NCDPI Unpacked Content
with
OCS Priority Standards Identified

NC Math 4

2022 Alignment

North Carolina Department of Public Instruction
INSTRUCTIONAL SUPPORT TOOLS
FOR ACHIEVING NEW STANDARDS

NC Math 4 ● Unpacked Contents
For the new Standard Course of Study that will be effective in all North Carolina schools in the 2020-21 School Year.

This document is designed to help North Carolina educators teach the **NC Math 4** Standard Course of Study. NCDPI staff are continually updating and improving these tools to better serve teachers and districts.

What is the purpose of this document?

The purpose of this document is to increase student achievement by ensuring educators understand the expectations of the new standards. This document may also be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. This document, along with on-going professional development, is one of many resources used to understand and teach the NC SCOS.

What is in the document?

This document includes a detailed clarification of each standard in the grade level along with a *sample* of questions or directions that may be used during the instructional sequence to determine whether students are meeting the learning objective outlined by the standard. These items are included to support classroom instruction and are not intended to reflect summative assessment items. The examples included may not fully address the scope of the standard. The document also includes a table of contents of the standards organized by domain with hyperlinks to assist in navigating the electronic version of this instructional support tool.

How do I send Feedback?
Link for: Feedback for NC's Unpacking Documents.

We will use your input to refine our unpacking of the standards. Thank You!

Just want the standards alone?
Link to: North Carolina Mathematics Standards

NC Math 4 Standards

Standards for Mathematical Practice

Number and Quantity

NC.M4.N.1 *Apply properties and operations with complex numbers.*
NC.M4.N.1.1
NC.M4.N.1.2

NC.M4.N.2 *Apply properties and operations with matrices and vectors.*
NC.M4.N.2.1
NC.M4.N.2.2

Algebra and Functions

NC.M4.AF.1 *Apply properties of function composition to build new functions from existing functions.*
NC.M4.AF.1.1
NC.M4.AF.1.2

NC.M4.AF.2 *Apply properties of trigonometry to solve problems.*
NC.M4.AF.2.1
NC.M4.AF.2.2
NC.M4.AF.2.3

NC.M4.AF.3 *Apply the properties and key features of logarithmic functions.*
NC.M4.AF.3.1
NC.M4.AF.3.2
NC.M4.AF.3.3

NC.M4.AF.4 *Understand the properties and key features of piecewise functions.*
NC.M4.AF.4.1
NC.M4.AF.4.2

NC.M4.AF.5 *Understand how to model functions with regression.*
NC.M4.AF.5.1
NC.M4.AF.5.2

Statistics and Probability

NC.M4.SP.1 *Create statistical investigations to make sense of real-world phenomena.*
NC.M4.SP.1.1
NC.M4.SP.1.2
NC.M4.SP.1.3
NC.M4.SP.1.4

NC.M4.SP.2 *Apply informal and formal statistical inference to make sense of, and make decisions in, meaningful real-world contexts.*
NC.M4.SP.2.1
NC.M4.SP.2.2
NC.M4.SP.2.3

NC.M4.SP.3 *Apply probability distributions in making decisions in uncertainty.*
NC.M4.SP.3.1
NC.M4.SP.3.2
NC.M4.SP.3.3
NC.M4.SP.3.4

Standards for Mathematical Practice

Practice	Explanation
Make sense of problems and persevere in solving them.	In NC Math 4, students solve real world problems using their knowledge of numbers, functions, and algebra. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?" Students also consider the reasonableness of intermediate results while applying processes to solve complex equations.
Reason abstractly and quantitatively.	In NC Math 4, students use algebraic, tabular, and graphical representations to reason about mathematical and real-world contexts. They examine patterns in their processes. Students contextualize to understand the meaning of the number or variable as related to the problem. They mathematize problem situations to manipulate symbolic representations by applying properties of operations.
Construct viable arguments and critique the reasoning of others.	In NC Math 4, students construct arguments using verbal or written explanations accompanied by expressions, equations, functions, graphs, and tables. They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They construct arguments to defend functions they have created to model contextual situations. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.
Model with mathematics.	In NC Math 4, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, generate functions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students determine whether the model and the constraints they have constructed makes sense given the context of the problem.
Use appropriate tools strategically.	In NC Math 4, students consider available tools when solving mathematical problems and decide when particular tools might be helpful. It is assumed that students will have access to graphing technologies (e.g., graphing calculator, Desmos) and spreadsheets. Students should examine results produced using technology to determine if the solution makes sense and be aware of issues that may arise when selecting an appropriate scale for a graph or mode to evaluate an expression. Students should also recognize when solving a problem by-hand is more efficient than a technology-assisted approach.
Attend to precision.	In NC Math 4, students use clear and precise language in their discussions with others and in their own reasoning. Students are aware of the effects of rounding on a solution. They recognize the importance and meaning of the symbols they use. They attend to units when solving real-world problems and appropriately label and interpret axes in graphs.
Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. In NC Math 4, students' reason about the solution and determine whether a simplified form is helpful for interpreting or using the result. Students are able to recognize key features of the graph of a function from its algebraic structure that may include asymptotes, end behavior, zeroes, amplitude, or period.
Look for and express regularity in repeated reasoning.	In NC Math 4, students use repeated reasoning to make generalizations about patterns and structures. By using repeated reasoning students are able to synthesize processes. For example, when examining a sequence of values students are able to develop a recursive function rule by identifying the repeated operations.
Use strategies and procedures flexibly.	In NC Math 4, students solve real world problems using their knowledge of numbers, functions, and algebra. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?" Students also consider the reasonableness of intermediate results while applying processes to solve complex equations.
Reflect on mistakes and misconceptions.	In NC Math 4, students use algebraic, tabular, and graphical representations to reason about mathematical and real-world contexts. They examine patterns in their processes. Students contextualize to understand the meaning of the number or variable as related to the problem. They mathematize problem situations to manipulate symbolic representations by applying properties of operations.

NC Math 4 Instructional Blueprint

NC DPI Resource: https://bit.ly/DPI_NCMath4

UNIT	CONCEPT	DURATION	OCS PRIORITY STANDARD(S)	SUPPORTING STANDARD(S)	NC DPI	Apex
1	Building Mathematical Community with ACT Prep (Complex Numbers, Matrices & Vectors)	7-9 Days	**NC.M4.N.2.2**	NC.M4.N.1.1 NC.M4.N.1.2 NC.M4.N.2.1	Unit 1: Lesson 1 Unit: ACT Prep	
2	Functions	7-9 Days	**NC.M4.AF.4.2** **NC.M4.AF.5.1**	NC.M4.AF.1.1 NC.M4.AF.1.2 NC.M4.AF.4.1 NC.M4.AF.5.2	Unit 2	Unit 1: 4
3	Logarithmic Functions	8-10 Days	**NC.M4.AF.3.3** **NC.M4.AF.5.1**	NC.M4.AF.3.1 NC.M4.AF.3.2	Unit 3	Unit 2: 4,5,8,9
4	Trigonometry	10-12 Days	**NC.M4.AF.2.3** **NC.M4.AF.5.1**	NC.M4.AF.2.1 NC.M4.AF.2.2	Unit 4	Unit 4: 4,9 Unit 5: 1,2,4
5	Exploratory Data Analysis	13-15 Days	**NC.M4.SP.1.2**	NC.M4.SP.1.1 NC.M4.SP.1.3 NC.M4.SP.1.4	Unit 5	
6	Probability Distributions	5-7 Days		NC.M4.SP.3.1 NC.M4.SP.3.2 NC.M4.SP.3.4	Unit 6: 1,2,3,4	Unit 8: 3,4 Unit 9: 2,3
7	Statistical Inference	13-15 Days	**NC.M4.SP.2.3** **NC.M4.SP.3.3**	NC.M4.SP.2.1 NC.M4.SP.2.2	Unit 6: 7 Unit 7: 1-7	Unit 10: 3, 4

Online Resources

- Mathematics | NC DPI NCDPI K-12 Mathematics Site.
- https://www.nc2ml.org/ (North Carolina Collaborative for Mathematics Learning, i.e. NC^{2}ML) - NC network of support for teachers. Provides resources, the ability to share best practices, and develop mathematical mindsets.

<table>
<tr><th colspan="9">Domain</th><th>Conceptual Category</th></tr>
<tr><th>K</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>HS</th></tr>
<tr><td>Counting and Cardinality</td><td colspan="5"></td><td colspan="2">Ratio and Proportions</td><td>Functions</td><td>Algebra</td></tr>
<tr><td colspan="6">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td><td>Functions</td></tr>
<tr><td colspan="6">Number and Operations Base Ten</td><td colspan="3" rowspan="2">The Number System</td><td rowspan="2">Number and Quantity</td></tr>
<tr><td colspan="3"></td><td colspan="3">Number and Operations Fractions</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="3">Statistics and Probability</td><td>Statistics and Probability</td></tr>
<tr><td colspan="9">Geometry</td><td>Geometry</td></tr>
</table>

Unit 1

Building Mathematical Community with ACT Prep
(Complex Numbers, Matrices & Vectors)

Unit 1: Building Mathematical Community with ACT Prep (Complex Numbers, Matrices & Vectors)

Source: NC^2ML Retrieved from: https://www.nc2ml.org/wp-content/uploads/2020/06/NC-Math-4-Instructional-FrameworkFINAL2.pdf

OCS Priority Standard(s):	***Supporting Standard(s):***
NC.M4.N.2.2 Execute procedures of addition, subtraction, and scalar multiplication on vectors.	**NC.M4.N.1.1** Execute procedures to add and subtract complex numbers. **NC.M4.N.1.2** Execute procedures to multiply complex numbers. **NC.M4.N.2.1** Execute procedures of addition, subtraction, multiplication, and scalar multiplication on matrices.

What is the Mathematics?

- In Math 2, students learned about complex numbers in the context of simplifying expressions with a negative value under a square root and solving quadratic equations. In this course, students will simplify complex numbers using addition, subtraction, and multiplication.
- Students will use the operations of addition, subtraction, multiplication and scalar multiplication to combine matrices.
- The connection can be made between operations of matrices and vectors because vectors can be represented as a matrix.

Important Considerations:

For success, significant time should be spent setting up the classroom culture. This includes:

- Developing classroom norms for communication (ex: non-verbal signals, listening and speaking expectations, talk moves for math discussions)
- Developing math routines
- Setting various expectations for the structure of the math block (ex: expectations for whole class instruction, cooperative learning, independent learning, effective integration of technology, etc.)
- Math discourse needs explicit modeling and practice. This includes students:
    - Sharing their thinking
    - Actively listening to the ideas of others
    - Connecting to others' ideas
    - Asking questions to clarify understanding
- Mathematical norms: http://www.youcubed.org/wp-content/uploads/Positive-ClassroomNorms2.pdf
- Develop mathematicians with positive attitudes about their ability to do mathematics by:
    - Creating opportunities to develop an appreciation for mistakes
    - Seeing mistakes as opportunities to learn
    - Teaching students to take responsibility for their learning

- Develop mathematicians who respect others by:
    - Demonstrating acceptance, appreciation, and curiosity for different ideas and approaches
    - Establishing procedures and norms for productive mathematical discourse
    - Consider other solution paths
- Develop mathematicians with a mindset for problem solving by:
    - Encouraging student authority and autonomy when problem solving
    - Emphasizing questioning, understanding, and reasoning about math, not just doing math for the correct answer
    - Asking follow-up questions when students are both right and wrong
- Allowing students to engage in productive struggle and moving them along by questioning, not telling. This unit is purely for the procedural knowledge of complex numbers, matrices, and vectors.
- These standards are included across fourth level courses in order to prepare students for the ACT due to state legislature.

Formative Assessments/Tasks:

- Neat Matrix Multiplication Task: https://www.openmiddle.com/matrix-multiplication/
- Complex Numbers: https://www.openmiddle.com/complex-number-products/
- Possible Homework Assignment: Add, Subtract, Multiply Complex Numbers Matrix Multiplication Task

Unit 1 Unpacking

Source: NC DPI Math 4 Unpacking Documents. Retrieved from https://www.dpi.nc.gov/nc-math-4-unpacking-rev-june-2022

<table>
<tr><th colspan="2">Supporting Standard: NC.M4.N.1.1</th></tr>
<tr><td colspan="2">NC.M4.N.1.1 Execute procedures to add and subtract complex numbers.</td></tr>
<tr><th>Clarification</th><th>Checking for Understanding</th></tr>
<tr><td>In NC Math 2 students use the quadratic formula to identify whether there are complex solutions to a quadratic equation. They express these complex solutions using $i.$ NC Math 2 students have also learned that all real numbers are complex numbers (NC.M2.N-CN.1).

In NC Math 4, students use properties to add and subtract complex numbers. They should recognize that the sum or difference of two complex numbers results in another complex number.
Please note the sum and difference algorithms refer to adding and subtracting, respectively, the like terms, combining the real parts and combining the imaginary parts.</td><td>Indicator: Evaluate the following expressions:
a. $(5+7i)+(-2+3i)$
b. $(9-i)-(4+6i)$
c. $\left(4-i\sqrt{2}\right)-\left(3+i\sqrt{2}\right)+8i$
Answers:
a. $3+10i$
b. $5-7i$
c. $1+\left(8-2\sqrt{2}\right)i$</td></tr>
<tr><th colspan="2">Supporting Standard: NC.M4.N.1.2</th></tr>
<tr><td colspan="2">NC.M4.N.1.2 Execute procedures to multiply complex numbers.</td></tr>
<tr><th>Clarification</th><th>Checking for Understanding</th></tr>
<tr><td>In Math 1 students multiply two binomial expressions like $(x+2)(x+3)$ to produce a polynomial, x^2+5x+6. Students also rewrite expressions that involve complex numbers as described in M4.N.1.1.
In NC Math 4, students are expected to be able to multiply two complex numbers with and without technology.
To multiply two complex numbers, you expand the product as you would with polynomials.</td><td>Indicator: Evaluate the following expressions:
a. $2i(13-9i)$
b. $(3+4i)(8-5i)$
c. $2(2+4i)(-3+i)$
d. $-3(2i+1)\cdot(5i)$
Answers:
a. $18+26i$
b. $44+17i$
c. $-20-20i$
d. $30-15i$</td></tr>
<tr><th colspan="2">Supporting Standard: NC.M4.N.2.1</th></tr>
<tr><td colspan="2">NC.M4.N.2.1 Execute procedures of addition, subtraction, multiplication, and scalar multiplication on matrices.</td></tr>
<tr><th>Clarification</th><th>Checking for Understanding</th></tr>
<tr><td>This standard represents students' first exposure to matrices in mathematics; however, students have been exposed to arrays in previous grades and</td><td>Indicator: Without the aid of technology, what is element e_{23} of the product of $G\cdot H$?</td></tr>
</table>

mathematics courses. Students will understand that the structure (rows and columns) of a matrix determines how matrices can be combined through addition, subtraction, and multiplication (including scalar multiplication) and use that structure to perform the aforementioned operations with matrices. This understanding includes when operations on matrices cannot be completed. Students are not expected to solve matrix equations.

$H = \begin{bmatrix} 7 & -1 & 0.5 \\ -5 & -3.5 & 4 \end{bmatrix} \quad G = \begin{bmatrix} 3 & 2 \\ 5 & 0.5 \end{bmatrix}$

Answer:
4.5

Indicator:

$$A = \begin{bmatrix} -4 & 7 \\ 1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -5 \\ 0 & 6 \end{bmatrix}, \quad C = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$$

Given the matrices A, B, and C above, evaluate the following expressions:

a. $A + B$ b. $B - A$ c. $A \cdot C$ d. $3B + 2A$ e. $B \cdot A$

Answers:

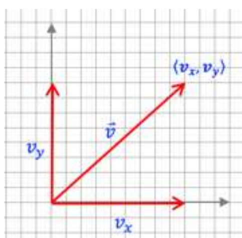
a. $\begin{bmatrix} -2 & 2 \\ 1 & 9 \end{bmatrix}$ b. $\begin{bmatrix} 6 & -12 \\ -1 & 3 \end{bmatrix}$ c. $\begin{bmatrix} -4 \\ 20 \end{bmatrix}$ d. $\begin{bmatrix} -2 & -1 \\ 2 & 24 \end{bmatrix}$ e. $\begin{bmatrix} -13 & -1 \\ 6 & 18 \end{bmatrix}$

OCS Priority Standard: NC.M4.N.2.2

NC.M4.N.2.2 Execute procedures of addition, subtraction, and scalar multiplication on vectors.

Clarification

Students have not previously learned how to add and subtract vectors.



In NC Math 4, students will recognize the component form of a vector as ***v*** = $< vx, vy >$ where v_x and v_y represent the horizontal and vertical components, respectively. Additionally, students will recognize that a vector in standard position has its initial point at the origin with a terminal point that represents the components of the vector.

Students should also be familiar with three different methods for adding and subtracting vectors:

Checking for Understanding

Indicator: Given the vectors $\boldsymbol{u}$, $\boldsymbol{v}$, and $\boldsymbol{w}$ below, evaluate the following expressions:

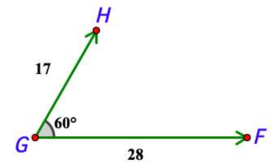
$$\boldsymbol{u} =< 3,7 > \quad \boldsymbol{v} =< 4,-5 > \quad \boldsymbol{w} =< -2,9 >$$

a. $w + v$ b. $v - u$ c. $8w$ d. $3u - 2v$

Answers:

a. $< 2,4 >$ b. $< 1,-12 >$ c. $< -16,72 >$ d. $< 1,31 >$

Indicator: Given vector GH and vector GF with an angle of 60 degrees between them, find the magnitude of the resultant vector.



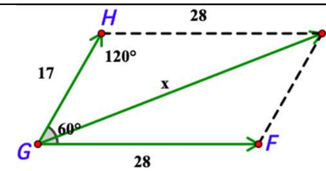
Answer:

- Adding vectors end-to-end - positioning the vectors (without changing their magnitudes and directions) so that the initial point of one vector coincides with the terminal point of the other vector
- Adding/subtracting corresponding components - add or subtract the corresponding components
- Using the parallelogram rule - a graphical method used for:
  - addition of two vectors,
  - subtraction of two vectors, and
  - resolution of a vector into two components in arbitrary directions.

Students should understand the process of finding the sum/difference of two vectors using any of the methods mentioned. They are not expected to know the name of the methods.

Students will also apply the properties when multiplying a vector by a scalar.

Students are not expected to execute procedures for vectors beyond 2-dimensional vectors.

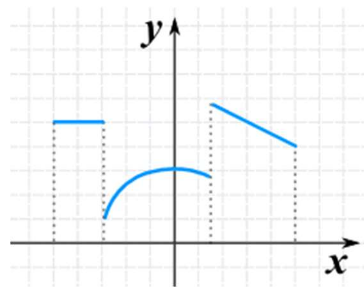


$x^2 = 17^2 + 28^2 - 2(17)(28) * \cos(120)$

$x = 39.36$

Unit 2

Functions (Parent Functions & Key Features, Piecewise, Composition of Functions & Regression)



Unit 2: Functions

Source: NC^2ML Retrieved from: https://www.nc2ml.org/wp-content/uploads/2020/06/NC-Math-4-Instructional-FrameworkFINAL2.pdf

OCS Priority Standard(s):	***Supporting Standard(s):***
NC.M4.AF.4.2 Construct piecewise functions to model a contextual situation. **NC.M4.AF.5.1** Construct regression models of linear, quadratic, exponential, logarithmic, & sinusoidal functions of bivariate data using technology to model data and solve problems.	**NC.M4.AF.1.1** Execute algebraic procedures to compose two functions. **NC.M4.AF.1.2** Execute a procedure to determine the value of a composite function at a given value when the functions are in algebraic, graphical, or tabular representations. **NC.M4.AF.4.1** Translate between algebraic and graphical representations of piecewise functions (linear, exponential, quadratic, polynomial, square root). **NC.M4.AF.5.2** Compare residuals and residual plots of non-linear models to assess the goodness-of-fit of the model.

What is the Mathematics?

- Recognize constant, linear, exponential, quadratic, square root, cubic, and absolute value functions from multiple representations including tables, graphs, function rules, and verbal descriptions.
- Discuss the key features (including domain, range, intervals where the function is increasing, decreasing, positive, or negative, end behavior, minimum and maximum) from Math 1-3 courses.
- Students will use functions that they have previously learned in Math 1-3 to model and build piecewise functions. Since they have already seen graphs and equations of piecewise functions in Math 3, students are now translating between algebraic and graphical representations. This means they can be given a graph of a piecewise function and create the function rule. This is an opportunity to build on the discussion of parent functions done in the previous unit. Students will continue discussing key features and can evaluate the piecewise function here. As students looked at domain and range in the first unit, the development of piecewise will emphasize the importance of domain.
- Students will also use these piecewise functions to model real-world situations. Students could look at financial situations (budgets, tax brackets, pricing, salaries, commission, parking rates, etc), temperatures, velocities, growth charts, roller coasters, etc.
- Students will use the knowledge of evaluating functions at a specific value to extend into composition of functions. They can build on the discussion of function families to compose and get new functions.
- The standard as written does not intend for students to use composition to assess inverse functions. This could become an extension for honors students if there is time available since they have discussed inverse functions in Math 3.

- Students will use regression to model bivariate data. As a statistical thinker, students need to understand part of the process when exploring data by first trying to get a look at what the data is doing. They will create a scatterplot of the data and try to describe the overall trend they are seeing. Then decide what types of functions might be appropriate to model the data. THEN, they will try to fit a model. This might be done on a graphing calculator or through other statistical technology. After students fit a model, they must decide how appropriate that model is for the data. It is extremely important for students to realize in the statistical process that this is part of explaining why and convincing others that their model is a good fit. In addition to the original scatterplot and the mathematical context, the residuals and residual plots are further evidence students will be using to justify that they have found the best fit model.
- Students will use residuals and residual plots because **r and r_2 do not apply** as measures of goodness-of-fit to all functions.
- Video - R_2 explained: https://www.youtube.com/watch?v=IMjrEeeDB-Y

Important Considerations:

- This unit provides students an opportunity to review functions in terms of piecewise-defined functions. An approach to rewriting these functions might include the use of parent functions and function transformations that they talked about extensively in Math 1-3.
- This unit provides opportunities to bring in financial literacy components (tax brackets, compound interest on investments, debt & savings, income calculation, hourly wages, payroll deductions, budgeting).
- For this unit, the recommended functions to use for piecewise, composition, and regression are linear, exponential, quadratic, polynomial, and square root functions.
    - Note: The use of step functions in piecewise could also be an honors extension if desired.
- Unit Progression: There is always flexibility in a course structure. Below addresses two ways that this unit could be placed in this course:
    - In the current progression with this unit serving as Unit 2, students will expand on the ideas with parent functions and continue discussing key features & characteristics while also using them to build new functions such as piecewise and composition. When introducing regression, this will be a review of what was previously taught in Math 1. Students will use the functions being addressed in this unit to explore appropriate regression models. This topic will continue to be spiraled throughout the course as students explore logarithmic and trigonometric functions later.
    - Regardless of where this unit is placed, the length of time allotted for this unit is because it is the capstone of students' study of functions discussed in NC Math 1-3. The length allows time for a deeper understanding of function types and how they can be applied to piecewise functions & composition.

Formative Assessments/Tasks:
• Parent Functions Card Sort http://www.mrseteachesmath.com/2014/12/parent-functionsmatching-activity.html • Parent Function Polygraph: https://teacher.desmos.com/polygraph/custom/560ad6907701c30306330608 • https://www.insidemathematics.org/sites/default/files/materials/sorting%20functions.pdf • Exponential Marble Slides: https://teacher.desmos.com/activitybuilder/custom/566b317b4e38e1e21a10aafb • Linear Marble Slides: https://teacher.desmos.com/activitybuilder/custom/566b31734e38e1e21a10aac8 • Quadratic Marble Slides: https://teacher.desmos.com/activitybuilder/custom/566b31784e38e1e21a10aade • Which One Doesn't Belong: http://wodb.ca/graphs.html Graphs 16, 22, 27 • Key Features of a Function exploration: https://teacher.desmos.com/activitybuilder/custom/564b8b6fa8e7fefa0bad36b7 • Piecewise Functions Exploration https://teacher.desmos.com/activitybuilder/custom/5bf2ee0cb5573d0c04696261 • Choosing a Regression Model http://bit.ly/regression_models

Unit 2 Unpacking

Source: NC DPI Math 4 Unpacking Documents. Retrieved from https://www.dpi.nc.gov/nc-math-4-unpacking-rev-june-2022

<table>
<tr><th colspan="2">Supporting Standard: NC.M4.AF.1.1</th></tr>
<tr><td colspan="2">NC.M4.AF.1.1 Execute algebraic procedures to compose two functions.</td></tr>
<tr><th>Clarification</th><th>Checking for Understanding</th></tr>
<tr><td>Students will extend their understanding of function notation and evaluating functions to working with compositions of two functions. This includes evaluating a composition of functions for specific values in the domain. Students will understand that the net effect of $f \circ g$ is $a \to g(a) \to f(g(a))$ for any real value, a. Students understand that a is in the domain of g and that $g(a)$ is in the domain of f. There is no limit as to the type of functions to be used when composing functions.

In problems involving roots in the denominator, it is not necessary to rationalize the denominator.</td><td>Indicator: Find $(f \circ g)(x)$ if $f(x) = -6x + 11$ and $g(x) = 3x - 5$.

Answer:
$-18x + 41$

Indicator: Given the two functions,
$f(x) = 2x - 3$ and $g(x) = x^2 + 1$, find the following:

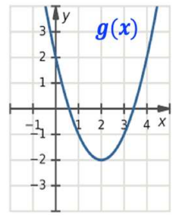
a. $f(g(x))$ b. $g(f(x))$ c. $(f \circ f)(x)$ d. $(g \circ g)(x)$

Answers:
a. $2x^2 - 1$ b. $4x^2 - 12x + 10$ c. $4x - 9$ d. $x^4 + 2x^2 + 2$

Indicator: Given
$f(x) = \frac{1}{x^2+4}$ and $g(x) = \sqrt{x+1}$. Find $f(g(x))$ and $g(f(x))$.

Answers:
$f(g(x)) = \frac{1}{(\sqrt{x+1})^2+4}$ and $g(f(x)) = \sqrt{\frac{x^2+5}{x^2+4}}$</td></tr>
<tr><th colspan="2">Supporting Standard: NC.M4.AF.1.2</th></tr>
<tr><td colspan="2">NC.M4.AF.1.2 Execute a procedure to determine the value of a composite function at a given value when the functions are in algebraic, graphical, or tabular representations.</td></tr>
<tr><th>Clarification</th><th>Checking for Understanding</th></tr>
<tr><td>Students have already evaluated functions for specific values in the domain, given a function in algebraic, graphical or tabular form. Building upon this skill, students will now apply it to compositions of functions in a variety of representations; this includes functions presented in two different representations.</td><td></td></tr>
</table>

For example, given the graph of the quadratic function $g(x)$ and the algebraic representation of the linear function $f(x) = 3x - 1$ students can evaluate $f(g(4))$ by finding the corresponding *y*-value (2) in $g(x)$ and then evaluating $f(2)$. So $f(g(4)) = 3(2) - 1 = 5$.



There is no limit as to the type of functions to be used when composing functions.

Indicator:

x	-2	-1	0	1	2	3	4	5
$f(x)$	5	2	1	2	5	10	17	26

x	-2	-1	0	1	2	3	4	5
$g(x)$	-1	1	3	5	7	9	11	13

Given the two functions above, find the following:

a. $f(g(-2))$ b. $g(f(2))$ c. $(f \circ g)(1)$ d. $(g \circ f)(-1)$

Answers:

a. 2 b. 13 c. 26 d. 7

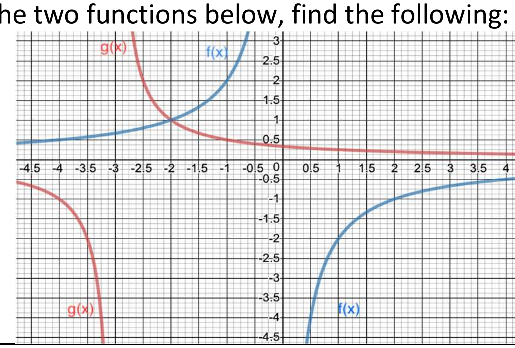
Indicator: Given the two functions,
$f(x) = 2x - 3$ and $g(x) = x^2 + 1$, find the following with the given values:

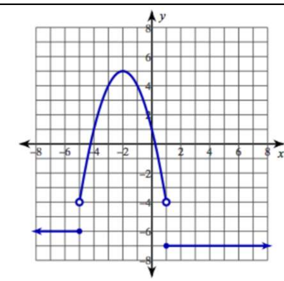
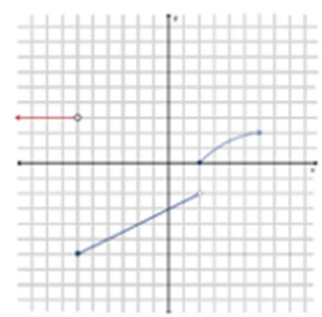
a. $f(g(1))$ b. $g(f(-1))$ c. $(f \circ f)(-3)$ d. $(g \circ g)(0)$

Answers:

a. 1 b. 26 c. -21 d. 2

Indicator: Given the two functions below, find the following:



	a. $f(g(-2))$ b. $g(f(2))$ c. $(f \circ g)(-4)$ d. $(g \circ f)(1)$ ***Answers:*** a. -2 b. 0.5 c. 2 d. 1
Supporting Standard: NC.M4.AF.4.1	
NC.M4.AF.4.1 Translate between algebraic and graphical representations of piecewise functions (linear, exponential, quadratic, polynomial, square root).	
Clarification	**Checking for Understanding**
This standard builds upon the interpreting functions (IF) standards from Math 1–3, where students have graphed functions given in algebraic form and written equations for graphical forms of functions. In NC Math 3, students analyzed the key features of piecewise-defined functions. They also interpret and evaluate piecewise functions. In NC Math 4, students will write algebraic representations of piecewise defined functions given in graphical form and graph algebraic representations of piecewise-defined functions.	**Indicator**: Create a piecewise-defined function for the graph to the right.  *Answer:* $f(x) = \begin{cases} -6 & if\ x \leq -5 \\ -x^2 - 4x + 1 & if\ -5 < x < 1 \\ -7 & if\ x \geq 1 \end{cases}$ **Indicator**: Graph the following function and determine the domain and range of the function. $f(x) = \begin{cases} 3 & if\ x < -6 \\ \frac{1}{2}x - 3 & if\ -6 \leq x < 2 \\ \sqrt{x-2} & if\ x \geq 2 \end{cases}$ *Answer: (Graph to the right).* Domain: All real numbers Range: $[-6, -2) \cup [0, \infty)$ 
OCS Priority Standard: NC.M4.AF.4.2	

NC.M4.AF.4.2 Construct piecewise functions to model a contextual situation.	
Clarification	**Checking for Understanding**
This standard requires students to write piecewise functions based on real-world context. Students understand that each part of the given context may represent a different function type and understand that the domain for each piece is restricted by the context of the problem.	**Indicator**: Write the rule for a piecewise function that models the following situation, the data plan for a smart phone costs $35 for 5 gigabytes of data and it costs $10 extra for each additional gigabyte of data used. *Answer:* $f(x) = \begin{cases} 35 & if\ x \leq 5 \\ 35 + 10(x - 5) & if\ x > 5 \end{cases}$ **Indicator**: A certain county has a tax code, where 10% tax is paid on all income up to the first $10,000, a 15% tax is paid for any income over $10,000 and up to $25,000, and a tax rate of 25% is paid on all income over $25,000. Find a piecewise function to calculate the total tax $T(x)$ on an income of $\boldsymbol{x}$ dollars. *Answer:* $t(x) = \begin{cases} .1x & if\ x \leq 10{,}000 \\ .15(x - 10{,}000) + 1{,}000 & if\ 10{,}000 < x \leq 25{,}000 \\ .25(x - 25{,}000) + 3{,}250 & if\ x > 25{,}000 \end{cases}$
OCS Priority Standard: NC.M4.AF.5.1	
NC.M4.AF.5.1 Construct regression models of linear, quadratic, exponential, logarithmic, & sinusoidal functions of bivariate data using technology to model data and solve problems.	
Clarification	**Checking for Understanding**

Students have previously worked with scatter plots and lines of best fit in 8th grade. In NC Math 1, students have done linear and exponential regression using technology. They have also examined residual plots to determine linearity.

In NC Math 4, students will also construct regression models for quadratic, logarithmic and sinusoidal functions of bivariate data. Additionally, students will be able to use regression models to solve real-world and mathematical problems.

Indicator:

a. Analyze the scatterplot of the cost of a 30-second advertisement for the Super Bowl from 1967-2011. Describe the patterns in the data.

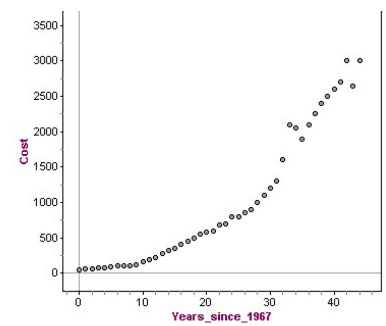
b. Calculate a quadratic regression model for *x* years after 1967.

c. Based on your model, what is the predicted cost of a 30-sec. advertisement in 2007? How accurate is your prediction model's cost compared to the actual cost?

d. Based on your model, what is the predicted cost for a 30-sec. advertisement in 2020?

Year of Super Bowl	Cost of 30-sec Advertisement (in thousands)	Year of Super Bowl	Cost of 30-sec Advertisement (in thousands)
1967	40	1990	700
1968	54	1991	800
1969	67.5	1992	800
1970	78.2	1993	850
1971	72	1994	900
1972	86	1995	1000
1973	103.5	1996	1100
1974	107	1997	1200
1975	110	1998	1300
1976	125	1999	1600
1977	162	2000	2100
1978	185	2001	2050
1979	222	2002	1900
1980	275	2003	2100
1981	324.3	2004	2250
1982	345	2005	2400
1983	400	2006	2500
1984	450	2007	2600
1985	500	2008	2700
1986	550	2009	3000
1987	575	2010	2650
1988	600	2011	3000
1989	675		

Answer:



a. The scatterplot is positive and nonlinear. The cost of a 30-second advertisement increases as the years increase.

b. $y = 1.916x^2 - 15.510x + 104.408$, x=years since 1967 and y=predicted cost of a 30-second advertisement.

c. In 2007, the predicted cost was roughly $2,550,000. This is fairly close to the actual recorded cost which was $2,600,000.

d. In 2020, the predicted cost is approximately $4,660,000.

Data obtained from: https://www.businessinsider.com/cost-super-bowl-ads-through-the-years-2011-2

Supporting Standard: NC.M4.AF.5.2	
NC.M4.AF.5.2 Compare residuals and residual plots of non-linear models to assess the goodness-of-fit of the model.	
Clarification	**Checking for Understanding**
In NC Math 1, students created scatter plots of data and determined whether a linear model was appropriate based on the residual plot. In NC Math 4, students will apply knowledge from Math 1 about interpolating and extrapolating and validity of predictions. They will now closely analyze the residual plots of non-linear models to determine the goodness-of-fit of a selected non-linear model. Students will understand that a residual plot with no discernable pattern indicates the selected model is a good fit for the data set.	**Indicator**: Answer the following questions Does the wealth of a country influence its citizens' life expectancy? Do you live longer in countries that have more money?

Country	GDP per Capita	Life Expectancy
Afghanistan	$1,900	51.7
Argentina	$20,700	77.3
Bolivia	$7,500	69.5
Brazil	$15,500	74
Cambodia	$4,000	64.9
Chad	$2,400	50.6
China	$16,600	75.7
Costa Rica	$17,200	78.7
Djibouti	$3,600	63.6
Ecuador	$11,200	77
Egypt	$13,000	73
Ghana	$4,600	67
Greece	$27,800	80.7
Haiti	$1,800	64.2
India	$7,200	68.8
Iran	$20,000	74
Iraq	$17,000	74.9
Ireland	$72,600	80.9
Korea, North	$1,700	70.7
Madagascar	$1,600	66.3
Mexico	$19,500	76.1
Mozambique	$1,300	53.7
Netherlands	$53,600	81.4
Niger	$1,200	55.9
Paraguay	$9,800	77.4
Peru	$13,300	74
Portugal	$30,300	79.4
Russia	$27,900	71
Saudi Arabia	$55,300	75.5
Singapore	$90,500	85.2
South Africa	$1,500	63.8
Spain	$38,200	81.8
Sweden	$51,300	82.1
Switzerland	$61,400	82.6
Turkey	$26,500	75
Uganda	$2,400	55.9
United Kingdom	$43,600	80.8
United States	$59,500	80
Zambia	$4,000	52.7
Zimbabwe	$2,300	60.4

a. Describe the relationship between the Gross Domestic Product per Capita for a country and its Life Expectancy. What trends do you notice?

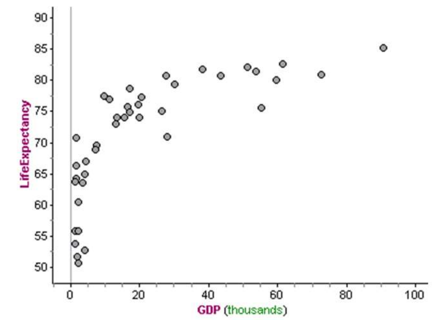
b. What type of function might model this well?

c. Using your calculator regression tools or statistical software, calculate a model for the data. Compare the residual plots as well. Record the model equation that best fits the data AND sketch the residual plot.

d. If Canada has a GDP per capita of $48,100, what would you predict to be their Life Expectancy? Look up how accurate this is based on today's life expectancy information on the World Factbook.

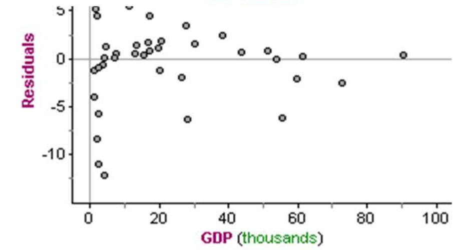
Answers:

a. The scatter plot is positive and nonlinear. As GDP increases, so does life expectancy. The life expectancy is increasing, but at a decreasing rate.



b. A logarithmic function might model this scatterplot the best.

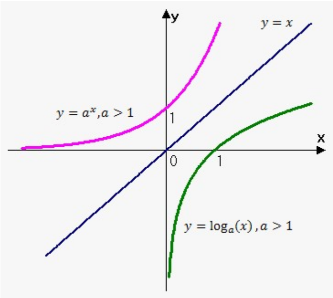
c. The best model found was a logarithmic one. The regression equation is $y = 11.77 + 6.40\ln(x)$ where x=GDP and y=predicted life expectancy. *This residual plot shows that there is more variation in the residuals for lower GDPs than higher GDPs.



d. Canada: $y = 11.77 + 6.40\ln(48100) = 80.8$ years. According to World Factbook (in 2018), the life expectancy for people in Canada is 82 years. The predicted value is fairly close to the actual life expectancy data for Canada.

Unit 3

Logarithmic Functions



Unit 3: Logarithmic Functions

Source: NC^2ML Retrieved from: https://www.nc2ml.org/wp-content/uploads/2020/06/NC-Math-4-Instructional-FrameworkFINAL2.pdf

OCS Priority Standard(s):	***Supporting Standard(s):***
NC.M4.AF.3.3 Interpret key features of a logarithmic function using multiple representations. **NC.M4.AF.5.1** Construct regression models of linear, quadratic, exponential, logarithmic, & sinusoidal functions of bivariate data using technology to model data and solve problems.	**NC.M4.AF.3.1** Execute properties of logarithms to rewrite expressions and solve equations algebraically. **NC.M4.AF.3.2** Implement properties of logarithms to solve equations in contextual situations.

What is the Mathematics?

- In Math 3, students solved exponential equations algebraically by converting them to logarithmic form, but they did not use properties of logarithms. They have also not seen natural logarithms or exponential equations of base *e*. In this course, students will develop the properties of logarithms, use those properties (of any base, including *e*) to solve logarithmic equations algebraically, and analyze the graph and key features of logarithmic functions.
    - "Students are **not** expected to know or use the properties of logarithms, *e*, or natural logs to solve problems. These can be extension topics but are beyond the scope of the NC Math 3 standards." NC.M3-F.LE.4
- Students will use rules of exponents to derive the properties of logs.
- After deriving the properties of logs, students will use those properties to solve equations. Students will also be expected to solve equations in the context of real-world problems.
- Students will need to understand and interpret the key features of log functions (intercepts, end behavior, domain, range, and intervals where the function is increasing and decreasing).
    - Continuing the ideas in Units 1 & 2.
- Students will use regression to model bivariate data. Just as in the Functions Unit, students will need to determine which type of function best fits the data by using scatter plots, residuals, and residual plots. This can be done using a graphing calculator or other statistical technology.
  Students should be able to justify why they have found the best fit model.
    - See example below in the Tasks section.

Important Considerations:

- AF.3.1 Make the conceptual connection between exponential rules and properties of logs.
- AF.3.3 Use various representations (tables and graphs) to develop the idea of inverses and key features. Trying to develop the concept that for y = log (x), it means y is the power of 10 that you need to find x. *See the Brief for Math 3 Unit 2 Exponential & Logarithmic Functions.*

- Since the first known instance of the slide rule in 1622, changing bases between logarithms to solve problems has been considered important procedural knowledge for math learners to master. In recent years, graphing technology has added the ability to complete change of base between logarithms without knowing this procedure. The use of technology now allows students to use a logarithm with any base as the TI-84 has a logBASE option and Desmos allows you to type any base subscript. We do not negate that it is important for students to understand the conceptual knowledge behind how graphing technology is programmed to complete this procedure but do suggest that the change of base property is not as important as it once was and the other properties of logarithms are a more relevant focus. For instance, students will need to understand how to condense logarithms into one log in order to solve equations but would still be able to solve exponential equations without needing change of base.
    - For example, students need condensing properties to solve $log_2(x) + log_2(x - 2) = 3$. Students do not need change of base to solve $3^{x-5} = 7$ because they can convert to logarithmic form as in Math 3 or can take log_3 of both sides.
- Please include **natural log** in problems that discuss logarithms and context of logs.
- This Unit could be a time to bring in those financial literacy components (tax brackets, compound interest on investments, debt and savings, income calculation, hourly wages, payroll deductions, budgeting).

Formative Assessments/Tasks:

- MVP Secondary Math 3 Module 2 Logarithmic Functions (Honors) covers much of the content of this unit
- Logarithmic Regression Example: http://bit.ly/Log-Regression

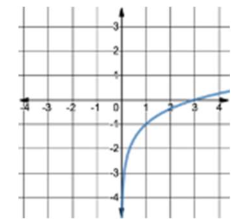
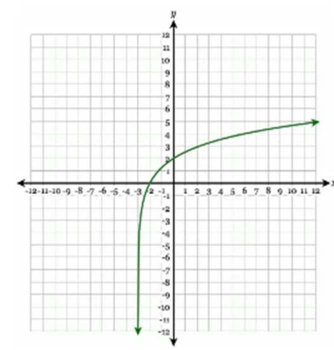
Unit 3 Unpacking

Source: NC DPI Math 4 Unpacking Documents. Retrieved from https://www.dpi.nc.gov/nc-math-4-unpacking-rev-june-2022

<table>
<tr><th colspan="2">Supporting Standard: NC.M4.AF.3.1</th></tr>
<tr><td colspan="2">NC.M4.AF.3.1 Execute properties of logarithms to rewrite expressions and solve equations algebraically.</td></tr>
<tr><td>Clarification</td><td>Checking for Understanding</td></tr>
<tr><td>In NC Math 3, students recognized the inverse relationship between exponential and logarithmic functions. They also translated between exponential and logarithmic forms of equations.
In NC Math 4, students will build upon their understanding of the properties of exponents when applying the properties of logarithms. This standard requires students to procedurally rewrite logarithmic expressions to solve logarithmic equations; including natural logs and e.</td><td>Indicator: Solve $log(x-3) + log(x+4) = 2log3$.

Answer: $\frac{-1+\sqrt{85}}{2} \approx 4.11$</td></tr>
<tr><th colspan="2">Supporting Standard: NC.M4.AF.3.2</th></tr>
<tr><td colspan="2">NC.M4.AF.3.2 Implement properties of logarithms to solve equations in contextual situations.</td></tr>
<tr><td>Clarification</td><td>Checking for Understanding</td></tr>
<tr><td>This standard requires students to use the skills developed in NC.M4.AF.3.1 to solve logarithmic equations that are based on contextual situations. Students should be able to determine when it is appropriate to apply logarithmic properties when presented with a problem based in a contextual situation; this may include creating an exponential equation and then solving the equation using properties of logarithms.
Students should be familiar with all logarithms, including natural logs.</td><td>Indicator: The formula $A = P(1+\frac{r}{n})^{nt}$ describes the accumulated value, $\boldsymbol{A}$, of a sum of money, $\boldsymbol{P}$, the principal, after $\boldsymbol{t}$ years at annual percentage rate $\boldsymbol{r}$ (in decimal form) compounded $\boldsymbol{n}$ times a year. How long will it take $50,000 to grow to $900,000 at 20% annual interest compounded quarterly?
Answer: $\approx$ 14.8 years

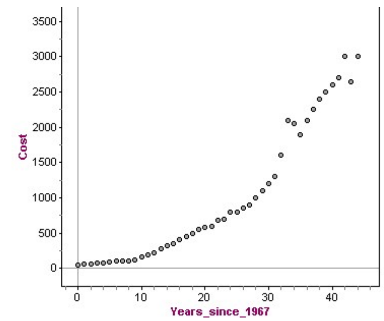
Indicator: The Smiths are saving for their son's college education. If they deposit $12,000 in an account bearing 6.4% interest compounded continuously, how much will be in the account when Evan goes to college in 12 years?
Answer: $25,865.41</td></tr>
<tr><th colspan="2">OCS Priority Standard: NC.M4.AF.3.3</th></tr>
<tr><td colspan="2">NC.M4.AF.3.3 Interpret key features of a logarithmic function using multiple representations.</td></tr>
<tr><td>Clarification</td><td>Checking for Understanding</td></tr>
<tr><td>In NC.M3, students develop an understanding of a logarithm through its inverse relationship with exponents. They also recognize that each constant and coefficient in a log equation or function represents a key feature of the</td><td>Indicator: Given the function $f(x) = \log_3 x - 1$, describe the key features. Include domain and range, intercepts, intervals of increasing or decreasing, and end behavior.</td></tr>
</table>

function. Additionally in NC.M3.F-IF.7, students analyze the key features of piecewise, absolute value, polynomial, exponential, rational, and trigonometric functions using different representations. In NC Math 4, students will need to understand and interpret the key features (intercepts, end behavior, domain, range, and intervals where the function is increasing and decreasing) of log functions.	*Answers:* Domain: $(0, \infty)$ Range: $(-\infty, \infty)$ x-intercept: 3 y-intercept: $\emptyset$ Increasing: $(0, \infty)$ Decreasing: $\emptyset$ EB(L): $x \to 0; y \to \infty$ EB(L): $x \to \infty; y \to \infty$  **Indicator**: Given the logarithmic function below, describe the key features. Include domain and range, intercepts, intervals of increasing or decreasing, and end behavior. *Answers:* Domain: $(-3, \infty)$ Range: $(-\infty, \infty)$ x-intercept: $(-2, 0)$ y-intercept: $(0, 2)$ Increasing: $(0, \infty)$ Decreasing: $\emptyset$ EB(L): $x \to -3; y \to \infty$ EB(L): $x \to \infty; y \to \infty$ 
OCS Priority Standard: NC.M4.AF.5.1	
NC.M4.AF.5.1 Construct regression models of linear, quadratic, exponential, logarithmic, & sinusoidal functions of bivariate data using technology to model data and solve problems.	
Clarification	**Checking for Understanding**
Students have previously worked with scatter plots and lines of best fit in 8th grade. In NC Math 1, students have done linear and exponential regression using technology. They have also examined residual plots to determine linearity. In NC Math 4, students will also construct regression models for quadratic, logarithmic and sinusoidal functions of bivariate data. Additionally, students will be able to use regression models to solve real-world and mathematical problems.	**Indicator**: a. Analyze the scatterplot of the cost of a 30-second advertisement for the Super Bowl from 1967-2011. Describe the patterns in the data

b. Calculate a quadratic regression model for *x* years after 1967.
c. Based on your model, what is the predicted cost of a 30-sec. advertisement in 2007? How accurate is your prediction model's cost compared to the actual cost?
d. Based on your model, what is the predicted cost for a 30-sec. advertisement in 2020?

Year of Super Bowl	Cost of 30-sec Advertisement (in thousands)		Year of Super Bowl	Cost of 30-sec Advertisement (in thousands)
1967	40		1990	700
1968	54		1991	800
1969	67.5		1992	800
1970	78.2		1993	850
1971	72		1994	900
1972	86		1995	1000
1973	103.5		1996	1100
1974	107		1997	1200
1975	110		1998	1300
1976	125		1999	1600
1977	162		2000	2100
1978	185		2001	2050
1979	222		2002	1900
1980	275		2003	2100
1981	324.3		2004	2250
1982	345		2005	2400
1983	400		2006	2500
1984	450		2007	2600
1985	500		2008	2700
1986	550		2009	3000
1987	575		2010	2650
1988	600		2011	3000
1989	675			

Answer:

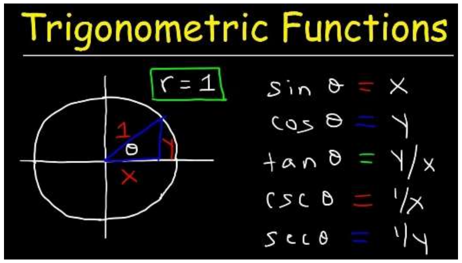


a. The scatterplot is positive and nonlinear. The cost of a 30-second advertisement increases as the years increase.
b. $y = 1.916x^2 - 15.510x + 104.408$, x=years since 1967 and y=predicted cost of a 30-second advertisement.
c. In 2007, the predicted cost was roughly $2,550,000. This is fairly close to the actual recorded cost which was $2,600,000.
d. In 2020, the predicted cost is approximately $4,660,000.

Data obtained from: https://www.businessinsider.com/cost-super-bowl-ads-through-the-years-2011-2

Unit 4

Trigonometry



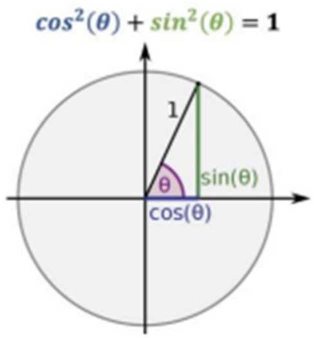
Unit 4: Trigonometry

Source: NC^2ML Retrieved from: https://www.nc2ml.org/wp-content/uploads/2020/06/NC-Math-4-Instructional-FrameworkFINAL2.pdf

OCS Priority Standard(s):	***Supporting Standard(s):***
NC.M4. AF.2.3 Interpret key features (amplitude, period, phase shift, vertical shifts, midline, domain, range) of models using sine and cosine functions in terms of a context. **NC.M4.AF.5.1** Construct regression models of linear, quadratic, exponential, logarithmic, & sinusoidal functions of bivariate data using technology to model data and solve problems.	**NC.M4.AF.2.1** Translate trigonometric expressions using the reciprocal and Pythagorean identities. **NC.M4.AF.2.2** Implement the Law of Sines and the Law of Cosines to solve problems.

What is the Mathematics?

- In Math 2, students worked with right triangle trig ratios. It is important that teachers take the time to develop the reciprocal and Pythagorean identities in this course. For AF.2.1 students will make connections to right triangle trig to develop the reciprocal identities (csc, sec, cot) as well as to develop Pythagorean Identities from the basis in Math 3 where they explored ($cosx, sinx$)



*Note: Connect to prior knowledge of NC.M3.F-TF.2a-b. Remember at this level we **do not expect students to memorize** the special coordinates of Unit Circle but rather we are using the understanding that the x-coordinate is the cosine value and y-coordinate is the sine value.*

- Students will extend their thinking of right triangle trig to establish relationships between the angles and side lengths for any triangle. Take the time here to facilitate an investigation of the law of sines and cosines to build conceptual understanding.
- In NC.M3.F-TF.5, students worked with key features (amplitude, period, vertical shifts, midline, domain, range) of sine functions in terms of a context. AF2.3 extends this learning to examine phase shifts and the cosine function. In this standard, students are interpreting the key features from graphs, tables, equations, and context. The context of this work is very important, and this is a good space to allow students to have fun with the mathematics in real-world applications.

- Students will use regression to model bivariate data. Just as in the Functions Unit, students will need to explore data and determine which function best fits the model of a scatter plot. This might be done on a graphing calculator or through other statistical technology. The residuals and residual plots ARE the evidence students will be using to prove they have found the best fit model. While this is the first point you have done regression for sinusoidal functions, this is the perfect place to spiral functions in from previous units.

Important Considerations:

- For trigonometric identities, going beyond the Pythagorean and reciprocal trig identities is not in the scope of this course. These trig identities are not meant to be complex multi-step precalculus or calculus level simplifying expressions problems. Instead the focus is on giving students opportunities to be exposed to the Pythagorean and reciprocal identities and also set them up for possibly taking Precalculus in the future. When this is being taught teachers should also connect back to students previously learned concepts and right triangle trigonometry and the Pythagorean theorem.

Formative Assessments/Tasks:

- Match My Trig Graph: https://teacher.desmos.com/activitybuilder/custom/56a26aada55a6568199d1c5c
- MVP Secondary Math Trig Functions, Equations, and Identities

Unit 4 Unpacking

Source: NC DPI Math 4 Unpacking Documents. Retrieved from https://www.dpi.nc.gov/nc-math-4-unpacking-rev-june-2022

Supporting Standard: NC.M4.AF.2.1

NC.M4.AF.2.1 Translate trigonometric expressions using the reciprocal and Pythagorean identities.

Clarification

Grade 8 students study the Pythagorean theorem to find the length of one side of a right triangle given the lengths of two sides. In Math 2 students learn right triangle trigonometry and are familiar with trigonometric ratios. However, this is the first time that students will encounter trigonometric identities. In NC Math 4, students will use algebraic reasoning to rewrite trigonometric expressions in simplified equivalent forms using the reciprocal and Pythagorean identities.

Checking for Understanding

Indicator: Rewrite the following trigonometric expression as a single trigonometric function: $\sec\theta - \sin\theta \cdot \tan\theta$

Answer: $\cos\theta$

Supporting Standard: NC.M4. AF.2.2

NC.M4. AF.2.2 Implement the Law of Sines and the Law of Cosines to solve problems.

Clarification

Students have previously worked with finding the missing sides and angles of *right* triangles using the Pythagorean Theorem and by applying the trigonometric ratios, respectively. Students will use the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles. They should be able to:

- Distinguish between situations that require the Law of Sines (ASA, AAS, SSA) and situations that require the Law of Cosines (SAS, SSS);
- represent real world problems with diagrams of non-right triangles and use them to solve for unknown side lengths and angle measures;
- Solve for missing side lengths and angles using Law of Sines and Law of Cosines.

Note: *The ambiguous case for oblique triangles is NOT an expectation in NC Math 4.*

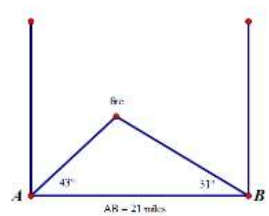
Checking for Understanding

Indicator: A plane is flying from city D to city B, which is 115 miles due north. After flying 45 miles, the pilot must change course and fly 15° west of north to avoid a thunderstorm.

a. If the pilot remains on this course for 25 miles, how far will the plane be from city B?
b. How many degrees will the pilot need to turn to the right to fly directly to city B?

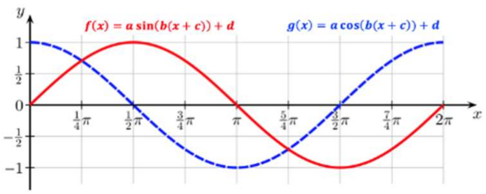
Answers:

a. $c \approx 46.3\ miles$
b. $measure\ of\ angle\ A \approx 157.02°$



Indicator: Two fire towers are 21 miles apart. Tower B is due East of Tower A. The rangers at Tower A spot a fire at 43°north of east. The rangers at Tower B spot the same fire 31° north of west. How far is Tower B from the fire?

Answer: $x \approx 14.899\ miles$

OCS Priority Standard: NC.M4. AF.2.3	
NC.M4. AF.2.3 Interpret key features (amplitude, period, phase shift, vertical shifts, midline, domain, range) of models using sine and cosine functions in terms of a context.	
Clarification	**Checking for Understanding**
This standard is an extension of standards in NC Math 3 which addressed the key features and parameters of the sine function. This standard extends to the key features of the cosine function. Additionally, phase shift for trigonometric functions is a new concept in NC Math 4. It is appropriate to make sure that students understand the relationship between the sine and cosine functions in relation to their key features. Students will be required to determine the meaning of variables and coefficients of trigonometric models and translate between different representational forms of trigonometric models (i.e. graphs, tables, and algebraic formats). The standard for sine and cosine functions: $f(x) = a\ sin\ (b(x+c)) + d$ $g(x) = a\ cos\ (b(x+c)) + d$ where $a, b, c\ and\ d$ are constants.  	**Indicator**: The depth of water near a boat dock is collected by a buoy. The data shows that the water level reached 11 ft. during high tide at 2:00 am and a level of 8 ft. during low tide at 9:00 am. The depth can be modeled by a sinusoidal function. a. How deep will the water be at noon? b. For approximately what times will the water level be less than 9 ft? *Answers:* a. $f(x) = 1.5\sin\left(\frac{\pi}{7}(x+1.5)\right) + 9.5\Big), f(12) = 9.17\ ft.$ b. $Approximately\ 6:00\ am\ to\ 12:00\ am, repeats\ every\ 14\ hours.$
OCS Priority Standard: NC.M4.AF.5.1	
NC.M4.AF.5.1 Construct regression models of linear, quadratic, exponential, logarithmic, & sinusoidal functions of bivariate data using technology to model data and solve problems.	
Clarification	**Checking for Understanding**
Students have previously worked with scatter plots and lines of best fit in 8th grade. In NC Math 1, students have done linear and exponential regression using technology. They have also examined residual plots to determine linearity. In NC Math 4, students will also construct regression models for quadratic, logarithmic and sinusoidal functions of bivariate data. Additionally, students will be able to use regression models to solve real-world and mathematical problems.	**Indicator**:

a. Analyze the scatterplot of the cost of a 30-second advertisement for the Super Bowl from 1967-2011. Describe the patterns in the data.

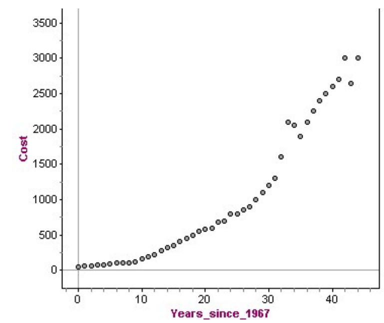
b. Calculate a quadratic regression model for *x* years after 1967.

c. Based on your model, what is the predicted cost of a 30-sec. advertisement in 2007? How accurate is your prediction model's cost compared to the actual cost?

d. Based on your model, what is the predicted cost for a 30-sec. advertisement in 2020?

Year of Super Bowl	Cost of 30-sec Advertisement (in thousands)	Year of Super Bowl	Cost of 30-sec Advertisement (in thousands)
1967	40	1990	700
1968	54	1991	800
1969	67.5	1992	800
1970	78.2	1993	850
1971	72	1994	900
1972	86	1995	1000
1973	103.5	1996	1100
1974	107	1997	1200
1975	110	1998	1300
1976	125	1999	1600
1977	162	2000	2100
1978	185	2001	2050
1979	222	2002	1900
1980	275	2003	2100
1981	324.3	2004	2250
1982	345	2005	2400
1983	400	2006	2500
1984	450	2007	2600
1985	500	2008	2700
1986	550	2009	3000
1987	575	2010	2650
1988	600	2011	3000
1989	675		

Answer:



a. The scatterplot is positive and nonlinear. The cost of a 30-second advertisement increases as the years increase.

b. $y = 1.916x^2 - 15.510x + 104.408$, x=years since 1967 and y=predicted cost of a 30-second advertisement.

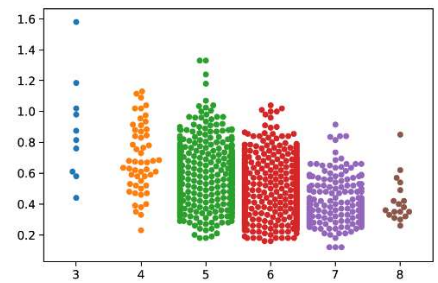
c. In 2007, the predicted cost was roughly $2,550,000. This is fairly close to the actual recorded cost which was $2,600,000.

d. In 2020, the predicted cost is approximately $4,660,000.

Data obtained from: https://www.businessinsider.com/cost-super-bowl-ads-through-the-years-2011-2

Unit 5

Exploratory Data Analysis



Unit 5: Exploratory Data Analysis

Source: NC^{2}ML Retrieved from: https://www.nc2ml.org/wp-content/uploads/2020/06/NC-Math-4-Instructional-FrameworkFINAL2.pdf

OCS Priority Standard(s):	***Supporting Standard(s):***
NC.M4.SP.1.2 Design sample surveys and comparative experiments using sampling methods to collect and analyze data to answer a statistical question.	**NC.M4.SP.1.1** Construct statistical questions to guide explorations of data in context. ***[For example, a question that: anticipates variability, is answerable with data, states the population under consideration, states the attribute under investigation, and is clear enough to guide the analysis of the data]*** **NC.M4.SP.1.3** Organize large datasets of real-world contexts (i.e. datasets that include 3 or more measures and have sample sizes >200) using technology (e.g., spreadsheets, dynamic data analysis tools) to determine: types of variables in the data set, possible outcomes for each variable, statistical questions that could be asked of the data, and types of numerical and graphical summaries could be used to make sense of the data. **NC.M4.SP.1.4** Interpret non-standard data visualizations from the media or scientific papers to make sense of real-world phenomena. ***[This standard is not referring to traditional graphs such as a histogram, dot plot, boxplot, bar graph, pie chart, or stem and leaf plot]***

What is the Mathematics?

- When teaching statistics, it is important to realize that statistics is generally considered a distinct, but heavily overlapping discipline, with mathematics. With these differences come some important considerations in teaching statistics compared to teaching mathematics. To begin, context is central to the practice of statistics. As a result, it is important that all work in the statistics units is situated in a context and ideally ones that are real and relevant to your students. Another big difference is the omnipresence of variability. Statistics involves studying data that vary meaning that it is important to consider variability in the practice of statistics, which comes from many sources including measurement, natural, induced, and sampling. This means in teaching statistics it is important to always describe the variability in a distribution using both numerical summaries such as standard deviation, Interquartile range, and the five-number summary, as well as graphical summaries that help you to see the variability in the data and look for patterns in that variability. Another difference that falls out of the omnipresence of variability is the consideration of uncertainty. In terms of teaching, this means that the language we use in statistics is much less deterministic, normative or certain. In other words, we do not talk about proof so much as creating well-reasoned arguments with databased evidence. The measurement of uncertainty is also where the field of probability comes into play which is mathematical in nature.

- Core to teaching statistics is that because statistics is a methodological discipline it should be taught as an investigative process that includes formulating statistical questions, collecting data or finding data that can be used to answer your questions, analyzing data to make sense of the data related to a question posed, and finally interpreting the data and data analysis in context to answer a statistical question. One of the main goals of this unit is to emphasize the statistical investigation cycle and build upon and make connections between the statistics concepts that students have learned in their previous mathematics coursework. Exploring real data and asking and answering statistical questions should be the drive of each lesson in this unit.
- In relation to statistical questions in the investigative cycle, In middle school (6.SP.1), students learn how to identify statistical questions as well as determine whether or not a question is a statistical question. Statistical questions are questions that allow for data collection and will have variability in the data. Issues that may arise in a question include vagueness, biased tones, wordiness, confusing answer choices, unreasonable or unrealistic responses, or deterministic responses. Additionally, in Math 3, students are asked what statistical question could you answer using the data. In this class, students will be expected to construct their own statistical questions, in which they identify their population of interest and the aspect of that population that they wish to study. This is one of the foundations of statistics. When creating statistical questions researchers suggest using the following criterion:
    - The variable(s) of interest is/are clear and available.
    - The population of interest is clear.
    - The intent is clear.
    - The question can be answered with the data.
    - The question is one that is worth investigating – is interesting, has a purpose.
    - The question allows for analysis to be made of the whole group. (Arnold, 2013, p.111)

    When having students create their own statistical questions it is important to give them opportunities to share with other students and have them evaluate the questions based on the criterion, which connects to SMP3.
- In relation to collecting data in the statistical investigation cycle a foundational pillar of statistics is a good experimental or sampling design for data collection. An important consideration in data collection is what is the population under consideration as well as how can a representative sample be drawn from that population such that statistical inference is possible. An important note here is that collecting data does not necessarily mean collecting raw data every class period. Collecting data also involves looking at data that has already been collected and evaluating how it could be used to address a statistical question posed. In teaching data collection, it is ideal to provide examples in contexts students are familiar with such as considering their school as a population and ways of sampling from that population such as stratified random samples such that grade levels are strata, or perhaps cluster sampling where classes during a particular period are clusters that are randomly selected. In collecting a representative sample, it is ideal to have a randomized sample to reduce bias and make statistical inferences. To highlight this to students it is useful to have them collect data in nonrandom ways to then see the bias in data collected and then show examples of how to collect data randomly. There are multiple ways to collect a randomized sample: simple random sampling, stratified random sampling, cluster random sampling, and systematic random sampling. There are also non-randomized sampling methods that should be avoided when possible, due to bias, but are necessary for some studies: convenience sampling, volunteer response bias, and non-response bias.
    - Simple Random Sampling: A simple random sample is a sample that is taken by randomly selecting individuals from a list that represents the population (university listserv, for example). This can be done by pulling names out of a hat, flipping a coin, using a random number generator, or using a random digit table. It is important to note that individuals cannot be repeated in a sample.

- Stratified Random Sampling: A stratified random sample is a sample that is taken by randomly selecting individuals from lists that represent strata. A strata is a group of individuals that has something in common with each other and different from every other strata. For instance, you could stratify a sample on grade level (freshman, sophomore, junior, or senior), where each individual in the freshman group is the same age and different ages than the other groups, and then you would randomly select individuals from each group. The idea here is that the sample needs to include members from each group, so you collect this type of sample to ensure that all members are represented.
- Cluster Random Sampling: A cluster random sample is a sample that is taken by randomly selecting clusters of individuals that represent the whole and (usually) surveying all of the individuals in that cluster. In this case, the groups (clusters) should be similar in make-up from cluster to cluster, and the individuals in the cluster should represent the population. An example of this would be to randomly select homerooms in a school and survey all of the individuals in the room.
- Systematic Random Sampling: A systematic random sample is a sample taken by randomly selecting a starting point on a list of all of the individuals in the population, and then taking individuals that are every nth number of individuals away from each other. For instance, a systematic random sample of published phone numbers may come from randomly selecting a starting point in the phone book and then using every tenth phone number. This type of sampling is generally being abandoned by statisticians but is still an efficient way of collecting a random sample.
- Non-randomized sampling methods that should be avoided, due to bias:
  - Convenience Sampling: A convenience sample is a sample taken by surveying individuals that are conveniently located. An example of this would be standing outside of a store in a shopping center and asking everyone that walks by. This type of sampling would most likely result in under-coverage bias because this is not guaranteed to capture a representative sample of the population of interest. For instance, you might miss all of the people who do all of their shopping online or don't go shopping.
  - Voluntary Response Bias: Volunteer response bias results from placing a survey in a location where individuals must volunteer to respond. This includes asking people to call in on a radio, asking them to go online and take the survey, asking them to send an email or leave a review. This type of sample relies on individuals actively working to respond to a survey, so it risks only having extreme responses.
  - Nonresponse Bias: This occurs when an attempt at a census (surveying every individual in the population) is done, and individuals who don't respond are part of a common group that doesn't get represented in the sample. An example of when this might occur is sending out an email blast that people need to respond to. Potentially, people who are very busy will not be represented because they will never have the time to respond, among other groups who might not respond.
- Randomized Experimental Design requires both the randomized sampling from above AND random assignment of treatments and placebos. This might look like: "For this experiment, a simple random sample was used to select 100 individuals, then the treatment was randomly assigned to fifty individuals and the placebo assigned to the rest."
- Other considerations in experimental design would be to design single-blind and double-blind situations, where either the individual is unaware of which treatment they are receiving, the researcher is unaware of which treatment each individual received, or both (double-blind).

- In terms of analyzing data most analysis should be done with the support of technology, which relates to SMP5. Most studies in this day and age collect large amounts of data, sometimes multiple years' worth of data values (see important considerations section for links to large data set archives), and then use programs to run statistical analyses. In this unit, students should learn how to input a large data set into one of these programs, such as

CODAP, Tuva, Excel, or Desmos, and then have those technologies produce the numerical summaries and graphical displays (which have been covered heavily in grades 6-12) that students have deemed appropriate for this data set.

  - When discussing the data set, students should be able to determine if a variable is quantitative (numerical, where an average would make sense) or categorical (qualitative, often in the form of words, but could be something like jersey numbers or zip codes, where an average would not make sense).
  - Then, they should further be able to determine if a quantitative variable is discrete or continuous and if a categorical variable is ordinal (ordered, such as rate it 1-5, with 1 at the lowest and 5 is the highest) or non-ordinal, and which numerical summaries and graphical displays would be reasonable to use to describe each type of variable data.
  - Numerical summaries for quantitative variables include but are not limited to mean, median, minimum, maximum, range, quartile 1, quartile 3, interquartile range (quartile 3-quartile 1), and standard deviation. Numerical summaries for categorical variables include but are not limited to frequencies (counts of individuals in each category), and relative frequencies (proportions of individuals in each category).
  - Graphical displays include but are not are not limited to box plots, histograms, bar charts, pie charts (not a statistical favorite), segmented bar charts, scatterplots, frequency plots, dot plots, etc.
  - This is also an opportunity for students to practice constructing appropriate statistical questions, but this time they would do so using the large data set, rather than designing their own data collection methods.

- In terms of interpreting data, it is important that all interpretations are based on specific evidence from the data. Furthermore, it is important to not overgeneralize!! At best, surveys and observational studies allow you to observe some relationships and possible differences that are occurring in the data. Experiments attempt to pinpoint specific relationships by controlling other factors that might affect the relationship, and potentially could allow for conversation about causation, but a good statistician will never state cause and effect because even the best experiments have the potential to miss an important feature affecting the relationship. In this unit all of the generalizations and conclusions should be descriptive and not go beyond the sample collected.
- In the media and workplaces, many different visual displays are used to describe data, and are often different from the standard representations that have been discussed previously. Students need to be able to analyze these different displays, based on the scales and descriptions that are provided on the image. In the important considerations section, there is a link to non-standard visual displays that have been used in the media recently, but feel free to explore your local newspapers and social media to find non-standard graphs and images to analyze.
- For further reading on the teaching of statistics we recommend you consult the Guidelines for Assessment and Instruction in Statistics Education from the American Statistical Association (https://www.amstat.org/asa/files/pdfs/GAISE/GAISEPreK-12_Full.pdf). Also, there are good resources on the NC2ML webpage including research briefs that would be good resources to consider.

Important Considerations:

- Consider that some of these standards are not intended to be a standalone day of instruction. For example, S.1.4 would be better done by incorporating it throughout the unit and having classroom discussions on these nontraditional data visualizations.

- Teachers do have access to statistical technology. Some suggestions are CODAP (free and online), Tuva (partially free and online), Excel, and Desmos (free and online). A neat option for technology-based schools is R: because this is a true technology platform used in the field of statistics and it's free and would fit well in conjunction with teaching computer sciences or computational thinking.
- **SP.1** is intended to ensure that, throughout every piece of this unit, teachers are helping students explicitly make connections to the other pieces of the statistical analysis cycle that they are doing. To learn more about this, go to the k-12 GAISE report (https://www.amstat.org/asa/files/pdfs/GAISE/GAISEPreK-12_Full.pdf) and read the first chapter (pages 11-21). You can also read NC2ML's research briefs for Statistical Reasoning and Literacy (https://www.nc2ml.org/wp-content/uploads/2019/01/BRIEF-67-stats-MS.pdf) and Math 1-3
    - Math 1: https://www.nc2ml.org/wp-content/uploads/2018/10/BRIEF-6-V2-1.pdf
    - Math 2: https://www.nc2ml.org/wp-content/uploads/2018/10/BRIEF-12-V2.pdf
    - Math 3: https://www.nc2ml.org/wp-content/uploads/2018/03/BRIEF-20.pdf)

Resources (Open Access):

- Amazing list of teaching statistics resources updated frequently https://www.amstat.org/asa/files/pdfs/EDU-CommonCoreResources.pdf
- Webinars on teaching statistics https://www.amstat.org/asa/education/K-12-StatisticsEducation-Webinars.aspx
- Real-life graphical and numerical displays for S.1.4
    - https://www.nytimes.com/column/whats-going-on-in-this-graph
    - https://www.amstat.org/ASA/Whats-Going-on-in-this-Graph.aspx
    - https://www.pollingreport.com/
    - https://www.gapminder.org/
    - https://fivethirtyeight.com/
- Large data sets:
    - https://www.ncdc.noaa.gov/data-access
    - https://www.dataquest.io/blog/free-datasets-for-projects/
    - https://github.com/awesomedata/awesome-public-datasets
    - https://ww2.amstat.org/censusatschool/
    - https://www.gapminder.org/
    - https://www.census.gov/data.html
    - https://collegescorecard.ed.gov/data/
    - https://www.cia.gov/library/publications/the-world-factbook/
    - https://www.ipums.org/
    - https://fivethirtyeight.com/

- https://www.pewresearch.org/

- Lesson Plans:
  - https://www.amstat.org/asa/education/stew/home.aspx
  - https://www.engageny.org/common-core-curriculum
  - https://tasks.illustrativemathematics.org/content-standards
  - https://www.mathematicsvisionproject.org/
  - https://www.introdatascience.org/
  - https://www.openintro.org/stat/index.php?stat_book=isrs

Unit 5 Unpacking

Source: NC DPI Math 4 Unpacking Documents. Retrieved from https://www.dpi.nc.gov/nc-math-4-unpacking-rev-june-2022

Supporting Standard: NC.M4.SP.1.1

NC.M4.SP.1.1 Construct statistical questions to guide explorations of data in context.

Clarification

Students were first introduced to statistical questions in 6th grade. They recognize statistical questions as questions that anticipate variability. The focus in NC.6.SP.1 is on the recognition of statistical questions and their distinction from non-statistical questions.

The GAISE Report for Statistics outlines a four-step process for problem-solving as students engage in statistical inquiry:

1. Formulating a statistical question that anticipates variability and can be answered by data.
2. ~~Designing and implementing a plan that collects appropriate data.~~
3. ~~Analyzing the data by graphical and/or numerical methods.~~
4. ~~Interpreting the analysis in the context of the original question.~~

This standard will focus on the first step of the statistical problem-solving process as students construct statistical questions that "anticipate an answer based on data that vary".

Checking for Understanding

Formative check: Determine if the question is a statistical question.

a. How has the number of live births changed over the last 30 years?
b. How many votes did the candidate that won Student Body President receive?
c. How do the heights of basketball players from two rival high schools compare?

Answers: a. Yes, b. No, c. Yes

Indicator: The prom committee is planning the Junior/Senior prom for this year. The committee wants to make sure that they honor the preferences of students in both the junior and senior class. Write a statistical question that can help the committee to determine the student's music preferences.

Possible Answers: What type of music is most popular among students at our school? How do the favorite types of music differ between classes?

OCS Priority Standard: NC.M4.SP.1.2

NC.M4.SP.1.2 Design sample surveys and comparative experiments using sampling methods to collect and analyze data to answer a statistical question.

Clarification

Students have experience with a variety of data collection and analysis tools (numerical summaries and graphical displays) based on the type of data collected (qualitative or quantitative). In NC Math 3, students investigate different methods of data collection (surveys, experiments, observations, etc.), study design and data analysis techniques to best answer a statistical question.

In NC.7.SP.1, students understand sampling and its usefulness in drawing inferences about a population under investigation.

Checking for Understanding

Indicator: The prom committee is planning the Junior/Senior prom for this year. The committee wants to make sure that they honor the preferences of students in both the junior and senior class.

a. Describe how you would collect data to answer the question: "What type of music is most popular among students at our school?"
b. Describe your plan to analyze the data.

Possible Answers:

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In NC Math 4, students will develop their understanding of sampling methods that are both biased and unbiased in relation to surveys and comparative experiments.
As stated in NC.M4.SP.1.1, the GAISE Report for Statistics outlines a four-step process for problem-solving as students engage in statistical inquiry.
~~1. Formulating a statistical question that anticipates variability and can be answered by data.~~
2. Designing and implementing a plan that collects appropriate data.
3. Analyzing the data by graphical and/or numerical methods.
~~4. Interpreting the analysis in the context of the original question.~~

This standard will focus on the second and third steps of the problem-solving process as students will design surveys and comparative experiments to plan the collection and analysis of the data stemming from the original statistical question.

Students will now combine their understanding of how to construct statistical questions and different sampling methods to create surveys and design experiments to answer statistical questions.

Given a real-world context, students can design an experiment based on random sampling to investigate the phenomenon. Students also understand unbiased probability sampling methods (e.g. SRS, stratified, cluster, systematic, multistage) and use appropriate methods when designing experiments or conducting surveys. Likewise, they are aware of biased sampling methods and avoid them when collecting data.
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a. Survey or questionnaire; the question will have 5 main choices (r&b, rock, rap, country or other)
b. A bar graph will be used to compare preferences; a table could also be created comparing favorite music type to class (junior or senior) to determine preferences by class.

Indicator: An agricultural researcher is investigating the effectiveness of three different brands of weed killer, brands X, Y, & Z. He has set up an area of weeds in a 4 by 4 zone as shown below. Describe how he can create an experiment to compare the effectiveness of the weed killers.

Answer:
The researcher can number the zones 1-16, put the numbers in a hat, and then randomly choose out 3 zones to be assigned to brand X weed killer. Then randomly choose another 3 numbers to assign the zones to brand Y. Again, randomly choose another 3 numbers to assign the zones to brand Z. The last remaining 3 zones will be left alone as a control and no weed killer will be assigned. The researcher will compare the amount of weeds killed in the zones to see which brand is the most effective.
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<td colspan="2">Supporting Standard: NC.M4.SP.1.3</td>
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<td colspan="2">NC.M4.SP.1.3 Organize large datasets of real-world contexts (i.e. datasets that include 3 or more measures and have sample sizes >200) using technology (e.g., spreadsheets, dynamic data analysis tools) to determine: types of variables in the data set, possible outcomes for each variable, statistical questions that could be asked of the data, and types of numerical and graphical summaries could be used to make sense of the data.</td>
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<tr>
<td>Clarification</td>
<td>Checking for Understanding</td>
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<td>Up until this point, students have worked with relatively small data sets so that they better understand the tools used for statistical analysis. Most statistical phenomena are based on real-world data which can include relatively larger data sets.</td>
<td>Indicator: Use the random sampler at the census@school (https://ww2.amstat.org/censusatschool/index.cfm) to draw a sample of 250 students from NC. First use technology to explore what variables are in the data set.</td>
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Building on the understanding that more sampling may yield more accurate results, students will now work with larger data sets and multiple measures. This will require the use of technology that has the capacity to efficiently output summary measurements and create graphical displays of data for very large data sets. This may include, but is not limited to, statistical software, spreadsheets, and online analytical tools:

- Spreadsheet software, such as Google Sheets and Microsoft Excel can be used at the very minimum.
- CODAP is a free open source software for data analysis.
- Cause Web also has a variety of data collection and analysis tools listed on their site.

a. Create a statistical question you could investigate using the data set.
b. Use technology to create appropriate visualizations of the data that help you to investigate the question you posed.
c. Use technology to find appropriate descriptive statistics to help you to investigate the question you posed.

Supporting Standard: NC.M4.SP.1.4

NC.M4.SP.1.4 Interpret non-standard data visualizations from the media or scientific papers to make sense of real-world phenomena.

Clarification

Exploratory data analysis includes the analysis of graphical displays of data. Traditionally, students have worked with "typical" statistical graphs…dot plots, line plots, box plots, histograms and scatter plots. However, these graphical displays are not widely used when examining real-world phenomena intended for the general public. For example, the following map was used to track the spread of the Coronavirus in the United States at the beginning of the pandemic.

Students understand the display, what information it conveys and how to translate the information into another form (e.g. a table, another graph, etc.).

A useful resource is the *New York Times*' What's Going On in This Graph?

Checking for Understanding

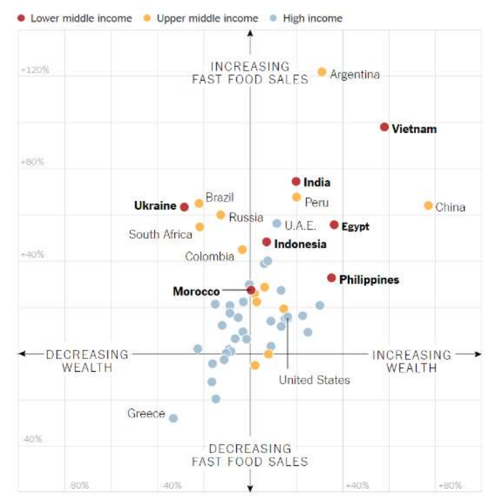
Indicator:

- **What do you notice?**
- **What do you wonder?**

What are you curious about that comes from what you notice in the graph?

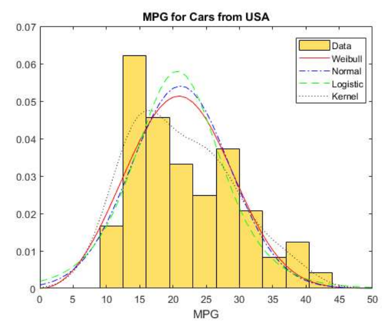
- **What might be going on in this graph?**

Write a catchy headline that captures the graph's main idea. If your headline makes a claim, explain what you noticed that supports your claim.



Unit 6

Probability Distributions



Unit 6: Probability Distributions

Source: NC^2ML Retrieved from: https://www.nc2ml.org/wp-content/uploads/2020/06/NC-Math-4-Instructional-FrameworkFINAL2.pdf

OCS Priority Standard(s):	***Supporting Standard(s):***
	NC.M4.SP.3.1 Implement discrete probability distributions to model random phenomena and make decisions (e.g., expected value of playing a game, etc.). **NC.M4.SP.3.2** Implement the binomial distribution to model situations and make decisions. **NC.M4.SP.3.4** Implement the normal distribution as a probability distribution to determine the likelihood of events occurring.

What is the Mathematics?

- In teaching probability distributions, it is helpful to begin with discrete probability distributions, which can be considered theoretically (based on assumptions about the physical world) and experimentally (based on collecting data from repeated trials). Oftentimes, students are pushed to the theoretical understanding in statistics too fast. It is helpful for students to start with a hands-on activity to create an initial understanding. Then, they can move to a simulation model. Because this is built on the idea of doing the activity many, many times, the simulation helps bridge to the theoretical because we cannot complete the hands-on activity a large (or infinite) amount of times. Lastly, students will move to the theoretical understanding.
- A discrete probability distribution is the distribution of probabilities for the outcomes of a discrete random variable, which is a variable that can take one of a countable list of distinct values. Students will need to understand that in a discrete probability distribution all probabilities are less than or equal to one and the sum of probabilities is equal to one. Also, the expected value of a discrete probability distribution is the mean, or what would happen, on average, over a long series of trials calculated by $\Sigma x_i p_i$, where x_i represents the value of outcome i and p_i represents the probability of outcome i. Students can also think of the expected value as a weighted average.
    - A special type of discrete random variable is a uniform random variable, which is one where the probability of every value is the same. For example, the value of rolling one fair six-sided die.
    - A special type of discrete random variable is a binomial random variable, which applies to situations where the variable describes a binomial experiment. A binomial experiment is an experiment consisting of a fixed number of independent trials (n) each with two possible outcomes, success and failure, and the same probability of success (p). For example, flipping a coin four times and recording the number of times it lands on heads would be a binomial experiment because there is a fixed number of trials (i.e. four), they are independent (i.e. the result of one flip does not impact the results of the next), there are only two outcomes (i.e. heads or tails), and the probability of success is the same for each trial (i.e. 0.5). Students will need to be able to recognize when a situation is binomial and use the binomial distribution to find the probability of

events. There are some nice probabilities to the binomial distribution including that the expected value is np and the standard deviation is sqrt(np(1-p)). In teaching the binomial distribution it is important to eventual show using simulation that as you continue to increase n, the shape of the distribution begins to create a bell-shaped curve which is a powerful basis for then moving into discussing the normal distribution, which is a continuous distribution but can model a binomial distribution under certain condition. An advanced connection for honors students that can be made is to Riemann sums and integrals since the area under a probability function is the probability of an event or events occurring.

  - When teaching discrete probability distributions, it is important to focus on what they tell us about a situation. Some good examples to use with students are what the expected value tells you in a financial situation such as paying to play a game of chance and also in terms of weighted averages such as their GPA. Also, when considering discrete random variables by hand it is important that the number of possible values the variable takes does not become too large (ideally less than 12 or so) as the calculations become tedious.

- A continuous probability distribution is one that tells the probability of intervals of values for a continuous random variable. A continuous random variable is one that can take any value in and interval of collection of intervals. A continuous probability distribution is generally modeled in the form of a smooth continuous curve where the area underneath the curve on an interval represents the probability of that interval of values occurring. There are many different continuous random variables, but the focus of this unit is on the normal distribution. Students will need to understand that the normal distribution is a continuous probability distribution and that the total area under the curve is 1. Teachers should use this opportunity to teach students about standardized scores (z-scores) and explain how standardized scores allow for comparisons when original scales are different (for example: ACT math scores and SAT math scores).
  - Introduce the Empirical Rule (68-95-99.7 Rule) as a visual of probability under the Normal curve with standard deviations. This leads to the Standardized Normal Distribution with a mean of 0 and standard deviation of 1. Students can find probabilities with 1, 2, or 3 standard deviations away from the mean. Then, this will lead into the question, "What happens when I'm at something other than 1, 2, or 3 standard deviations? What about 1.5?" Therefore, there is a need for a z-score (standardized score) and technology to calculate these probabilities. (WE ARE NOT using a z-table for this at all. Technology has taken the place of this outdated way of calculating the probabilities.) *[See video resources below] **Desmos will allow students to do NormalDist. This does provide a nice visual to build conceptual understanding.*
  - It is helpful for students to sketch the normal curve with appropriately shaded regions when you are calculating the probability. This will reinforce the connection and understanding of the probability to the area under the shaded region of the normal curve. Students will use technology to calculate normal curve probabilities.
- ***Teacher Instructional Support:***
  - How to use the Empirical Rule: https://youtu.be/cgxPcdPbujI
  - Z-scores and Normal Distributions: https://youtu.be/NY2zWGBXBhU
  - Using TI-84 for NormalCDF and invNorm (SUPER Important since we don't want the teachers to use the Z-table) https://youtu.be/S5n0pJ1gPDQ

- Binomial Distribution: Explains what a Binomial Distribution is and how the binomial formula is used (gives a pretty detailed explanation but is somewhat dry. I wouldn't necessarily show it to students, but it can teach the teacher.) https://youtu.be/qIzC1-9PwQo
- How to do Binomial PDF and CDF on a TI-84. (Does not explain what to do when you need to find X>n) https://youtu.be/IngLJs6T1Qw

Important Considerations:

- S.3.1 Decision making is important here and can possibly connect to financial math
(gambling - in casino games the expected value is always negative, ensuring that the casino will make money over time).
- A good resource would be to use Desmos to calculate the binomial probability. While the TI84 will let them find a binomialpdf and a binomialcdf, there are sometimes struggles for students in understanding that the binomcdf only produces area equivalent to less than or equal to a number. So Desmos would make this easier, since students can set the bounds.
- S.3.4 Ensure that students understand that the normal distribution is the z-distribution, that students know how to calculate a z-score, and then how to interpret the probability that is produced by the calculator (normalcdf).

Formative Assessments/Tasks:

- **https://tasks.illustrativemathematics.org/content-standards/HSS/MD/A/2/tasks/1023**

Unit 6 Unpacking

Source: NC DPI Math 4 Unpacking Documents. Retrieved from https://www.dpi.nc.gov/nc-math-4-unpacking-rev-june-2022

Supporting Standard: NC.M4.SP.3.1

NC.M4.SP.3.1 Implement discrete probability distributions to model random phenomena and make decisions (e.g., expected value of playing a game, etc.).

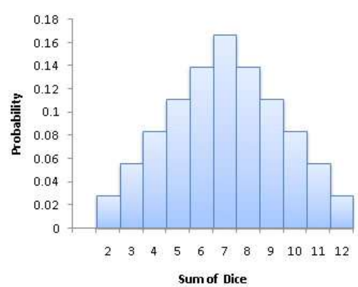
Clarification

Discrete probability distributions are based on **discrete random variables**, where there is a finite number of possible outcomes.

A **discrete probability distribution** lists each outcome and its associated probability. Students can create a probability distribution table listing each outcome and their probabilities.

Students understand that a probability distribution can be used to find the expected value or average weighted outcome of discrete random variables. They also understand expected value in determining fairness of a game.

A **probability histogram** is a graphical display that graphs each outcome on the horizontal axis and its probability on the vertical axis.
For example, the probability histogram on the right represents the sum of the rolls of 2 six-sided dice and the probability of obtaining each sum. The probabilities in a discrete probability distribution should add to 1.



Checking for Understanding

Indicator: The NC lottery has a game called Pick 3 which involves picking three numbers in exact order. It costs a dollar to play and if you win you get $500. What is the expected value of playing this game many times in the long run? Is it a wise investment to play the NC pick 3 in the long run?

Answer:

Possible Outcome	*-$1*	*+$499*
Probability	$\frac{999}{1000}$	$\frac{1}{1000}$

Expected Value = $-1\left(\frac{999}{1000}\right) + 499\left(\frac{1}{1000}\right) = -0.50$

In the long run, a player is expected to lose $0.50 per play. Since the expected value is less than zero, the game isn't fair.

Supporting Standard: NC.M4.SP.3.2

NC.M4.SP.3.2 Implement the binomial distribution to model situations and make decisions.

Clarification

Students understand the four main criteria for a binomial setting:

1. A fixed number of observations, n.
2. Each observation (n) is independent.

Checking for Understanding

Indicator: A test has 10 multiple choice questions each of which has four choices and only one right answer. A student decides to use a spinner with four equivalent sectors on it to randomly choose which answer to pick for

<table>
<tr><td>3. Each observation falls into one of two possible categories, known as success or failure.
4. The probability of success (p) is the same for each observation.

Students also understand that the distribution of the count of X successes in the binomial setting is called the binomial distribution with parameters n and p where $\boldsymbol{n}$ represents the number of observations and $\boldsymbol{p}$ represents the probability of success; abbreviated as X is $B(n, p)$.

Students understand that when n is large, the distribution of X is normal, $N(np, \sqrt{np(1-p)})$ and the normal approximation can be used when *n* and *p* satisfy the following conditions: 1) $np \geq 10 \text{ and } n(1-p) \geq 10$.
*Technology use is at the discretion of the teacher.</td><td>each question. What is the likelihood the students gets the following number of questions correct:
a. 5
b. 10
c. At least 5
d. No more than 2
e. Between 7 and 9, inclusive

Answers:
a. *0.058*
b. *0.0000009*
c. *0.078*
d. *0.244*
e. *0.0035*</td></tr>
<tr><td colspan="2">**Supporting Standard: NC.M4.SP.3.4**</td></tr>
<tr><td colspan="2">**NC.M4.SP.3.4** Implement the normal distribution as a probability distribution to determine the likelihood of events occurring.</td></tr>
<tr><td>**Clarification**</td><td>**Checking for Understanding**</td></tr>
<tr><td>Students know when it is appropriate to use the normal distribution and how to determine probabilities using the normal distribution. They also know how to find an observation from a given probability based on an assumption of the normal distribution.

Students understand that the normal curve is a density curve, where the area under the curve is exactly one and above the horizontal axis.

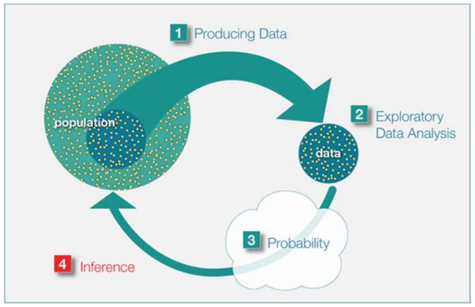
Students understand the empirical (68-95-99.7) rule and how it applies to the normal curve.

Students know that the standard normal distribution $N(\mu, \sigma)$ has mean, μ ,and standard deviation, σ.

Students understand z-scores and know how to find the standardized value of *x* using $z = \frac{x-\mu}{\sigma}$, where z represents the distance that a value is from the mean, μ , in standard deviation, σ , units.</td><td>**Indicator**: During the production of a new part for a turbine, parts must be tested and assessed for quality control. The mean of this new part should be 1.4 cm with a standard deviation of 0.006 cm.
a. What is the probability that one of the new parts produced will be over 1.412 cm long?
b. What is the probability that one of the new parts produced will be under 1.395 cm long?
Answers:
a. *0.025*
b. *0.2023*</td></tr>
</table>

Unit 7

Statistical Inference



Unit 7: Statistical Inference

Source: NC^2ML Retrieved from: https://www.nc2ml.org/wp-content/uploads/2020/06/NC-Math-4-Instructional-FrameworkFINAL2.pdf

OCS Priority Standard(s):	***Supporting Standard(s):***
NC.M4.SP.2.3 Implement a one proportion z-test to determine if an observed proportion is significantly different from a hypothesized proportion. **NC.M4.SP.3.3** Recognize from simulations of sampling distributions of sample means and proportions that a normal distribution can be used as an approximate model in certain situations.	**NC.M4.SP.2.1** Design a simulation to make a sampling distribution that can be used in making informal statistical inferences. **NC.M4.SP.2.2** Construct confidence intervals of population proportions in the context of the data.

What is the Mathematics?

- In considering sampling distributions we need to remember that they are a distribution of statistics of samples of the same size, they are not distributions of individuals like what we have been looking at up until this point. In simulating repeated sampling to create a sampling distribution we will initially use the value of a sample statistic to answer the question, “What are the possible values of the population parameter?” To answer this question the focus initially will be to build on students' understanding of simulation from Math III to focus now on constructing a simulation to create a sampling distribution. Sampling distributions can be conceptually difficult for students to understand and this unit will continue to build on students' previous experiences with them in Math III. Because we are focusing on proportions, one challenge conceptually is that the sampling distribution of a categorical variable is quantitative. To help students understand simulating repeated sampling to create a sampling distribution, it would be helpful to begin with a physical simulation. For example, ask your students to respond to a question that will provide categorical data such as, “How would you rate the importance of washing your hands frequently: not important, somewhat important, or very important?” Have them respond on a sticky note and then have them post their sticky notes on the board and as a class organize the sticky notes in such a way you could see how many students responded for each of the possible outcomes (i.e. some kind of bar graph). Next you want to shift students thinking from just describing the class to making inferences to a larger population such as the school. To do so we must consider a proportion rather than a count. Furthermore, we would expect there to be variation from sample to sample so to estimate what is happening in the population we would need to understand how much variability we might expect by chance. To be able to answer the statistical question of “What proportion of students at ____ High School believe it is very important to wash their hands frequently?” We can begin by treating our sample as a population and then collecting a random sample with replacement of the same size as what our sample was that we started with. Then record the proportion of successes in the case of this example the proportion of responses in the resample that are “very important.” Next draw a sample by random again, and again record the sample proportion. Continue this process several times for students to then see this is tedious and begin to think about how technology could be used here instead. You could then have students think about how to come up with another way to physically simulate drawing a sample (i.e. table of random numbers, drawing responses from a hat, etc.). Then build toward students thinking about how they might use technology such as Excel, CODAP, or Desmos to simulate drawing random samples.
    - Applet for simulations http://www.rossmanchance.com/applets/OneSample.html
    - CODAP works well for simulating repeated sampling for creating a sampling distribution (https://codap.concord.org/).

- For many good resources specific to simulation see pages 32 and 33 of https://www.amstat.org/asa/files/pdfs/EDU-CommonCoreResources.pdf

- For considering when sampling distributions can be modeled by the normal distribution simulations are useful to visualize what a distribution of sample statistics would look like and to then see the conditions necessary for using the normal distribution to model a sampling distribution.
  - We are focused only on recognizing that a sampling distribution can be modeled by the Normal distribution when the sample size is sufficiently large ($np \geq 10$ and $n(1-p) \geq 10$) and observations are independent, but do not expect students to start doing probability with a sampling distribution. This is in AP Statistics. In this course, once students recognize that the sampling distribution would be approximately Normal, they could explain the difference between the original population, the sample measure collected, and the sampling distribution. Furthermore, the focus in this course is not to know what to do when a situation could not be modeled by a normal distribution. Such cases are the focus of college level statistics courses and beyond the scope of what is necessary here.
- In Math 3, students constructed a margin of error using a given formula: 2(standard deviation of a sampling distribution) or by approximating how far from the center they would have to go to capture approximately 95% of the samples in a sampling distribution. You can start with informally building confidence intervals from the simulated sampling distributions created earlier in the unit to build conceptually what a confidence interval is. To construct a confidence interval, students will build on this knowledge by taking a sample statistic and adding and subtracting that margin of error. Confidence intervals tell us how confident we can be that a population *parameter* (the true proportion) is included in the constructed interval. For instance, if our sample proportion, p-hat, is 0.4, the standard deviation of the statistic, phat, could be calculated using $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$. If the sample size were $n = 100$, the standard deviation would be about 0.049. In the previous unit, students learned 1) approximately 95% of the area under a normal curve is within 2 standard deviations of the mean, and 2) a sampling distribution for p-hat is approximately normal with mean p-hat and standard deviation $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$. Thus a 95% confidence interval for p-hat would be 0.4 ± 0.049 or (0.351, 0.449). We would say that "we are 95% confident that the true population proportion is between 0.351 and 0.449". It is important to emphasize that the confidence is in the method used, not in this particular interval. Since the interval created from the sample proportion and there is natural variability from sample to sample, the interval will fluctuate from sample to sample. If we took many samples, approximately 95% of the intervals constructed would capture the true population proportion. It would be INCORRECT to say that the probability that the population parameter falls between 0.351 and .0449 is .95.
  - PBS Crash Course to understanding Confidence Intervals https://youtu.be/yDEvXB6ApWc
- For more support on this:
  - https://istats.shinyapps.io/ExploreCoverage/ (interactive site to explore CI)
- While we use a confidence interval to estimate a population parameter, we use significance tests to evaluate the evidence provided by data about some claim concerning the population parameter. Introducing students to hypothesis testing helps them to understand what we mean by "statistically significant". A simple introductory activity would be to tell students that you are going to walk around the room and flip a coin for each of them: tails they are not selected, heads they get some reward. As you walk around, flip the coin and announce the result (always "Tails") without allowing students to actually see the coin. After a certain amount of time, students are going to start to question the fairness of the coin. This will prompt a conversation

about "at what point would you be convinced the coin is not fair, that this phenomenon is not occurring just due to chance?" The p-value of a hypothesis test is the probability of getting sample results as extreme or more extreme than the sample results, assuming the null hypothesis is true. One general rule of thumb is that if the p-value is less than 5%, we would be convinced that it is highly unlikely for this outcome to have happened just due to chance and we, therefore, have evidence against the null hypothesis in favor or the alternative hypothesis.

- It is important to emphasize to students that we never prove a null hypothesis is true and we never accept an alternative hypothesis. We only have evidence against the null hypothesis (in which case we reject the null hypothesis) or we do not have evidence against the null hypothesis (in which case we fail to reject the null hypothesis).
- It is also important to emphasize that the level of significance is not always .05. Depending on the context in a real-life application, it might be desirable that the p-value be .01, for example, in order to reject the null hypothesis.

Important Considerations:

- It is important that you understand how to create a simulation that is able to demonstrate repeated sampling in order to build a sampling distribution.
- Informally check the conditions and/or make sure to use the conditions language, such as "the conditions have been met." The conditions are: 1) the sample is a random sample from the population of interest (this is to avoid bias: systematically overestimating or underestimating the population proportion), 2) the population is at least 10 times the sample size (this allows us to use the formula for standard deviation of the sample proportion; since we are sampling without replacement, having such a large population to sample from means that the probability of success does not really change significantly from one draw to the next), 3) np (or n$*$ p-hat) and n(1-p) (or n$*$ (1-phat)) are both at least 10 (in the last unit we learned that the sampling distribution of p-hat is approximately normal if np and n(1-p) are both at least 10).
- When calculating a standardized test statistic for a one-proportion z-test, we assume the null hypothesis is true: $z = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o(1-p_o)}{n}}}$ where p_o is the proportion from the null hypothesis.
- Friday Institute has 2 Massive Open Online Courses that can be taken for free online that help teach statistics conceptually.
  - https://www.fi.ncsu.edu/projects/teaching-statistics-through-inferential-reasoningmooc-ed/
  - https://www.fi.ncsu.edu/projects/teaching-statistics-through-data-investigations-mooced/
- Against All Odds Videos - https://www.learner.org/series/against-all-odds-inside-statistics/
- S.3.3 Use simulations of sampling distributions!!! And begin to develop a conceptual understanding of a sampling distribution.

Formative Assessments/Tasks:

- **https://www.statsmedic.com/intro-day2 (**The "1 in 6 wins" activity is an introduction to the thinking behind a one proportion z test)

Unit 7 Unpacking

Source: NC DPI Math 4 Unpacking Documents. Retrieved from https://www.dpi.nc.gov/nc-math-4-unpacking-rev-june-2022

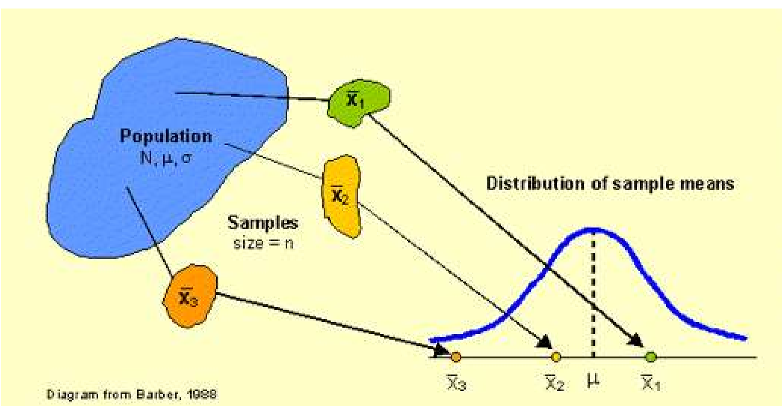
Supporting Standard: NC.M4.SP.2.1

NC.M4.SP.2.1 Design a simulation to make a sampling distribution that can be used in making informal statistical inferences.

Clarification

Formal statistical inference is new to students in NC Math 4. Students were briefly introduced to simulation in the middle grades (7.SP.2). They understand that random sampling (or simulated samples) can be used to gauge variation and to make statistical inferences. Students also understand the difference between a **statistic** and a **parameter** and how statistics are used to estimate population parameters.

This standard explicitly has students design a simulation and use the information to create a sampling distribution to make informal inferences. Students will understand that a sampling distribution is a graph of a statistic generated from multiple samples of the same size from the population of interest (see figure below).



Students understand that sampling distributions can be created by collecting sample means or sample proportions.

Checking for Understanding

Indicator: Collect the heights of all the students in your math class measured in centimeters. Also record each students sex. Find the average height of males in your class and of females.

a. Would you expect to get the same average heights if you collect height and sex data from another class in your school?
b. How could you simulate collecting height data from 100 classes with 25 students in them using the sample you collected from your class?

Carry out your simulation for the height of females and approximate what the average height of females is based on your sample. The Center for Disease Control reported the average height of women ages 20 and over in 2015-2016 to be 161.5 cm
(source: https://www.cdc.gov/nchs/data/nhsr/nhsr122-508.pdf).

c. How likely is it to get the average height of women reported by the CDC given your sample?
d. Explain why you might be seeing the results you are seeing, including some of the limitations in making inferences from your sample.

Supporting Standard: NC.M4.SP.2.2	
NC.M4.SP.2.2 Construct confidence intervals of population proportions in the context of the data.	
Clarification	**Checking for Understanding**
Students have worked with statistical measurements for center and variability. They have graphed data and analyzed summary statistics. Students understand that the sampling distribution of the sample proportions takes on the shape of the normal distribution for larger sample sizes and as the number of samples from the population increases. Yates, Moore, and Starnes (2002) state that "**a confidence interval** uses sample data to estimate an unknown population parameter with an indication of how accurate the estimate is and of how confident we are that the result is correct." Students will understand the connection between repeated sampling and the construction of the confidence interval. Students will use $\hat{p} \pm z * \sqrt{\frac{p(1-p)}{n}}$ to construct the confidence interval; students understand all constants and coefficients of the formula and can interpret confidence intervals in the context of the problem. Students also understand the conditions by which it is appropriate to use a sample to construct a confidence interval for a given sample. Students in NC Math 4 will construct confidence intervals for population proportions only. In NC Math 4, calculation of the margin of error becomes formalized; extending from informal understanding based on simulation in NC Math 3.	**Indicator**: Devise and carry out a plan to investigate, "what proportion of students at your school plan on going to a college or university after graduating." After collecting your data create a 95% confidence interval of the proportion of students reporting planning on attending a college or university after graduating and interpret what that interval tells you relative to the statistical question under investigation. *Possible Answer:* Create a survey to determine the post-secondary plans of all students in your school. It is impossible to do a consensus because there are 1300 students in your school and you do not have the resources or the time to get information from all of them. Since you want to make sure that you capture a representation of all students in the school, you decide to do stratified sampling by grade level. You ask students to complete your survey during grade level assemblies organized by the administration and you collect the surveys. After collecting the surveys, you find that 29 of the 75 students completing the survey indicate that they're going to college or university after graduating. $n = 75;\ \hat{p} = .39\ z* = 1.96$ $n \cdot 10 = 75 \cdot 10 = 750; 750 < 1300$ $n \cdot p = 75(.39) = 29.25; n \cdot (1-p) = 75(.61) = 53.25$ $\hat{p} \pm z * \sqrt{\frac{p(1-p)}{n}} \to .39 \pm (1.96)\sqrt{\frac{(.39).61}{75}}; 0.39 \pm .1104$ *We are 95% confident that the percent of students going to college or university after high school from this school lies between 28% and 50%.*
OCS Priority Standard: NC.M4.SP.2.3	
NC.M4.SP.2.3 Implement a one proportion z-test to determine if an observed proportion is significantly different from a hypothesized proportion.	
Clarification	**Checking for Understanding**
Hypothesis testing is the use of statistics to determine the probability that a given hypothesis is true. Students understand hypothesis testing as a formal statistical procedure for determining the true population parameter; in this	**Indicator**: According to the CDC the relative frequency of males born in the U.S. is .512 (source: https://www.cdc.gov/nchs/data/nvsr/nvsr53/nvsr53_20.pdf) In a random sample of 200 babies born in North Carolina 120 were males.

case students will be determining the probability that a sample proportion is the true population proportion based on a one-proportion z-test. In order to fully understand the process of hypothesis testing, students should engage in the entire process and relate each step to the context of the problem. A basic process includes the following steps: ***Step 1:*** *State the null and alternative hypotheses using symbols and in the context of the problem.* $H_0: p = p_0$ $H_a: p \neq p_0$ or $H_a: p < p_0$ or $H_a: p > p_0$ ***Step 2:*** *Verify the conditions of the inference procedure (one-proportion z-test).* *If $np_0 \geq 10$ and $n(1-p_0) \geq 10$, then we can use the normal distribution* ***Step 3:*** *Carry out the procedure by hand or using technology. Find the z-statistic and the corresponding p-value in the z-table.* $$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$ ***Step 4:*** *Interpret the results in the context of the problem; stating whether to reject or to fail to reject the null hypothesis. Measure statistical significance against a predetermined significance level (0.05 unless otherwise stated).* • *If $p - value \leq a$; reject H_0: the difference between the proportions is statistically significant.* • *If $p - value > a$; fail to reject H_0: the difference between the proportions is NOT statistically significant.*	a. Is this unusual? b. Is 120 out of 200 babies significantly different than the proportion that the CDC reports? *Answers:* a. $\hat{p} = .60$; This is not usual because there can be a lot of variation in individual samples. b. S1: $H_0: p = .512$ (51.2% of babies born in the US are male); $H_a: p \neq .512$ (the percentage of males born in the US is something other than 51.2%) S2: $200(.512) = 102.4; 102.4 \geq 10$ and $200(.488) = 97.6; 97.6 \geq 10$ S3: $z = \frac{.6-.512}{\sqrt{\frac{.6(.4)}{200}}} = \frac{.088}{.0346} = 2.54; p = .9945$ S4: $.0128 \leq .05$ so we reject the null hypothesis; There is enough evidence to say that the proportion of males born in the US is something different than what the CDC reports.

OCS Priority Standard: NC.M4.SP.3.3

NC.M4.SP.3.3 Recognize from simulations of sampling distributions of sample means and proportions that a normal distribution can be used as an approximate model in certain situations.

Clarification	Checking for Understanding
Students understand that a sampling distribution is a graphical display of a statistic for sample data. They also understand that the sampling distribution is a probability distribution that is obtained from the collection of a large number of samples of the same size. It represents a wide range of possible outcomes of a statistic for a given population.	**Indicator**: In a recent NPR/Ipsos poll of 1,007 U.S. adults conducted March 21-22, 2019 "78% of Americans support schools teaching about climate change" (source:https://www.ipsos.com/en-us/news-polls/four-five-parents-want-schools-teach-about-climate-change-04-22-2019). a. How could you simulate a sampling distribution of sample proportions based on this information?

Students understand that there is less variability in larger sample sizes than in smaller sample sizes (less than 30). So, the sampling distribution becomes more *like* the normal distribution as sample size increases and more samples are drawn. Students understand that they can use the normal approximation, $N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$, for the probability distribution of **sample proportions** when: 1. The population is at least 10 times as larger as the sample 2. $np > 10$ and $n(1-p) > 10$ Students understand that they can use the normal approximation, $N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$, for the probability distribution of **sample means** when the population is at least 10 times as larger as the sample.	b. If we assume this sample is representative of the people of the United States, what probability distribution could be used to approximate the sampling distribution? c. Discuss what information you are relying on to make this conclusion. *Answers:* a. *Collect 100 samples of* $n = 50$ *and create a sampling distribution of the sample proportions and estimate the center, shape, and spread of the sampling distribution.* b. *A normal distribution with a mean* $u_p = .78$ *and standard deviation* $\sigma_p = \sqrt{\frac{p(1-p)}{n}} = .013$*;* c. *The* a *US population is larger than 10,070 and* $1007(.78) \geq 10$ *and* $1007(.22) \geq 10$*. So, the normal approximation can be used.*

TOOL 2

HESS COGNITIVE RIGOR MATRIX (MATH-SCIENCE CRM):

Applying Webb's Depth-of-Knowledge Levels to Bloom's Cognitive Process Dimensions

Revised Bloom's Taxonomy	Webb's DOK Level 1 Recall & Reproduction	Webb's DOK Level 2 Skills & Concepts	Webb's DOK Level 3 Strategic Thinking/Reasoning	Webb's DOK Level 4 Extended Thinking
Remember Retrieve knowledge from long-term memory, recognize, recall, locate, identify	o Recall, observe, & recognize facts, principles, properties o Recall / identify conversions among representations or numbers (e.g., customary and metric measures)	Use these Hess CRM curricular examples with most mathematics or science assignments or assessments.		
Understand Construct meaning, clarify, paraphrase, represent, translate, illustrate, give examples, classify, categorize, summarize, generalize, infer a logical conclusion), predict, compare/contrast, match like ideas, explain, construct models	o Evaluate an expression o Locate points on a grid or number on number line o Solve a one-step problem o Represent math relationships in words, pictures, or symbols o Read, write, compare decimals in scientific notation	o Specify and explain relationships (e.g., non-examples/examples; cause-effect) o Make and record observations o Explain steps followed o Summarize results or concepts o Make basic inferences or logical predictions from data/observations o Use models /diagrams to represent or explain mathematical concepts o Make and explain estimates	o Use concepts to solve non-routine problems o Explain, generalize, or connect ideas using supporting evidence o Make and justify conjectures o Explain thinking/reasoning when more than one solution or approach is possible o Explain phenomena in terms of concepts	o Relate mathematical or scientific concepts to other content areas, other domains, or other concepts o Develop generalizations of the results obtained and the strategies used (from investigation or readings) and apply them to new problem situations
Apply Carry out or use a procedure in a given situation; carry out (apply to a familiar task), or use (apply) to an unfamiliar task	o Follow simple procedures (recipe-type directions) o Calculate, measure, apply a rule (e.g., rounding) o Apply algorithm or formula (e.g., area, perimeter) o Solve linear equations o Make conversions among representations or numbers, or within and between customary and metric measures	o Select a procedure according to criteria and perform it o Solve routine problem applying multiple concepts or decision points o Retrieve information from a table, graph, or figure and use it solve a problem requiring multiple steps o Translate between tables, graphs, words, and symbolic notations (e.g., graph data from a table) o Construct models given criteria	o Design investigation for a specific purpose or research question o Conduct a designed investigation o Use concepts to solve non-routine problems o Use & show reasoning, planning, and evidence o Translate between problem & symbolic notation when not a direct translation	o Select or devise approach among many alternatives to solve a problem o Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze Break into constituent parts, determine how parts relate, differentiate between relevant-irrelevant, distinguish, focus, select, organize, outline, find coherence, deconstruct	o Retrieve information from a table or graph to answer a question o Identify whether specific information is contained in graphic representations (e.g., table, graph, T-chart, diagram) o Identify a pattern/trend	o Categorize, classify materials, data, figures based on characteristics o Organize or order data o Compare/ contrast figures or data o Select appropriate graph and organize & display data o Interpret data from a simple graph o Extend a pattern	o Compare information within or across data sets or texts o Analyze and draw conclusions from data, citing evidence o Generalize a pattern o Interpret data from complex graph o Analyze similarities/differences between procedures or solutions	o Analyze multiple sources of evidence o Analyze complex/abstract themes o Gather, analyze, and evaluate information
Evaluate Make judgments based on criteria, check, detect inconsistencies or fallacies, judge, critique	"UG" – unsubstantiated generalizations = stating an opinion without providing any support for it!		o Cite evidence and develop a logical argument for concepts or solutions o Describe, compare, and contrast solution methods o Verify reasonableness of results	o Gather, analyze, & evaluate information to draw conclusions o Apply understanding in a novel way, provide argument or justification for the application
Create Reorganize elements into new patterns/structures, generate, hypothesize, design, plan, produce	o Brainstorm ideas, concepts, or perspectives related to a topic	o Generate conjectures or hypotheses based on observations or prior knowledge and experience	o Synthesize information within one data set, source, or text o Formulate an original problem given a situation o Develop a scientific/mathematical model for a complex situation	o Synthesize information across multiple sources or texts o Design a mathematical model to inform and solve a practical or abstract situation