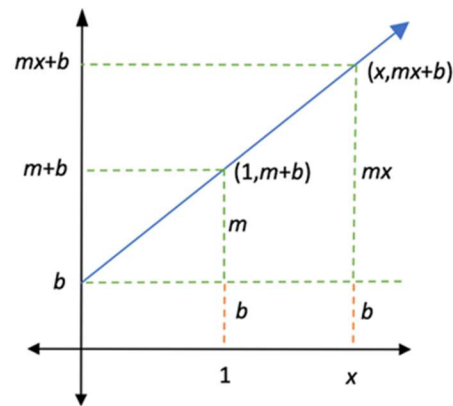


# Unit 3

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## Linear Relationships: Slope, Linear Equations, and Systems

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## Unit 3: Linear Relationships: Slope, Linear Equations, and Systems

Source: NCSCOS 6-8 Mathematics. Retrieved from: <https://files.nc.gov/dpi/documents/curriculum/mathematics/scos/current/6-8.pdf>

<i>OCS Priority Standard(s):</i>	<i>Supporting Standard(s):</i>
<p><b>NC.8.EE.7</b> Solve real-world and mathematical problems by writing and solving equations and inequalities in one variable.</p> <ul style="list-style-type: none"><li>• Recognize linear equations in one variable as having one solution, infinitely many solutions, or no solutions.</li><li>• Solve linear equations and inequalities including multi-step equations and inequalities with the same variable on both sides.</li></ul> <p><b>NC.8.EE.8</b> Analyze and solve a system of two linear equations in two variables in slope-intercept form.</p> <ul style="list-style-type: none"><li>• Understand that solutions to a system of two linear equations correspond to the points of intersection of their graphs because the point of intersection satisfies both equations simultaneously.</li><li>• Solve real-world and mathematical problems leading to systems of linear equations by graphing the equations. Solve simple cases by inspection.</li></ul> <p><b>NC.8.F.4</b> Analyze functions that model linear relationships.</p> <ul style="list-style-type: none"><li>• Understand that a linear relationship can be generalized by <math>y = mx + b</math>.</li><li>• Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two <math>(x, y)</math> values or a graph.</li><li>• Construct a graph of a linear relationship given an equation in slope-intercept form.</li><li>• Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and <math>y</math>-intercept of its graph or a table of values.</li></ul>	

## Unit 3 Unpacking

Source: NC DPI 8<sup>th</sup> Grade Math Unpacking Document Revised June 2022. Retrieved from <https://www.dpi.nc.gov/nc-8th-grade-math-unpacking-rev-june-2022>

### OCS Priority Standard: **NC.8.EE.7**

#### Analyze and solve linear equations and inequalities.

**NC.8.EE.7** Solve real-world and mathematical problems by writing and solving equations and inequalities in one variable.

- Recognize linear equations in one variable as having one solution, infinitely many solutions, or no solutions.
- Solve linear equations and inequalities including multi-step equations and inequalities with the same variable on both sides.

#### Clarification

In 7<sup>th</sup> grade, students learned to solve multistep one-variable equations and inequalities, with the variable on one side. In 8<sup>th</sup> grade, students will build upon this understanding to solve one-variable equations and inequalities with the same variable on both sides.

Students recognize and explain when linear equations have one solution, infinitely many solutions, or no solution without completing the solving process.

Linear inequalities may have infinitely many solutions or no solution. For example,

$x + 5 < 2x + 8 - x$  has no solution.

This inequality has no solution as 5 is not greater than 8.

Students justify their answer with mathematical reasoning, including the use of the properties of equality.

#### Checking for Understanding

Determine the number of solutions for each of the following. If there is only one solution, determine the solution.

a)  $\frac{1}{2}(2p + 9) = -p + 5$

b)  $5 - 3q = 4 - \frac{2}{3}(4.5q - 1.5)$

c)  $8 - 2(n + 3) = n + 7 - 3n$

d)  $\frac{1}{5}g + \frac{2}{5} = 1\frac{1}{5}g - 2\left(4 + \frac{2}{5}\right)$

In the following equation,  $a$  and  $b$  represent integers.

$$2x + a = 5 - bx$$

What values of  $a$  and  $b$  would create an equation with just one solution?

What values of  $a$  and  $b$  would create an equation with no solutions?

What values of  $a$  and  $b$  would create an equation with infinitely many solutions?

Solve the following and graph the solutions on a number line:

a)  $3x - 2 > 9 + 5x$

b)  $\frac{5+2y}{4} \geq \frac{y+3}{2}$

c)  $\frac{2}{3}h + 9 < 8\left(\frac{1}{3}h - 2\right)$

d)  $\frac{1}{5}(13 - 20x) \leq -14 - 4x$

Two companies are competing for a contract to make the programs for the high school football games. Howie's Printing charges a \$19.99 fee for printing and \$0.25 for each program printed. Mint Print charges a \$29.99 fee for printing and \$0.10 for each program printed.

For what number of printed programs will Howie's Printing cost more than Mint Print?

- a) Write and solve an inequality to describes this situation.
- b) Describe what your solution means.
- c) If you anticipate needing 75 programs for a football game, which company is the cheaper choice?

**OCS Priority Standard: NC.8.EE.8**

**Analyze and solve pairs of simultaneous linear equations.**

**NC.8.EE.8** Analyze and solve a system of two linear equations in two variables in slope-intercept form.

- Understand that solutions to a system of two linear equations correspond to the points of intersection of their graphs because the point of intersection satisfies both equations simultaneously.
- Solve real-world and mathematical problems leading to systems of linear equations by graphing the equations. Solve simple cases by inspection.

**Clarification**

Students find the solution to a system of two linear equations by graphing. In 8<sup>th</sup> grade, the linear equations will be limited to slope-intercept form.

**Understand that solutions to a system of two linear equations correspond to the points of intersection of their graphs because the point of intersection satisfies both equations simultaneously.**

Students can explain and demonstrate that the solution to the system of equations must be a solution to each of the equations of the system. By comparing the graphs of a system to the corresponding equations in the system, students can discover characteristics of systems that have no solutions, one solution, and infinite solutions.

**Solve real-world and mathematical problems leading to systems of linear equations by graphing the equations. Solve simple cases by inspection.**

Students write a system of equations to represent a word problem. Word problems will be presented in a way that students can write the equation directly into slope-intercept form. Students recognize the structure of equations and by inspection recognize when the equations will produce one solution, no solution or infinitely many solutions.

**Checking for Understanding**

Plant A and Plant B are on different watering schedules. This affects their rate of growth. Plant A started at a height of 4 inches and grows 2 inches per week. Plant B started at 2 inches and grows at 4 inches per week.

- a) Create a table that represents the height of each plant for each week.
- b) Plot the graph of each table on the same coordinate plane.
- c) Where do the graphs of each plant growth intersect?
- d) What does this intersection represent?
- e) Write an equation to represent each plant's growth per week.
- f) What is the relationship between the point of intersection and the equation for each plant's growth?

Find the solution to each of the following system of equations.

a)  $b = \frac{2}{3}a + 1$

$b = -\frac{1}{3}a + 7$

b)  $y = \frac{3}{7}x - 4$

$y = \frac{3}{7}x + 1$

**OCS Priority Standard: NC.8.F.4****Use functions to model relationships between quantities.****NC.8.F.4** Analyze functions that model linear relationships.

- Understand that a linear relationship can be generalized by  $y = mx + b$ .
- Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two  $(x, y)$  values or a graph.
- Construct a graph of a linear relationship given an equation in slope-intercept form.
- Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and  $y$ -intercept of its graph or a table of values.

**Clarification**

Analyzing linear functions is a major focus for 8<sup>th</sup> grade. In previous grades, students have worked with proportional relationships, which are a subset of linear functions. This means that some of the properties of a proportional relationship can be generalized to linear functions, while some properties are specific to proportional relationships.

**Understand that a linear relationship can be generalized by  $y = mx + b$ .**

In previous grades, students created tables, graphs, and equations from proportional relationships. Students can build from this previous knowledge to the larger generalization of linear functions.

In 7<sup>th</sup> grade, students learned that in a proportional relationship, the  $y$ -coordinate when  $x = 1$  is the unit rate and built an understanding of the equation of a proportional relationship (see NC.7.RP.2).

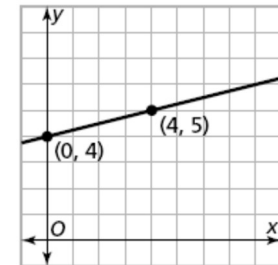
$x$ (input)	$y$ (output)
0	$b$
1	$m + b$
2	$m \cdot 2 + b$
3	$m \cdot 3 + b$
...	...
$x$	$m \cdot x + b$

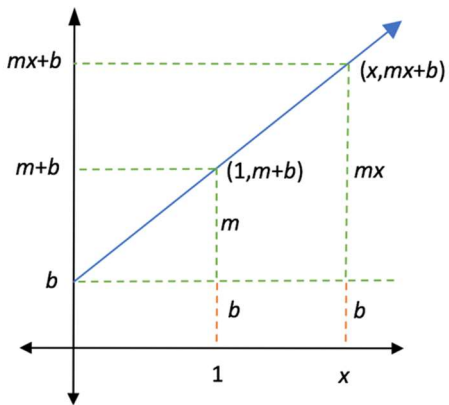
**Checking for Understanding**

Write an equation that models the linear relationship in the table.

$x$	$y$
-2	8
0	2
1	-1

Write an equation that models the linear relationship in the graph.





In 8<sup>th</sup> grade, students build on this understanding to develop the linear equation in slope-intercept form. The initial small triangle starts with sides of 1 and  $m$ . As the triangle is scaled by a factor of  $x$ , the sides of the new triangle become  $x$  and  $mx$ . However, in order to find the coordinate that is on the line, the  $y$ -coordinate

must be increased by  $b$ , since the triangle is starting off the  $x$ -axis. This gives the coordinate of  $(x, mx + b)$ . This means that when the input is  $x$ , the output is  $mx + b$ , producing the function,  $y = mx + b$ . The same generalization can be seen in the table.

In 8<sup>th</sup> grade, the only required form of a linear equation is slope-intercept form. Students will not be asked to work with other forms of a linear equation or be asked to change the other forms to slope-intercept form.

**Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two  $(x, y)$  values or a graph.**

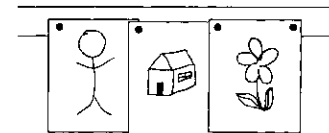
Students find the rate of change between two points. These two points may be given in a table, on the coordinate plane, or as ordered pairs. Students will also find the initial value in a linear function represented as an equation, table or graph. Having found the rate of change and the initial value, students should be able to write a linear function that models the situation given.

This standard does not imply that students should memorize the slope formula or use a purely algebraic approach to find an equation. Students should be able to use patterns and other representation to find the initial value and rate of change.

Write an equation for the line that has a rate of change of  $\frac{1}{2}$  and passes through the point  $(-2, 5)$ .

The company charges \$45 a day for the car as well as charging a one-time \$25 fee for the car's navigation system (GPS). Write a function for the cost in dollars,  $c$ , and the number of days,  $d$ , the car was rented.

Children's pictures are hung in a line as seen in the picture. Pictures that are hung next to each other share a tack.



- Describe the rate of change based on this context.
- How many tacks would be needed for 28 pictures?

Adapted from NAEP – Released Item (1992) **Question ID:** 1992-8M7 #8 M045201

A leaf falls 18 feet from a branch to the ground at a rate of 5 feet every 2 seconds. Determine whether each statement about the leaf is true. If it is false, change the statement so that it is true.

Statement	T/F
The initial height of the leaf is 18 feet.	
The leaf falls at a rate of $\frac{2}{5}$ foot every 1 second.	
The leaf is 3 feet above the ground after 6 seconds.	

Adapted from SBAC Mathematics Practice Test Scoring Guide Grade 8 p. 11

**Construct a graph of a linear relationship given an equation in slope-intercept form.**

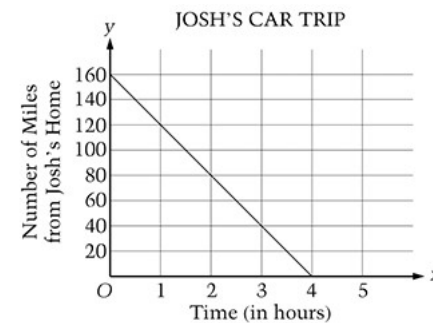
Student create the graphical representation of linear function, given a linear equation in slope-intercept form, using the initial value and rate of change. Students will choose an appropriate scale to graph a linear function.

**Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and y-intercept of its graph or a table of values.**

Students give meaning to the rate of change and the initial value of a linear function based on a context. The linear function can be given in a variety of representations, including verbal descriptions, tables, graphs, and equations.

Students use terms such as slope and y-intercept to describe a graphical representation of a linear function and correlate their meaning to the rate of change and initial value, where the input is 0. Students should use the units from a context appropriately in their description of rate of change and initial value.

The linear graph below describes Josh's car trip from his grandmother's home directly to his home.



- What is the distance from Josh's grandmother's home to his home?
- How long did it take for Josh to make the trip?
- What was Josh's average speed for the trip? Explain how you know.

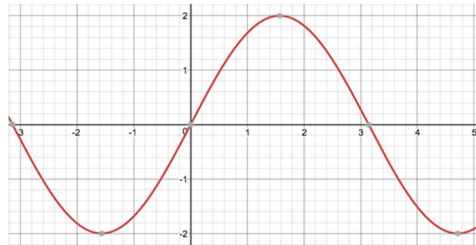
Adapted from NAEP – Released Item (2011) Question ID: 2011-8M8 #15 M169901

# Unit 4

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## Functions: Linear and Non-Linear Relationships

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## Unit 4: Functions: Linear and Non-Linear Relationships

Source: NCSCOS 6-8 Mathematics. Retrieved from: <https://files.nc.gov/dpi/documents/curriculum/mathematics/scos/current/6-8.pdf>

<b>OCS Priority Standard(s):</b>	<b>Supporting Standard(s):</b>
<p><b>NC.8.F.2</b> Compare properties of two linear functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).</p> <p><b>NC.8.F.4</b> Analyze functions that model linear relationships.</p> <ul style="list-style-type: none"><li>• Understand that a linear relationship can be generalized by <math>y = mx + b</math>.</li><li>• Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two <math>(x, y)</math> values or a graph.</li><li>• Construct a graph of a linear relationship given an equation in slope-intercept form.</li><li>• Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and <math>y</math>-intercept of its graph or a table of values.</li></ul>	<p><b>NC.8.F.1</b> Understand that a function is a rule that assigns to each input exactly one output.</p> <ul style="list-style-type: none"><li>• Recognize functions when graphed as the set of ordered pairs consisting of an input and exactly one corresponding output.</li><li>• Recognize functions given a table of values or a set of ordered pairs.</li></ul> <p><b>NC.8.F.3</b> Identify linear functions from tables, equations, and graphs.</p> <p><b>NC.8.F.5</b> Qualitatively analyze the functional relationship between two quantities.</p> <ul style="list-style-type: none"><li>• Analyze a graph determining where the function is increasing or decreasing; linear or non-linear.</li><li>• Sketch a graph that exhibits the qualitative features of a real-world function.</li></ul>

## Unit 4 Unpacking

Source: NC DPI 8<sup>th</sup> Grade Math Unpacking Document Revised June 2022. Retrieved from <https://www.dpi.nc.gov/nc-8th-grade-math-unpacking-rev-june-2022>

### Supporting Standard: **NC.8.F.1**

**Define, evaluate, and compare functions.**

**NC.8.F.1** Understand that a function is a rule that assigns to each input exactly one output.

- Recognize functions when graphed as the set of ordered pairs consisting of an input and exactly one corresponding output.
- Recognize functions given a table of values or a set of ordered pairs.

#### Clarification

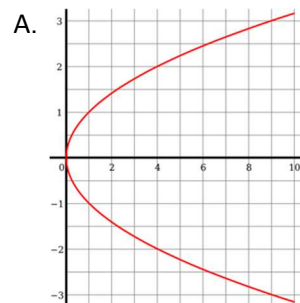
This standard starts to develop the definition of a function as a rule that produces a unique output for each input. This understanding has been building through students work with ratio and proportional relationships. In 8<sup>th</sup> grade, functions will be represented as tables, graphs, and equations. Function notation is not an expectation in 8<sup>th</sup> grade.

#### Recognizing functions from graphs, tables, or a set of ordered pairs.

Students identify graphs, tables or a set of ordered pairs that represent a function. These graphs, tables, and ordered pairs can represent nonlinear functions as well as linear functions. Students justify their reasoning using the definition of a function and characteristics from different representations of the function. If the students are exposed to the vertical line test, the students are expected to explain why it works.

#### Checking for Understanding

Which of the following are functions? Explain your reasoning for each.



B. 

$x$	$y$
0	0
3	2
5	4
7	6
9	4

C.  $(-5,2), (3,-5), (-3,2), (0,6), (-3,-2), (-5,6)$

### OCS Priority Standard: **NC.8.F.2**

**Define, evaluate, and compare functions.**

**NC.8.F.2** Compare properties of two linear functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

#### Clarification

This standard focuses on comparing linear functions that may be represented in different ways.

Keeping consistent with other function standards:

- Linear functions represented algebraically must be given in slope-intercept form.
- Functions represented with a verbal description must be able to be written directly into slope-intercept form.

#### Checking for Understanding

Sandra is looking to buy decorative doughnuts for a party. She is considering two local bakers. She looks up two bakers online to find how much each one charges. Here are the advertisements she saw online.

- a) What is the rate of change and the boxing fee for each doughnut shop?
- b) If the total cost was Sandra's biggest concern, who should she buy from? Justify your answer mathematically.

Students may convert functions to different representations to assist in their comparison.

Students describe the comparison of linear functions qualitatively (with words) and quantitatively (with numbers) by discussing and analyzing:

- Intercepts
- Rates of change
- Points of intersection
- Other points important to the context

**Ms. Spellings Donut Shop**

Try out this week's special!  
All decorative donuts are on sale for \$1.62 each with an \$8 boxing fee!

**Phil'd Up Doughnuts**  
*Boxing fee included in price!*

Decorative Doughnuts	Price
2	\$9
4	\$13
6	\$17
8	\$21
10	\$25
12	\$29

**Supporting Standard: NC.8.F.3**

**Define, evaluate, and compare functions**  
**NC.8.F.3** Identify linear functions from tables, equations, and graphs.

**Clarification**

Students identify a linear function from a variety of representations, including tables, equations and graphs. As with all functions, students should view a linear function as describing a relationship between quantities. A linear function is unique from other functions because of the characteristics of its relationships, namely having a constant additive rate of change.

Identifying a linear function comes from considering the rate of change between points that are a part of the function. Students build an understanding of linear functions by:

- Knowing that the rate of change of the function is the ratio of change in the output to the change in the input.
- Finding the rate of change between all points and showing that these rates of change are equivalent.
- Finding the rate of change between equally spaced points and showing that these rates are the same and form an additive pattern for each quantity (constant additive rate of change).

**For example:** Compare the function  $y = \frac{1}{2}x - 3$  with the function  $y = x^2 + 1$  to determine if the functions represent a linear relationship.

**Checking for Understanding**

Janice and Kim noticed that both proportional relationships and linear functions form a straight line when graphed. Janice claims that all linear functions are also proportional relationships. Kim disagreed and tells Janice that she has it backwards, that all proportional relationships are linear functions.

- Who is correct? Explain.
- A counterexample is when a specific example is given that disproves a claim. Create a counterexample to disprove the false claim. Explain how your example disproves the claim.

Devon is looking at the following table. He is supposed to determine if the table represents a linear function. Devon believes that it is a linear function because the numbers in the y column form a pattern of decreasing by 5 from one row to the next. However, this table does not represent a linear function.

x	y
-3	0
-1	-5
0	-10
1	-15

Sample answer: Using a table to compare.

$y = \frac{1}{2}x - 3$		$y = x^2 + 1$	
$x$	$y$	$x$	$y$
0	-3	0	1
2	-2	3	10
4	-1	6	37
6	0	9	82

In the first function,  $y = \frac{1}{2}x - 3$ , the  $x$ 's increase by 2, the  $y$ 's increase by the constant pattern of +1. This gives a constant additive rate of change of 1 unit of  $y$  for each 2 units of  $x$ . This function is a linear function.

*Note: As long as the  $x$ 's change by a constant additive pattern, the  $y$ 's will change at a constant additive pattern.*

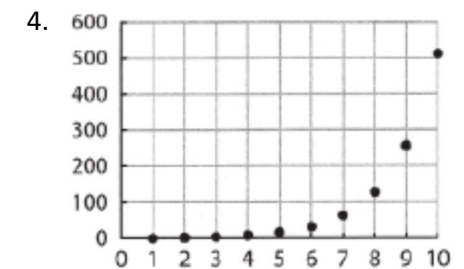
In the second function,  $y = x^2 + 1$ , as the  $x$ 's increase by 3, the  $y$ 's increase but not at a constant additive pattern. This function is nonlinear.

Students develop ways to see these patterns in the rate of change in both tables and graphs and understand how a constant additive rate of change would be represented in a linear equation. Students may see other patterns in the rates of change as other families of functions have different patterns. In 8<sup>th</sup> grade, students identify all mathematical functions that are not linear as nonlinear functions.

- Explain to Devon why the table does not represent a linear function.
- Describe how you could change the values in the table so that it would represent a linear function.

Determine if the functions listed below are linear or non-linear. Explain your reasoning.

- $y = -2x^2 + 3$
- $y = 0.25 + 0.5(x - 2)$
- $y = x(3 - x) + 1$



### OCS Priority Standard: NC.8.F.4

**Use functions to model relationships between quantities.**

**NC.8.F.4** Analyze functions that model linear relationships.

- Understand that a linear relationship can be generalized by  $y = mx + b$ .
- Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two  $(x, y)$  values or a graph.
- Construct a graph of a linear relationship given an equation in slope-intercept form.
- Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and  $y$ -intercept of its graph or a table of values.

**Clarification**

**Checking for Understanding**

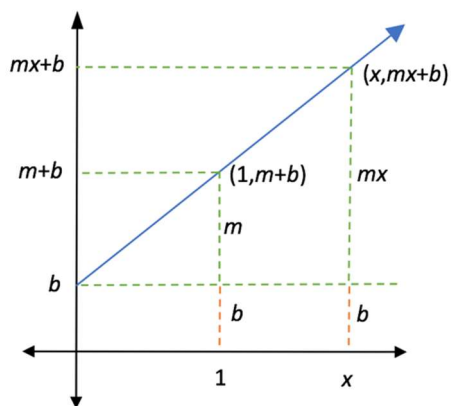
Analyzing linear functions is a major focus for 8<sup>th</sup> grade. In previous grades, students have worked with proportional relationships, which are a subset of linear functions. This means that some of the properties of a proportional relationship can be generalized to linear functions, while some properties are specific to proportional relationships.

**Understand that a linear relationship can be generalized by  $y = mx + b$ .**

In previous grades, students created tables, graphs, and equations from proportional relationships. Students can build from this previous knowledge to the larger generalization of linear functions.

In 7<sup>th</sup> grade, students learned that in a proportional relationship, the  $y$ -coordinate when  $x = 1$  is the unit rate and built an understanding of the equation of a proportional relationship (see NC.7.RP.2).

$x$ (input)	$y$ (output)
0	$b$
1	$m + b$
2	$m \cdot 2 + b$
3	$m \cdot 3 + b$
...	...
$x$	$m \cdot x + b$



In 8<sup>th</sup> grade, students build on this understanding to develop the linear equation in slope-intercept form. The initial small triangle starts with sides of 1 and  $m$ . As the triangle is scaled by a factor of  $x$ , the sides of the new triangle become  $x$  and  $mx$ . However, in order to find the coordinate that is on the line, the  $y$ -coordinate must

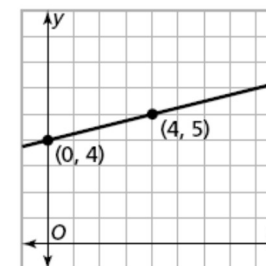
be increased by  $b$ , since the triangle is starting off the  $x$ -axis. This gives the coordinate of  $(x, mx + b)$ . This means that when the input is  $x$ , the output is  $mx + b$ , producing the function,  $y = mx + b$ . The same generalization can be seen in the table.

In 8<sup>th</sup> grade, the only required form of a linear equation is slope-intercept form. Students will not be asked to work with other forms of a linear equation or be asked to change the other forms to slope-intercept form.

Write an equation that models the linear relationship in the table.

$x$	$y$
-2	8
0	2
1	-1

Write an equation that models the linear relationship in the graph.



Write an equation for the line that has a rate of change of  $\frac{1}{2}$  and passes through the point  $(-2, 5)$ .

The company charges \$45 a day for the car as well as charging a one-time \$25 fee for the car's navigation system (GPS). Write a function for the cost in dollars,  $c$ , and the number of days,  $d$ , the car was rented.

**Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two (x, y) values or a graph.**

Students find the rate of change between two points. These two points may be given in a table, on the coordinate plane, or as ordered pairs. Students will also find the initial value in a linear function represented as an equation, table or graph. Having found the rate of change and the initial value, students should be able to write a linear function that models the situation given.

This standard does not imply that students should memorize the slope formula or use a purely algebraic approach to find an equation. Students should be able to use patterns and other representation to find the initial value and rate of change.

**Construct a graph of a linear relationship given an equation in slope-intercept form.**

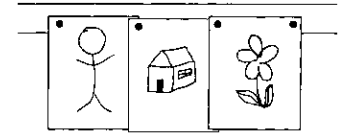
Student create the graphical representation of linear function, given a linear equation in slope-intercept form, using the initial value and rate of change. Students will choose an appropriate scale to graph a linear function.

**Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and y-intercept of its graph or a table of values.**

Students give meaning to the rate of change and the initial value of a linear function based on a context. The linear function can be given in a variety of representations, including verbal descriptions, tables, graphs, and equations.

Students use terms such as slope and y-intercept to describe a graphical representation of a linear function and correlate their meaning to the rate of change and initial value, where the input is 0. Students should use the units from a context appropriately in their description of rate of change and initial value.

Children's pictures are hung in a line as seen in the picture. Pictures that are hung next to each other share a tack.



- Describe the rate of change based on this context.
- How many tacks would be needed for 28 pictures?

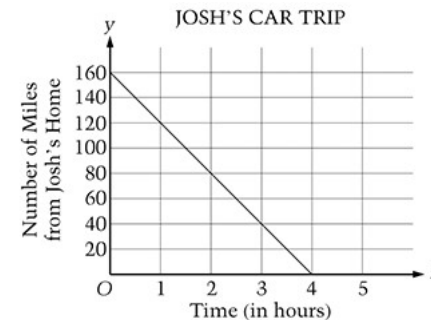
Adapted from NAEP – Released Item (1992) **Question ID: 1992-8M7 #8 M045201**

A leaf falls 18 feet from a branch to the ground at a rate of 5 feet every 2 seconds. Determine whether each statement about the leaf is true. If it is false, change the statement so that it is true.

Statement	T/F
The initial height of the leaf is 18 feet.	
The leaf falls at a rate of $2/5$ foot every 1 second.	
The leaf is 3 feet above the ground after 6 seconds.	

Adapted from SBAC Mathematics Practice Test Scoring Guide Grade 8 p. 11

The linear graph below describes Josh's car trip from his grandmother's home directly to his home.



- a) What is the distance from Josh’s grandmother’s home to his home?
  - b) How long did it take for Josh to make the trip?
  - c) What was Josh’s average speed for the trip? Explain how you know.
- Adapted from NAEP – Released Item (2011) Question ID: 2011-8M8 #15 M169901

**Supporting Standard: NC.8.F.5**

**Use functions to model relationships between quantities.**  
**NC.8.F.5** Qualitatively analyze the functional relationship between two quantities.

- Analyze a graph determining where the function is increasing or decreasing; linear or non-linear.
- Sketch a graph that exhibits the qualitative features of a real-world function.

**Clarification**

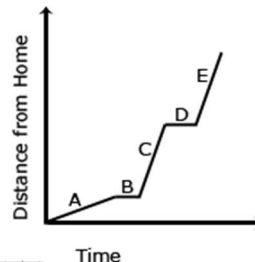
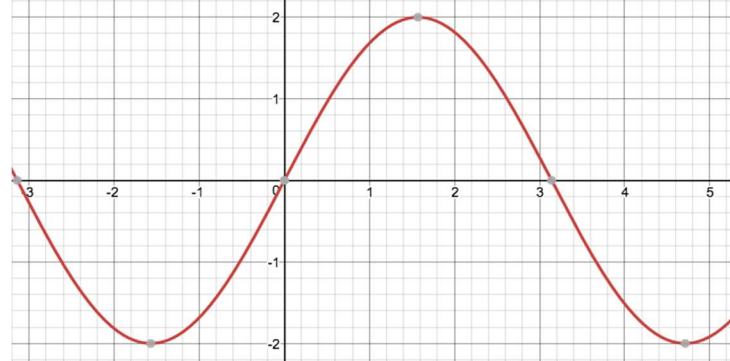
Students focus on describing the characteristics of linear and nonlinear real-world functions.

**Analyze a graph determining where the function is increasing or decreasing; linear or non-linear.**  
 Students describe specific sections of a graph over which the output is increasing, decreasing or remaining the same. Students describe which sections of the graph are linear and which are non-linear. Students use verbal descriptions in terms of the independent variable to define sections of a graph. For example, a student may say, “The function increases when x is between 3 and 7,” or “The average temperature rose between March and July.” Students are not expected to use compound inequalities in 8<sup>th</sup> grade when describing sections of a graph. Students support their claims with mathematical reasoning.

**Sketch a graph that exhibits the qualitative features of a real-world function.**  
 Students sketch a graph based on the information and context provided. These graphs may be composed of sections of different linear relationships with corresponding rates of change. Information will be provided giving either the location of the boundary for each section or the duration in which a particular rate of change should be applied.

**Checking for Understanding**

The graph shows John’s trip to school. He walks to Sam’s house and, together, they ride a bus to school. The bus stops once before arriving at school. Describe how each part A – E of the graph relates to the story.

Looking at this graph, describe:

- a) Where the graph is increasing,
- b) Where the graph is decreasing,
- c) Where the graph is not increasing or decreasing,
- d) Any areas that appear to be linear.

# Unit 5

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## Integer Exponents: Properties and Scientific Notation

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$$(x^3)^2(x^4)^3$$

$$(x^3)^2(x^4)^3$$

$$= x^6 \cdot x^{12}$$

$$= x^{18}$$



## Unit 5: Integer Exponents: Properties and Scientific Notation

Source: NCSCOS 6-8 Mathematics. Retrieved from: <https://files.nc.gov/dpi/documents/curriculum/mathematics/scos/current/6-8.pdf>

<i>OCS Priority Standard(s):</i>	<i>Supporting Standard(s):</i>
<p><b>NC.8.EE.1</b> Develop and apply the properties of integer exponents to generate equivalent numerical expressions.</p> <p><b>NC.8.EE.4</b> Perform multiplication and division with numbers expressed in scientific notation to solve real-world problems, including problems where both decimal and scientific notation are used.</p>	<p><b>NC.8.EE.3</b> Use numbers expressed in scientific notation to estimate very large or very small quantities and to express how many times as much one is than the other.</p>

## Unit 5 Unpacking

Source: NC DPI 8<sup>th</sup> Grade Math Unpacking Document Revised June 2022. Retrieved from <https://www.dpi.nc.gov/nc-8th-grade-math-unpacking-rev-june-2022>

### OCS Priority Standard: *NC.8.EE.1*

#### *Work with radicals and integer exponents.*

**NC.8.EE.1** Develop and apply the properties of integer exponents to generate equivalent numerical expressions.

Clarification	Checking for Understanding
<p>Students first worked with whole number exponents in 6<sup>th</sup> grade. At this grade level, students will build upon that knowledge to understand the properties of integer exponents and numerical bases and patterns of repeated multiplication and division. Students use their understanding of exponents as repeated multiplication to develop and create equivalent expressions and justify the following properties:</p> <ul style="list-style-type: none"> <li>• <math>5^3 \cdot 5^4 = 5^{3+4} = 5^7</math></li> <li>• <math>\frac{5^3}{5^4} = 5^{3-4} = 5^{-1} = \frac{1}{5}</math></li> <li>• <math>(5^3)^4 = 5^{3 \cdot 4} = 5^{12}</math></li> <li>• <math>5^3 \cdot 2^3 = (5 \cdot 2)^3 = (10)^3</math></li> <li>• <math>5^0 = 1</math></li> <li>• <math>5^{-3} = \frac{1}{5^3} = \frac{1}{125}</math></li> </ul> <p>Eighth grade is the first time students raise negative numbers to a power. Students recognize that negative numbers raised to an even power produce different products when parenthesis are used. For example, <math>-4^2</math> and <math>(-4)^2</math> have products of <math>-16</math> and <math>16</math> respectively.</p> <p>Students are not expected to know the names of the properties of exponents.</p>	<p>Rewrite the following expressions so that each expression does not contain an exponent.</p> <div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> <p>a) <math>\frac{2^3}{5^2}</math></p> <p>c) <math>6^0</math></p> <p>e) <math>(3^2)(3^4)</math></p> <p>g) <math>-\left(\frac{2}{3}\right)^2</math></p> <p>i) <math>\frac{(3^2)^4}{(3^2)(3^3)}</math></p> </div> <div style="width: 45%;"> <p>b) <math>\frac{2^2}{2^6}</math></p> <p>d) <math>\frac{3^{-2}}{2^4}</math></p> <p>f) <math>(4^3)^2</math></p> <p>h) <math>\left(-\frac{5}{4}\right)^2 \cdot \left(\frac{2}{5}\right)^{-3}</math></p> <p>j) <math>12^7 \cdot 12^{-7}</math></p> </div> </div>
<h3>Supporting Standard: <i>NC.8.EE.3</i></h3>	
<h4><i>Work with radicals and integer exponents.</i></h4>	
<p><b>NC.8.EE.3</b> Use numbers expressed in scientific notation to estimate very large or very small quantities and to express how many times as much one is than the other.</p>	
Clarification	Checking for Understanding

<p>Students use their knowledge of the base ten number system and exponents to rewrite numbers using scientific notation. Students interpret scientific notation generated when using technology. Students compare numbers written in scientific notation and express the multiplicative relationship between the numbers.</p> <p><b>For example:</b> Which of the following represents a larger number?</p> <p>a) <math>1.5 \times 10^9</math> b) <math>7.5 \times 10^7</math></p> <p><i>Solution: <math>1.5 \times 10^9</math> is the larger number</i></p> <p>For your answer, how many times larger is your answer than the smaller number?</p> <p><i>Solution: <math>1.5 \times 10^9</math> is 20 times larger than <math>7.5 \times 10^7</math></i></p> <p><i>Notice that 1.5 is .2 times 7.5. Looking at just the 10s, <math>10^9</math> is 100 times larger than <math>10^7</math>.</i></p> <p><i><math>(0.2)(100) = 20</math></i></p> <p><i>Students should see more than just a division problem but should see the multiplicative relationships that are unique to scientific notation.</i></p>	<p>Write the following into scientific notation:</p> <p>a) The distance between the sun and the Earth is 93,000,000 miles. c) A type of fairyfly is the smallest known flying insect and is only 0.0059 inches long.</p> <p>b) The distance between the sun and Neptune is 2,795,000,000 miles. d) An average bacterium is about 0.00004 inches long.</p> <p>Use the information from above to answer the following questions:</p> <p>e) In astronomy, the distance between the sun and the Earth is known as 1 AU, or astronomical unit. Measured in AUs, what is the distance between the sun and Neptune? f) If average sized bacteria were placed in a straight line, how many of bacteria would be needed to equal the length of the smallest known flying insect, a fairyfly?</p>
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**OCS Priority Standard: NC.8.EE.4**

**Work with radicals and integer exponents.**  
**NC.8.EE.4** Perform multiplication and division with numbers expressed in scientific notation to solve real-world problems, including problems where both decimal and scientific notation are used.

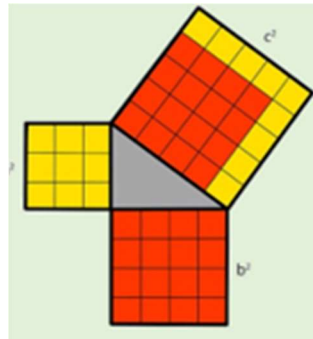
<b>Clarification</b>	<b>Checking for Understanding</b>
<p>Students use the laws of exponents to multiply and divide expressions containing numbers written in scientific and decimal notation to solve real-world problems.</p>	<p>Write the answer to the following in both scientific and decimal notation.</p> <p>a) Patrice works at a museum giving tours. Patrice would like to know how many words she speaks in a year giving tours at her job. The average person speaks about 150 words per minute. Patrice led tours that were 25 minutes long, 6 times per day. About how many words would Patrice have spoken in a year?</p> <p>b) Jensen is building a snow fort. Each block in the fort weighs about 1 kilogram. Jensen hopes to make about 40 blocks for the fort. If a snowflake weighs about <math>3 \times 10^{-3}</math> grams, approximately how many snowflakes will be in the fort?</p>

# Unit 6

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## Real Numbers: Rational Numbers, Irrational Numbers, and the Pythagorean Theorem

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# Unit 6: Real Numbers: Rational Numbers, Irrational Numbers, and the Pythagorean Theorem

Source: NCSCOS 6-8 Mathematics. Retrieved from: <https://files.nc.gov/dpi/documents/curriculum/mathematics/scos/current/6-8.pdf>

<i>OCS Priority Standard(s):</i>	<i>Supporting Standard(s):</i>
<p><b>NC.8.NS.2</b> Use rational approximations of irrational numbers to compare the size of irrational numbers and locate them approximately on a number line. Estimate the value of expressions involving:</p> <ul style="list-style-type: none"><li>• Square roots and cube roots to the tenths.</li><li>• <math>\pi</math> to the hundredths.</li></ul> <p><b>NC.8.G.7</b> Apply the Pythagorean Theorem and its converse to solve real-world and mathematical problems.</p> <p><b>NC.8.G.9</b> Understand how the formulas for the volumes of cones, cylinders, and spheres are related and use the relationship to solve real-world and mathematical problems.</p>	<p><b>NC.8.NS.1</b> Understand that every number has a decimal expansion. Building upon the definition of a rational number, know that an irrational number is defined as a non-repeating, non-terminating decimal.</p> <p><b>NC.8.EE.2</b> Use square root and cube root symbols to:</p> <ul style="list-style-type: none"><li>• Represent solutions to equations of the form <math>x^2 = p</math> and <math>x^3 = p</math>, where <math>p</math> is a positive rational number.</li><li>• Evaluate square roots of perfect squares and cube roots of perfect cubes for positive numbers less than or equal to 400.</li></ul> <p><b>NC.8.G.6</b> Explain the Pythagorean Theorem and its converse.</p> <p><b>NC.8.G.8</b> Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.</p>

## Unit 6 Unpacking

Source: NC DPI 8<sup>th</sup> Grade Math Unpacking Document Revised June 2022. Retrieved from <https://www.dpi.nc.gov/nc-8th-grade-math-unpacking-rev-june-2022>

### Supporting Standard: **NC.8.NS.1**

**Know that there are numbers that are not rational and approximate them by rational numbers.**

**NC.8.NS.1** Understand that every number has a decimal expansion. Building upon the definition of a rational number, know that an irrational number is defined as a non-repeating, non-terminating decimal.

Clarification	Checking for Understanding
<p>In 6<sup>th</sup> grade students were introduced to integers and rational numbers. In 7<sup>th</sup> grade, students formalized the definition of rational numbers. Students build on this knowledge to complete their understanding of the Real Number System by recognizing irrational numbers and their relationship to rational numbers. Students understand that an irrational number, when represented as a decimal is non-repeating and non-terminating and that irrational numbers cannot be written as a rational number. It is important for students to understand that distinction between fractional form and a rational number, as irrational numbers are often written in fractional form. For example, <math>\frac{3\pi}{4}</math>, is an irrational number written in fractional form. Students are able to identify irrational numbers.</p>	<p>Create a graphic organizer to show the relationships within the real number system, including natural numbers, whole numbers, integers, rational numbers and irrational numbers. Include examples that are exclusively within each type of number.</p>

### OCS Priority Standard: **NC.8.NS.2**

**Know that there are numbers that are not rational and approximate them by rational numbers.**

**NC.8.NS.2** Use rational approximations of irrational numbers to compare the size of irrational numbers and locate them approximately on a number line.

Estimate the value of expressions involving:

- Square roots and cube roots to the tenths.
- $\pi$  to the hundredths.

Clarification	Checking for Understanding
<p>Students estimate the value of an irrational number and use that estimate to compare an irrational number to other numbers and to place irrational numbers on a number line.</p>	<p>Graph the following on a number line: <math>\frac{3}{2}, \sqrt{2}, 1.\bar{3}, -\frac{7}{8}, -\frac{\sqrt{3}}{2}</math></p> <p>Estimate the following expressions to the tenths.</p>



**Understand and apply the Pythagorean Theorem.**

**NC.8.G.6** Explain the Pythagorean Theorem and its converse.

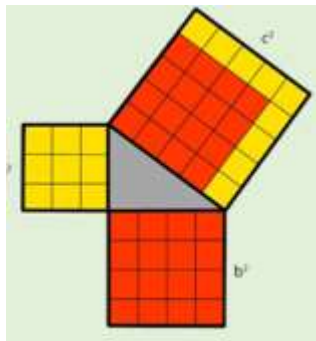
**Clarification**

This focus of this standard is on examining different models of the Pythagorean Theorem and its converse and showing understanding of how the models support the theorem. Students are NOT expected to prove the Pythagorean Theorem. However, students should be able to explain a proof provided that is within the scope of middle school mathematics.

**Pythagorean Theorem:** The sum of the areas of the two squares on the legs ( $a$  and  $b$ ) equals the area of the square on the hypotenuse ( $c$ ).

Students can explain the Pythagorean Theorem using models. Students understand the connection between the Pythagorean Theorem and area.

**For example,** students understand that the area of the squares that form the legs is equivalent to the area of the square created by the hypotenuse. Students can explain verbally or rearrange the area of tiles of the smaller square to create the larger square. There are a variety of ways that students can explain understanding.

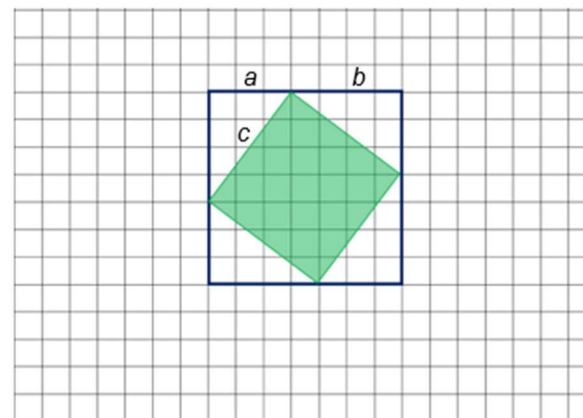


**The Converse of the Pythagorean Theorem:** If a triangle has sides of length  $a$ ,  $b$ , and  $c$  and if  $a^2 + b^2 = c^2$  then the angle opposite the side of length  $c$  is a right angle.

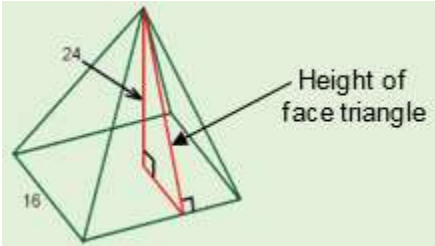
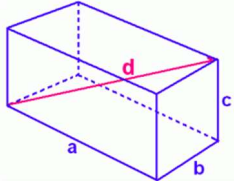
Students are able to determine whether a triangle is a right triangle by examining the relationship between the sides of a triangle. This standard should build from work with triangles in the 7<sup>th</sup> grade, where students

**Checking for Understanding**

How does the following diagram support the Pythagorean Theorem? Explain.

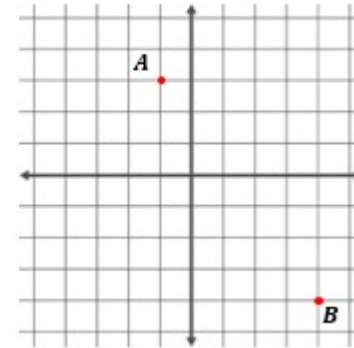
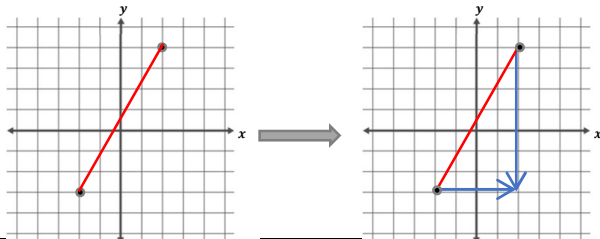




determined if a triangle exists based on the relationship between the sides (NC.7.G.2).	
<b>OCS Priority Standard: NC.8.G.7</b>	
<b>Understand and apply the Pythagorean Theorem.</b>	
<b>NC.8.G.7</b> Apply the Pythagorean Theorem and its converse to solve real-world and mathematical problems.	
<b>Clarification</b>	<b>Checking for Understanding</b>
<p>This standard focuses on the application of the Pythagorean Theorem. Students will apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in both two and three-dimensional objects.</p> <p>For example, The Pythagorean Theorem can be used to find the height of the triangles on the faces of the square pyramid, so that the surface area of the pyramid can be calculated.</p> 	<p>The Irrational Club wants to build a tree house. They have a 9-foot ladder that must be propped diagonally against the tree. If the base of the ladder is 5 feet from the bottom of the tree, how high will the tree house be off the ground?</p> <p>Find the length of <math>d</math> in the figure to the right if <math>a = 8</math> in., <math>b = 3</math> in. and <math>c = 4</math> in.</p>  <p>The distance from Jonestown to Maryville is 180 miles, the distance from Maryville to Elm City is 300 miles, and the distance from Elm City to Jonestown is 240 miles. Do the three towns form a right triangle? Why or why not?</p>
<b>Supporting Standard: NC.8.G.8</b>	
<b>Understand and apply the Pythagorean Theorem.</b>	
<b>NC.8.G.8</b> Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	
<b>Clarification</b>	<b>Checking for Understanding</b>
<p>This standard uses the Pythagorean Theorem as an application to find the distance between two non-vertical and non-horizontal points on a line. Students build on work from 6<sup>th</sup> grade where they found distances between vertical and horizontal lines in the coordinate plane. Given two points, students are able to draw a line connecting the points and create a right triangle using the points. Students understand that the line segment between the two points is the length of the hypotenuse of the right</p>	<p>Draw the right triangle where <math>\overline{AB}</math> is the hypotenuse? What is the length of <math>\overline{AB}</math>?</p>

triangle that can be formed. They also understand that the third vertex of the triangle is the intersection of the vertical and horizontal lines.

Note: The distance formula is NOT an expectation.



**OCS Priority Standard: NC.8.G.9**

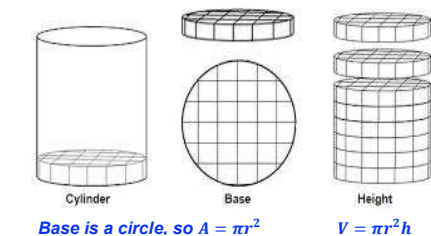
*Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.*

**NC.8.G.9** Understand how the formulas for the volumes of cones, cylinders, and spheres are related and use the relationship to solve real-world and mathematical problems.

**Clarification**

This standard focuses on the volume formulas for 3-dimensional shapes related to circles (cones, cylinders and spheres). Students have already worked with volume of cubes and rectangular prisms. They apply these same understandings beginning with cylinders.

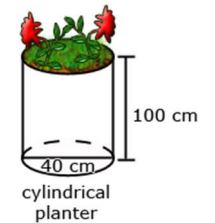
**For a cylinder**, students understand that the volume represents a “stack” or layers of area of the base. So, they apply the generalized formula  $V = Bh$ , where  $B$  represents the area of the base and  $h$  represents the height of the stack.

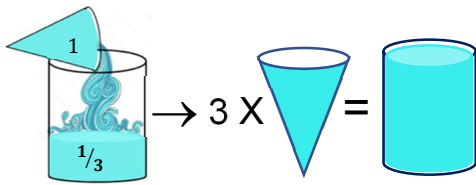


<https://www.texasgateway.org/resource/determining-volume-cones-and-cylinders>

**Checking for Understanding**

James wanted to plant pansies in his new planter. He wondered how much potting soil he should buy to fill it. Use the measurements in the diagram below to determine the planter’s volume.



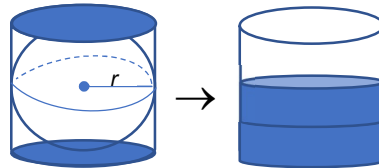


**For a cone**, students understand that the volume of a cylinder is 3 times the volume of a cone

having the same base area and height OR that the volume of a cone is  $\frac{1}{3}$  the volume of a cylinder having the same base area and height.

Therefore,  $V_{cone} = \frac{1}{3}\pi r^2 h$  OR  $V_{cone} = \frac{\pi r^2 h}{3}$ .

**A sphere** can be enclosed within a cylinder, which has the same radius and height of the sphere. If the sphere is flattened, it will fill  $\frac{2}{3}$  of the cylinder.



**Note:** The height of the cylinder is twice the radius of the sphere.

Based on this model, students

understand that the volume of a sphere is  $\frac{2}{3}$  the volume of a cylinder with the same radius and height. The height of the cylinder is the same as the diameter of the sphere or  $2r$ .

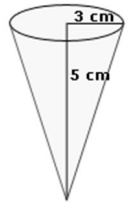
We can derive the volume formula for a sphere:

Beginning with the *cylinder* formula  $V = \pi r^2 h$ ;

Multiply the *cylinder* formula by  $\frac{2}{3}$   $V = \frac{2}{3}\pi r^2 h$ ;

Substitute  $2r$  for the height  $V = \frac{2}{3}\pi r^2 (2r)$ ; Therefore,  $V_{sphere} = \frac{4}{3}\pi r^3$ .

How much yogurt is needed to fill the cone to the right? Express your answers in terms of Pi.



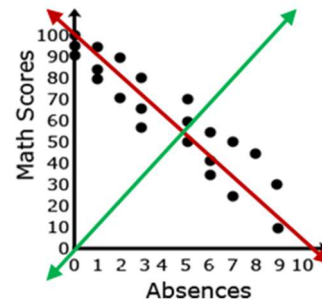
Approximately, how much air would be needed to fill a soccer ball with a radius of 14 cm?

# Unit 7

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## Statistics: Two-Variable Data and Fitting a Linear Model

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## Unit 7: Statistics: Two-Variable Data and Fitting a Linear Model

Source: NCSCOS 6-8 Mathematics. Retrieved from: <https://files.nc.gov/dpi/documents/curriculum/mathematics/scos/current/6-8.pdf>

<i>OCS Priority Standard(s):</i>	<i>Supporting Standard(s):</i>
<p><b>NC.8.SP.2</b> Model the relationship between bivariate quantitative data to:</p> <ul style="list-style-type: none"><li>• Informally fit a straight line for a scatter plot that suggests a linear association.</li><li>• Informally assess the model fit by judging the closeness of the data points to the line.</li></ul> <p><b>NC.8.SP.3</b> Use the equation of a linear model to solve problems in the context of bivariate quantitative data, interpreting the slope and y-intercept.</p> <p><b>NC.8.F.4</b> Analyze functions that model linear relationships.</p> <ul style="list-style-type: none"><li>• Understand that a linear relationship can be generalized by <math>y = mx + b</math>.</li><li>• Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two <math>(x, y)</math> values or a graph.</li><li>• Construct a graph of a linear relationship given an equation in slope-intercept form.</li><li>• Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and y-intercept of its graph or a table of values.</li></ul>	<p><b>NC.8.SP.1</b> Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Investigate and describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.</p> <p><b>NC.8.SP.4</b> Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.</p> <ul style="list-style-type: none"><li>• Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.</li><li>• Use relative frequencies calculated for rows or columns to describe possible association between the two variables.</li></ul>

# Unit 7 Unpacking

Source: NC DPI 8<sup>th</sup> Grade Math Unpacking Document Revised June 2022. Retrieved from <https://www.dpi.nc.gov/nc-8th-grade-math-unpacking-rev-june-2022>

## Supporting Standard: *NC.8.SP.1*

### *Investigate patterns of association in bivariate data.*

**NC.8.SP.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Investigate and describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

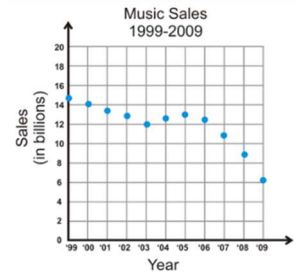
### Clarification

This standard focuses on graphing and interpreting two-variable (bivariate) qualitative/measurement data. Students will build on previous experiences, such as graphing ordered pairs on the coordinate plane and analyzing the relationships between quantitative variables.

Students will extend their understanding by investigating patterns of association between the variables. This includes recognizing associations that are both linear and non-linear, which differ from no association. Students also recognize when clusters and/or gaps are present in the data. Finally, students can identify points that are deviations from associated data noting them as outliers.

### Checking for Understanding

The music sales (in billions) for the years 1999-2009 is displayed in the scatter plot. What are the pattern(s) of association for the music sales in the decade between 1999 and 2009? Explain.

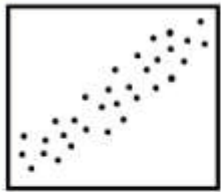


Source: CNN

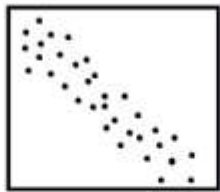
Data for 10 students' Math and Science scores are provided in the chart. Describe what the data says about the association between the Math and Science scores.

Student #	1	2	3	4	5	6	7	8	9	10
Math Score	64	50	85	34	56	24	72	63	42	93
Science Score	68	70	83	33	60	27	74	63	40	96

**Patterns of Association in Bivariate Data**



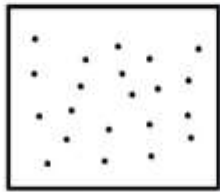
positive linear association



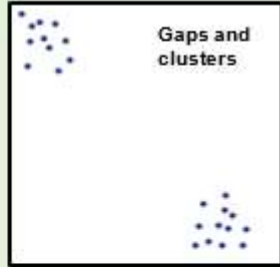
negative linear association



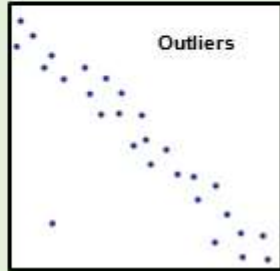
nonlinear association



no association



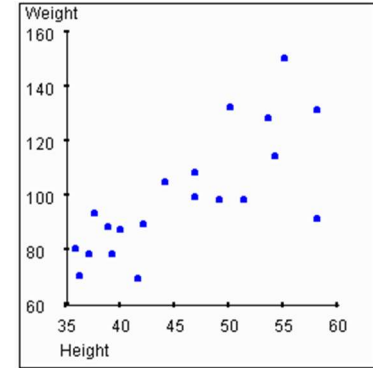
Gaps and clusters



Outliers

Students can construct graphs by hand, using calculators or through the use of computer software programs (i.e. Excel, GeoGebra, CODAP, etc.). Online tools, such as those at the [National Center for Educational Statistics](http://www.illustrativemathematics.org/HSIndex), can also be used to create a graph or generate data sets.

The scatter plot represents the height and weight of a class of a sample of individuals from a local doctor's office. Describe the association any associations in the data, noting whether there are clusters, gaps or outliers.



**OCS Priority Standard: NC.8.SP.2**

**Investigate patterns of association in bivariate data.**

**NC.8.SP.2** Model the relationship between bivariate quantitative data to:

- Informally fit a straight line for a scatter plot that suggests a linear association.
- Informally assess the model fit by judging the closeness of the data points to the line.

**Clarification**

This standard is a modeling standard. Students model linear relationships using graphs. Building on their understanding of statistical variation of univariate data, students extend this understanding to bivariate data.

Students understand that a straight line can represent a scatter plot with what appears to be a linear association. If a linear relationship is suspected,

**Checking for Understanding**

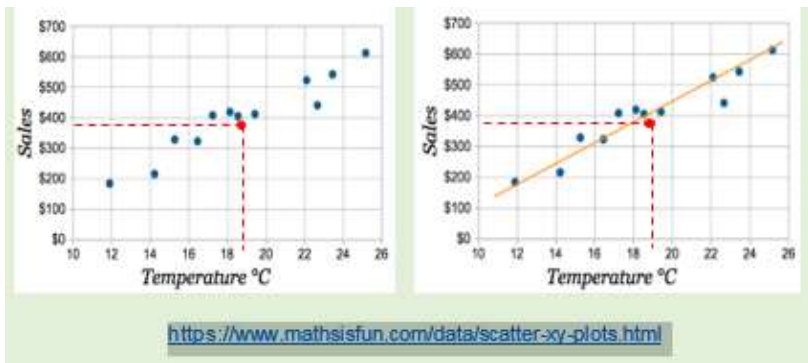
Given data from students' math scores and absences:

- Make a scatterplot.
- Draw a linear model paying attention to the closeness of the data points on either side of the line.

students draw the linear model that represents the *direction* of the association in the scatter plot minimizing the distance between the actual y-value and the predicted y-value (represented on the line) for each point.

The centroid (mean of x-values, mean of y-values) is a point on the estimated model line. Using this point as a pivot point can help students to determine the placement of the line. This is an informal understanding to assist students in the construction of the line. The line may or may not go through any or all of the data points. The use of linear regression is NOT expected.

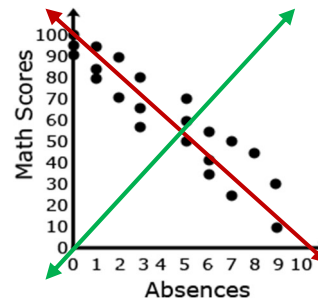
**For example,** the following graph represents the ice cream sales of a local ice cream shop versus the noon temperature for 12 days. The association appears to be positive meaning that as the temperature increased, the ice cream sales increased also. The red dot (Figure 1) represents the average temperature and sales (18.7, 402). The line is drawn to reflect the direction (positive) of the association and as close as possible to all data points in the scatterplot.



Furthermore, students notice any data values that fall outside the general pattern of associated data.

Absences	Math Scores
3	65
5	50
1	95
1	85
3	80
6	34
5	70
3	56
0	100
7	24
8	45
2	71
9	30
0	95
6	55
6	42
2	90
0	92
5	60
7	50
9	10
1	80

Given the scatter plot from students' math scores and absences, which line best models the association of the data? Explain.

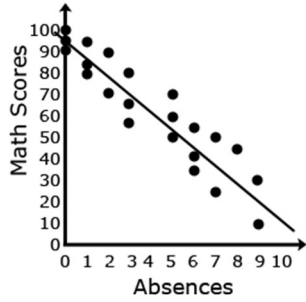


**OCS Priority Standard: NC.8.SP.3**

*Investigate patterns of association in bivariate data.*

**NC.8.SP.3** Use the equation of a linear model to solve problems in the context of bivariate quantitative data, interpreting the slope and y-intercept.



Clarification	Checking for Understanding
<p>This standard is a modeling standard. Students model linear relationships using linear equations. Students can interpret the coefficient (slope) and constant (<math>y</math>-intercept) of the equation in the context of the problem.</p> <p>This standard extends understandings from previous grade levels where students have graphed and created equations of quantitative relationships (NC.6.EE.9, NC.7.RP.2c).</p> <p>Students have also interpreted the meaning of points <math>(x, y)</math> and quantities (rates) within proportional relationships (NC.7.RP.2d). Additionally, students apply this same understanding to non-proportional relationships (NC.8.F.4) where all points are collinear. This standard is extended to work with scatter plots, where the points generally are non-collinear, and students have to select appropriate points to write the equation of the line.</p>	<p>Given the scatter plot and line for student math scores and absences, select two points to write the linear equation for the line. Interpret the slope and <math>y</math>-intercept in context of the problem.</p> 

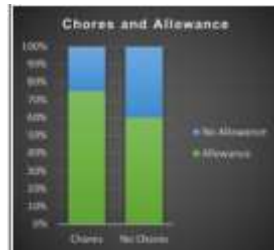
Supporting Standard: NC.8.SP.4
<p><b>Investigate patterns of association in bivariate data.</b></p> <p><b>NC.8.SP.4</b> Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.</p> <ul style="list-style-type: none"> <li>• Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.</li> <li>• Use relative frequencies calculated for rows or columns to describe possible association between the two variables.</li> </ul>

Clarification	Checking for Understanding																					
<p>This standard extends understandings of bivariate numerical data to bivariate <i>categorical</i> data. Students understand that a two-way table provides a way to organize data between two categorical variables.</p> <p>Students have experience with categorical data from elementary school and relative frequencies in 7<sup>th</sup> grade. They will extend this understanding to developing two-way tables of frequencies and relative frequencies.</p> <p>Students will examine patterns of association in categorical data by examining the relative frequencies in a two-way table. Students recognize that similar proportions indicate that there is no association indicating similarity between populations.</p>	<p>Kayla asked 10 students in her class whether they owned a dog, a cat or both. Complete the table below with the frequencies using the following relative frequencies:</p> <ul style="list-style-type: none"> <li>• 40% of the students own a dog</li> <li>• 30% of the students own a cat</li> <li>• 10% of the students own both</li> </ul> <table border="1" data-bbox="1564 1096 1984 1307"> <tr> <td colspan="2" rowspan="2"></td> <th colspan="2">Dog</th> <th rowspan="2">Totals</th> </tr> <tr> <th>Yes</th> <th>No</th> </tr> <tr> <th rowspan="2">Cat</th> <th>Yes</th> <td></td> <td></td> <td></td> </tr> <tr> <th>No</th> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <th>Totals</th> <td></td> <td></td> <td>10</td> </tr> </table> <p><i>Adapted from SBAC Released Items (Item #767, Grade 8)</i></p>			Dog		Totals	Yes	No	Cat	Yes				No					Totals			10
				Dog			Totals															
		Yes	No																			
Cat	Yes																					
	No																					
	Totals			10																		

**For example**, the tables below represent data from a survey of 25 students to determine if there is an association between allowance and chores. Since there is a difference in the relative frequencies, there appears to be an association between doing chores and receiving allowance.

	Allowance	No Allowance	Total
Chores	15	5	20
No Chores	3	2	5
Total	18	7	25

	Allowance	No Allowance
Chores	$\frac{15}{20} = .75$	$\frac{5}{20} = .25$
No Chores	$\frac{3}{5} = .60$	$\frac{2}{5} = .40$



The segmented bar graph is a visual display of the data. Since the green bar decreases for those not doing chores, there appears to be an association between doing chores and receiving allowance for this group of students. Additionally, not doing chores is associated with not receiving allowance.

*Students DO NOT need to create segmented bar graphs; this visual is used to aid in understanding.*

**OCS Priority Standard: NC.8.F.4**

**Use functions to model relationships between quantities.**

**NC.8.F.4** Analyze functions that model linear relationships.

- Understand that a linear relationship can be generalized by  $y = mx + b$ .

All the students at a middle school were asked to identify their favorite academic subject and whether they were in 7th grade or 8th grade. Here are the results:

Grade	English	History	Math/Science	Other	Total
7 <sup>th</sup> Grade	38	36	28	14	116
8 <sup>th</sup> Grade	47	45	72	18	182
Totals	85	81	100	32	298

Is there an association between favorite academic subject and grade for students at this school? Support your answer by calculating appropriate row relative frequencies using the given data.

[\*Illustrative Mathematics: What's Your Favorite Subject?\*](#)

- Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two  $(x, y)$  values or a graph.
- Construct a graph of a linear relationship given an equation in slope-intercept form.
- Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and  $y$ -intercept of its graph or a table of values.

### Clarification

Analyzing linear functions is a major focus for 8<sup>th</sup> grade. In previous grades, students have worked with proportional relationships, which are a subset of linear functions. This means that some of the properties of a proportional relationship can be generalized to linear functions, while some properties are specific to proportional relationships.

**Understand that a linear relationship can be generalized by  $y = mx + b$ .**

In previous grades, students created tables, graphs, and equations from proportional relationships. Students can build from this previous knowledge to the larger generalization of linear functions.

In 7<sup>th</sup> grade, students learned that in a proportional relationship, the  $y$ -coordinate when  $x = 1$  is the unit rate and built an understanding of the equation of a proportional relationship (see NC.7.RP.2).

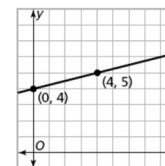
$x$ (input)	$y$ (output)
0	$b$
1	$m + b$
2	$m \cdot 2 + b$
3	$m \cdot 3 + b$
...	...
$x$	$m \cdot x + b$

### Checking for Understanding

Write an equation that models the linear relationship in the table.

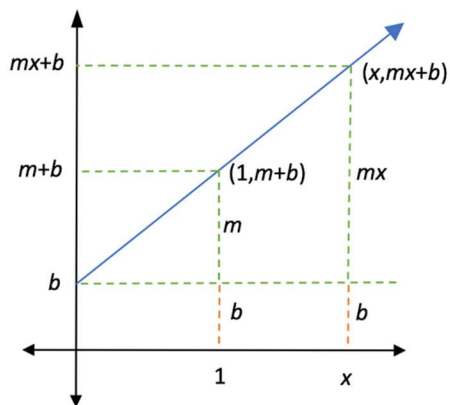
$x$	$y$
-2	8
0	2
1	-1

Write an equation that models the linear relationship in the graph.



Write an equation for the line that has a rate of change of  $\frac{1}{2}$  and passes through the point  $(-2, 5)$ .

The company charges \$45 a day for the car as well as charging a one-time \$25 fee for the car's navigation system (GPS). Write a function for the cost in dollars,  $c$ , and the number of days,  $d$ , the car was rented.



In 8<sup>th</sup> grade, students build on this understanding to develop the linear equation in slope-intercept form. The initial small triangle starts with sides of 1 and  $m$ . As the triangle is scaled by a factor of  $x$ , the sides of the new triangle become  $x$  and  $mx$ . However, in order to find the coordinate that is on the line, the  $y$ -coordinate must be increased by  $b$ , since the triangle is starting off the  $x$ -axis. This give the coordinate of

$(x, mx + b)$ . This means that when the input is  $x$ , the output is  $mx + b$ , producing the function,  $y = mx + b$ . The same generalization can be seen in the table.

In 8<sup>th</sup> grade, the only required form of a linear equation is slope-intercept form. Students will not be asked to work with other forms of a linear equation or be asked to change the other forms to slope-intercept form.

**Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two  $(x, y)$  values or a graph.**

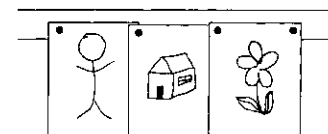
Students find the rate of change between two points. These two points may be given in a table, on the coordinate plane, or as ordered pairs.

Students will also find the initial value in a linear function represented as an equation, table or graph. Having found the rate of change and the initial value, students should be able to write a linear function that models the situation given.

This standard does not imply that students should memorize the slope formula or use a purely algebraic approach to find an equation. Students should be able to use patterns and other representation to find the initial value and rate of change.

**Construct a graph of a linear relationship given an equation in slope-intercept form.**

Children's pictures are hung in a line as seen in the picture. Pictures that are hung next to each other share a tack.



- Describe the rate of change based on this context.
- How many tacks would be needed for 28 pictures?

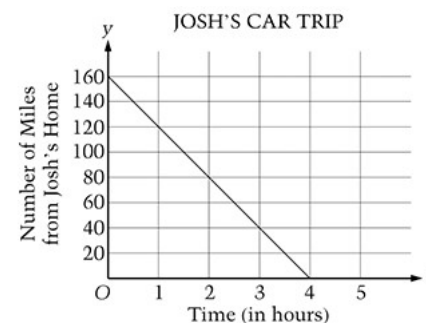
Adapted from NAEP – Released Item (1992) **Question ID:** 1992-8M7 #8 M045201

A leaf falls 18 feet from a branch to the ground at a rate of 5 feet every 2 seconds. Determine whether each statement about the leaf is true. If it is false, change the statement so that it is true.

Statement	T/F
The initial height of the leaf is 18 feet.	
The leaf falls at a rate of $\frac{2}{5}$ foot every 1 second.	
The leaf is 3 feet above the ground after 6 seconds.	

Adapted from SBAC Mathematics Practice Test Scoring Guide Grade 8 p. 11

The linear graph below describes Josh's car trip from his grandmother's home directly to his home.



Student create the graphical representation of linear function, given a linear equation in slope-intercept form, using the initial value and rate of change. Students will choose an appropriate scale to graph a linear function.

**Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and y-intercept of its graph or a table of values.**

Students give meaning to the rate of change and the initial value of a linear function based on a context. The linear function can be given in a variety of representations, including verbal descriptions, tables, graphs, and equations. Students use terms such as slope and y-intercept to describe a graphical representation of a linear function and correlate their meaning to the rate of change and initial value, where the input is 0. Students should use the units from a context appropriately in their description of rate of change and initial value.

- a) What is the distance from Josh's grandmother's home to his home?
- b) How long did it take for Josh to make the trip?
- c) What was Josh's average speed for the trip? Explain how you know.

Adapted from NAEP – Released Item (2011) Question ID: 2011-8M8 #15  
M169901



## HESS COGNITIVE RIGOR MATRIX (MATH-SCIENCE CRM): Applying Webb's Depth-of-Knowledge Levels to Bloom's Cognitive Process Dimensions

Revised Bloom's Taxonomy	Webb's DOK Level 1 Recall & Reproduction	Webb's DOK Level 2 Skills & Concepts	Webb's DOK Level 3 Strategic Thinking/Reasoning	Webb's DOK Level 4 Extended Thinking
<b>Remember</b> Retrieve knowledge from long-term memory, recognize, recall, locate, identify	<ul style="list-style-type: none"> <li>o Recall, observe, &amp; recognize facts, principles, properties</li> <li>o Recall/ identify conversions among representations or numbers (e.g., customary and metric measures)</li> </ul>	Use these Hess CRM curricular examples with most mathematics or science assignments or assessments.		
<b>Understand</b> Construct meaning, clarify, paraphrase, represent, translate, illustrate, give examples, classify, categorize, summarize, generalize, infer a logical conclusion, predict, compare/contrast, match like ideas, explain, construct models	<ul style="list-style-type: none"> <li>o Evaluate an expression</li> <li>o Locate points on a grid or number on number line</li> <li>o Solve a one-step problem</li> <li>o Represent math relationships in words, pictures, or symbols</li> <li>o Read, write, compare decimals in scientific notation</li> </ul>	<ul style="list-style-type: none"> <li>o Specify and explain relationships (e.g., non-examples/examples; cause-effect)</li> <li>o Make and record observations</li> <li>o Explain steps followed</li> <li>o Summarize results or concepts</li> <li>o Make basic inferences or logical predictions from data/observations</li> <li>o Use models/diagrams to represent or explain mathematical concepts</li> <li>o Make and explain estimates</li> </ul>	<ul style="list-style-type: none"> <li>o Use concepts to solve non-routine problems</li> <li>o Explain, generalize, or connect ideas using supporting evidence</li> <li>o Make and justify conjectures</li> <li>o Explain thinking/reasoning when more than one solution or approach is possible</li> <li>o Explain phenomena in terms of concepts</li> </ul>	<ul style="list-style-type: none"> <li>o Relate mathematical or scientific concepts to other content areas, other domains, or other concepts</li> <li>o Develop generalizations of the results obtained and the strategies used (from investigation or readings) and apply them to new problem situations</li> </ul>
<b>Apply</b> Carry out or use a procedure in a given situation; carry out (apply to a familiar task), or use (apply) to an unfamiliar task	<ul style="list-style-type: none"> <li>o Follow simple procedures (recipe-type directions)</li> <li>o Calculate, measure, apply a rule (e.g., rounding)</li> <li>o Apply algorithm or formula (e.g., area, perimeter)</li> <li>o Solve linear equations</li> <li>o Make conversions among representations or numbers, or within and between customary and metric measures</li> </ul>	<ul style="list-style-type: none"> <li>o Select a procedure according to criteria and perform it</li> <li>o Solve routine problem applying multiple concepts or decision points</li> <li>o Retrieve information from a table, graph, or figure and use it solve a problem requiring multiple steps</li> <li>o Translate between tables, graphs, words, and symbolic notations (e.g., graph data from a table)</li> <li>o Construct models given criteria</li> </ul>	<ul style="list-style-type: none"> <li>o Design investigation for a specific purpose or research question</li> <li>o Conduct a designed investigation</li> <li>o Use concepts to solve non-routine problems</li> <li>o Use &amp; show reasoning, planning, and evidence</li> <li>o Translate between problem &amp; symbolic notation when not a direct translation</li> </ul>	<ul style="list-style-type: none"> <li>o Select or devise approach among many alternatives to solve a problem</li> <li>o Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results</li> </ul>
<b>Analyze</b> Break into constituent parts, determine how parts relate, differentiate between relevant-irrelevant, distinguish, focus, select, organize, outline, find coherence, deconstruct	<ul style="list-style-type: none"> <li>o Retrieve information from a table or graph to answer a question</li> <li>o Identify whether specific information is contained in graphic representations (e.g., table, graph, T-chart, diagram)</li> <li>o Identify a pattern/trend</li> </ul>	<ul style="list-style-type: none"> <li>o Categorize, classify materials, data, figures based on characteristics</li> <li>o Organize or order data</li> <li>o Compare/ contrast figures or data</li> <li>o Select appropriate graph and organize &amp; display data</li> <li>o Interpret data from a simple graph</li> <li>o Extend a pattern</li> </ul>	<ul style="list-style-type: none"> <li>o Compare information within or across data sets or texts</li> <li>o Analyze and draw conclusions from data, citing evidence</li> <li>o Generalize a pattern</li> <li>o Interpret data from complex graph</li> <li>o Analyze similarities/differences between procedures or solutions</li> </ul>	<ul style="list-style-type: none"> <li>o Analyze multiple sources of evidence</li> <li>o Analyze complex/abstract themes</li> <li>o Gather, analyze, and evaluate information</li> </ul>
<b>Evaluate</b> Make judgments based on criteria, check, detect inconsistencies or fallacies, judge, critique	"UG" – unsubstantiated generalizations = stating an opinion without providing any support for it!		<ul style="list-style-type: none"> <li>o Cite evidence and develop a logical argument for concepts or solutions</li> <li>o Describe, compare, and contrast solution methods</li> <li>o Verify reasonableness of results</li> </ul>	<ul style="list-style-type: none"> <li>o Gather, analyze, &amp; evaluate information to draw conclusions</li> <li>o Apply understanding in a novel way, provide argument or justification for the application</li> </ul>
<b>Create</b> Reorganize elements into new patterns/structures, generate, hypothesize, design, plan, produce	<ul style="list-style-type: none"> <li>o Brainstorm ideas, concepts, or perspectives related to a topic</li> </ul>	<ul style="list-style-type: none"> <li>o Generate conjectures or hypotheses based on observations or prior knowledge and experience</li> </ul>	<ul style="list-style-type: none"> <li>o Synthesize information within one data set, source, or text</li> <li>o Formulate an original problem given a situation</li> <li>o Develop a scientific/mathematical model for a complex situation</li> </ul>	<ul style="list-style-type: none"> <li>o Synthesize information across multiple sources or texts</li> <li>o Design a mathematical model to inform and solve a practical or abstract situation</li> </ul>