Calculus Summer Assignment: Prerequisites for Calculus

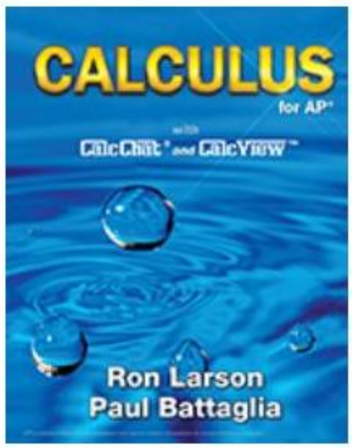
Name:______________________________Grade:__________ Due: Sept, 2024 (First Week of Class)

Welcome to Calculus! These exercises will help you get ready for class. Explanations are provided, and there are problems for you to practice. They will be due sometime during the first week of class (not the first day). There will be a quiz during the second week of class on this material.

Materials you will need for this course: Textbook: *Calculus* Seventh Edition, Larson, Hostetler, Edwards – Houghton Mifflin ISBN 0-618-14918-x

Text: Calculus for AP with CalcChat and CalcView,Larson and Battaglia
Cengage Learning

Textbook Image:



(Don't worry that the textbook is for AP - this is not an AP class but the textbook is still relevant to us)

Other supplies: Graph paper spiral notebook
Pencils, eraser, TI-83/84 Calculator

Note: Students will need a TI-84+ CE calculator for this course. **Students will not be able to use a graphing calculator that can download documents from computers such as the TI-nspire CX or any graphing calculator that says CAS (Computer algebra system)**

Please PRINT this document and show all work ON THESE PAGES; do NOT turn in any other papers or scratch work. Also please keep these pages IN ORDER. Email me if you have any questions: jdesantis@dcds.edu.

◆ Skill A Writing an equation of a line in slope-intercept form

Recall The slope-intercept form of a line is $y = mx + b$.

$$y = \underset{\text{slope}}{\underset{\uparrow}{m}}x + \underset{y\text{-intercept}}{\underset{\uparrow}{b}}$$

◆ **Example**

Write an equation for each line.

a. containing (0, 1) and with a slope of −2

b. containing (3, −4) and (9, 0)

◆ **Solution**

a. The slope, m, is given as −2. The line contains (0, 1), so this point is the y-intercept, or b is 1. Substituting these numbers into the equation gives $y = -2x + 1$.

b. First find the slope. $m = \frac{-4 - 0}{3 - 9} = \frac{-4}{-6} = \frac{2}{3}$

Then subtitute the coordinates of one of the given points into the equation and solve for b.

For the point (9, 0): $0 = \frac{2}{3}(9) + b$

$0 = 6 + b$

$b = -6$

Substituting this number for b and $\frac{2}{3}$ for m into the equation $y = mx + b$ gives the equation $y = \frac{2}{3}x - 6$.

For each equation, find the slope and the *y*-intercept.

1. $y = 3x - 1$ ______________

2. $y = \frac{1}{2}x + 2$ ______________

3. $y = -x + \frac{1}{2}$ ______________

Write an equation in slope-intercept form for each line.

4. with a slope of 2 and a y-intercept of −1 ______________________________

5. containing (0, −3) and with a slope of $\frac{1}{3}$ ______________________________

Write an equation in slope-intercept form for the line that contains each pair of points.

6. (1, 1) and (3, 5) ______________

7. (2, −4) and (−1, 5) ______________

8. (2, 4) and (−4, 1) ______________

9. (1, 0) and (3, 2) ______________

◆ Skill B Writing an equation of a line in point-slope form

Recall The point-slope form for an equation of a line is $y - y_1 = m(x - x_1)$.

◆ **Example**

Write an equation for the line through $(1, -1)$ and $(-1, 5)$

a. in point-slope form.

b. in slope-intercept form.

◆ **Solution**

a. First find m.

$$m = \frac{\text{difference in } y\text{-values}}{\text{difference in } x\text{-values}} = \frac{-1 - 5}{1 - (-1)} = \frac{-6}{2} = -3$$

Substitute the slope and one of the points into the point-slope equation.

$y - y_1 = m(x - x_1)$

$y - (-1) = -3(x - 1)$ Use the point $(1, -1)$.

$y + 1 = -3(x - 1)$ Simplify.

b. Rewrite the equation in the form $y = mx + b$.

$y + 1 = -3(x - 1)$

$y + 1 = -3x + 3$ Distributive Property

$y = -3x + 2$ Subtract 1 from each side.

Write an equation for each line in point-slope form.

1. containing $(4, -1)$ and with a slope of $\frac{1}{2}$ ____________________

2. crossing the x-axis at $x = -3$ and the y-axis at $y = 6$ ____________________

3. containing the points $(-6, -1)$ and $(3, 2)$ ____________________

Rewrite each equation in slope-intercept form.

4. the line from Exercise **1.** ____________________

5. the line from Exercise **2.** ____________________

6. the line from Exercise **3.** ____________________

7. In what situations would you find it easier to use point-slope form, and in what situations would you find it easier to use slope-intercept form? ____________________

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◆ **Skill C** Factoring trinomials by choosing factor pairs of the constant

Recall Another way to factor a trinomial, such as $x^2 - 5x - 6$, is to first make a list of the pairs of factors of the constant. Then choose the right combination to complete the factors of the trinomial.

◆ **Example**

Use the constant's factor pairs to factor $x^2 - 5x - 6$.

◆ **Solution**

List each pair of factors of -6 along with their sum.

Factors of -6	Sum of the factors
6 and -1	5
3 and -2	1
2 and -3	-1
1 and -6	-5

The sum of 1 and -6 is -5. Use the combination of 1 and -6 to form the factors. Thus, $x^2 - 5x - 6 = (x + 1)(x - 6)$.

Factor each trinomial. If the trinomial cannot be factored, write *prime*.

1. $x^2 - x - 2$

2. $x^2 + 3x - 4$

3. $x^2 + 4x + 3$

4. $x^2 - 4x + 3$

5. $x^2 + 2x - 8$

6. $x^2 + x - 20$

7. $x^2 + 2x - 15$

8. $x^2 - 3x + 10$

9. $x^2 - x - 12$

◆ **Skill D** Find the zeros of a polynomial function by factoring

Recall The zeros of a function are the values of x that make y equal to 0.

◆ **Example 1**

Find the zeros of the function $y = (x - 2)(x + 5)$.

◆ **Solution**

Let $y = 0$. Then use the Zero-Product Property to solve for x.

$(x - 2)(x + 5) = 0$

$(x - 2) = 0$ or $(x + 5) = 0$

$x = 2$ or $x = -5$

The zeros of $y = (x - 2)(x + 5)$ are 2 and -5.

Recall A quadratic polynomial can be factored into two binomials.

◆ **Example 2**

Solve the equation $x^2 - x - 6 = 0$.

◆ **Solution**

Since $x^2 - x - 6$ can be factored into $(x + 2)(x - 3)$, you can rewrite $x^2 - x - 6 = 0$ as $(x + 2)(x - 3) = 0$. Solve the equation $(x + 2)(x - 3) = 0$.

$x + 2 = 0$ or $x - 3 = 0$

$x = -2$ or $x = 3$

The solutions to $x^2 - x - 6 = 0$ are -2 and 3.

Solve by factoring.

1. $x^2 - 4x - 12 = 0$

____

2. $x^2 - 6x + 9 = 0$

____

3. $x^2 - 9x + 14 = 0$

____

4. $x^2 + 6x + 5 = 0$

____

5. $x^2 - 7x + 10 = 0$

____

6. $x^2 - 36 = 0$

____

7. $x^2 + 8x + 16 = 0$

8. $x^2 - x - 12 = 0$

◆ **Skill H** Writing and evaluating functions

Recall The value of $f(x) = x^2 + 5$ depends on the value of x.

◆ **Example 1**

Sarah uses an internet server which charges $12.50 per month plus $0.60 for each hour over 20 hours that she uses it during the month. Write this relation in function notation. How much will she be charged for using the service for 38 hours in April?

◆ **Solution**

Let h = number of hours over 20. Thus, the function is as follows.

$$f(h) = 12.50 + 0.60h$$
$$f(18) = 12.50 + 0.60(18) \quad \textit{where } h = 18$$
$$f(18) = 23.30$$

The charge for April will be $35.30.

◆ **Example 2**

If $g(x) = x^2 + 3x$, find $g(-5)$.

◆ **Solution**

$g(-5)$ means replace x with the value -5 and evaluate $g(x)$.

$$g(-5) = (-5)^2 + 3(-5)$$
$$= 25 - 15$$
$$= 10$$

Thus, $g(-5) = 10$.

Let $f(x) = 5 - \frac{2x}{3}$ and $g(x) = \frac{1}{2}x^2 + 3x$. Evaluate each function.

1. $f(6)$ ____________

2. $f(0)$ ____________

3. $f\left(\frac{1}{2}\right)$ ____________

4. $g(1)$ ____________

5. $g(-2)$ ____________

6. $g\left(\frac{1}{2}\right)$ ____________

7. $f(1) + g(0)$ ____________

8. $g(4) - f(5)$ ____________

9. $f(0) \cdot g(0)$ ____________

10. $g(-6) \cdot f(-6)$ ____________

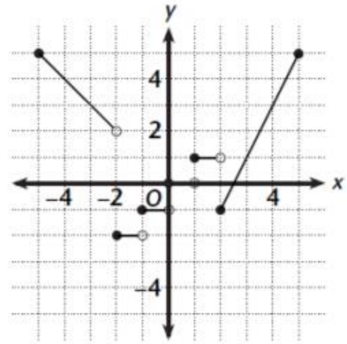
◆Skill L Graphing piecewise, step, and absolute-value functions

Recall A piecewise function in x is a function defined by different expressions in x on different intervals for x.

◆ **Example**

Graph this piecewise function.

$$f(x) = \begin{cases} |x|, & \text{if } -5 \le x < -2 \\ [x], & \text{if } -2 \le x < 2 \\ 2x - 5, & \text{if } 2 \le x \le 5 \end{cases}$$



◆ **Solution**

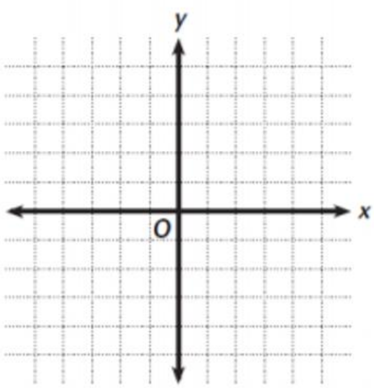
x	-5	-4	-3	-2.5
$y = \|x\|$	5	4	3	2.5

x	-2	-1.5	-1	-0.5	0	1
$y = [x]$	-2	-2	-1	-1	0	1

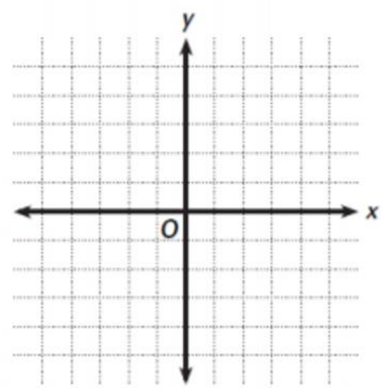
x	2	2.5	3	4	5
$y = 2x - 5$	2	0	1	3	5

Graph each function.

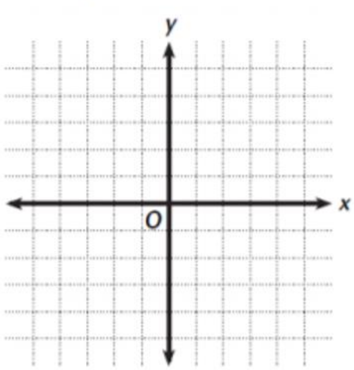
1. $f(x) = \begin{cases} x + 3, \text{ if } x < 0 \\ -2x + 5 \text{ if } x \ge 0 \end{cases}$



2. $f(x) = \begin{cases} \frac{1}{2}x \text{ if } -4 \le x \le 2 \\ 2x - 3 \text{ if } x > 2 \end{cases}$

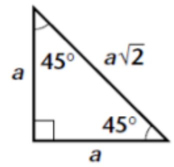


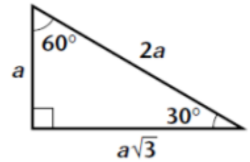
3. $f(x) = \begin{cases} |x| \text{ if } x \le 1 \\ 2 - |x - 2| \text{ if } x > 1 \end{cases}$



◆Skill S Solving 45-45-90 and 30-60-90 triangles

Recall

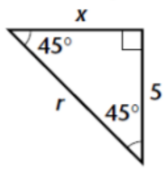




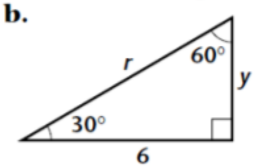
◆ **Example**

Find the lengths of the other 2 sides in each right triangle.

a.



b.



◆ **Solution**

a. $a = 5$

$x = a = 5$

$r = a\sqrt{2} = 5\sqrt{2}$

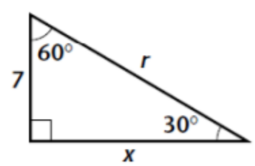
b. $a\sqrt{3} = 6 \rightarrow a = \frac{6}{\sqrt{3}}$ (or $2\sqrt{3}$)

$y = a = \frac{6}{\sqrt{3}}$ (or $2\sqrt{3}$)

$r = 2a = \frac{12}{\sqrt{3}}$ (or $4\sqrt{3}$)

Find the missing side lengths in each right triangle.

1.



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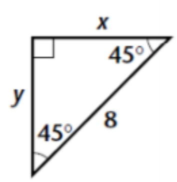
2.



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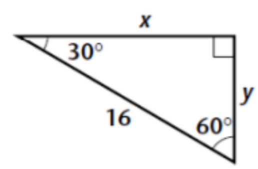
3.



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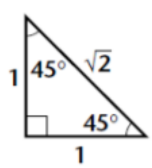
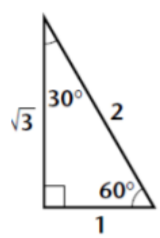
◆Skill T Finding exact values of the trigonometric functions for an angle whose reference angle is 30°, 45°, or 60°

Recall

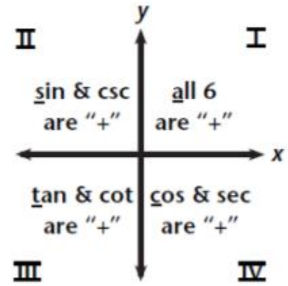




θ	30°	45°	60°
$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

A **reference angle** is the positive acute angle between the terminal side of a given angle and the x-axis.

One mnemonic for remembering which functions are **positive** in each quadrant is "<u>A</u>ll <u>s</u>tudents <u>t</u>ake <u>c</u>alculus."



◆ **Example**

Find each exact value.

a. sin 315° **b.** cos 240° **c.** tan 210°

◆ **Solution**

a. 315° is in Quadrant IV, where sine is negative. The reference angle is 45°.

$\sin 315° = -\sin 45°$

$= -\frac{1}{\sqrt{2}}$

b. 240° is in Quadrant III, where cosine is negative. The reference angle is 60°.

$\cos 240° = -\cos 60°$

$= -\frac{1}{2}$

c. 210° is in Quadrant III, where tangent is positive. The reference angle is 30°.

$\tan 210° = \tan 30°$

$= \frac{1}{\sqrt{3}}$

Find each trigonometric value. Give exact answers.

1. sin 120° ______ **2.** cos 330° ______ **3.** tan 225° ______ **4.** cos 150° ______

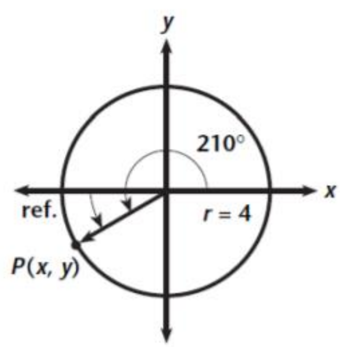
5. sin 240° ______ **6.** sin 150° ______ **7.** tan 315° ______ **8.** cos 225° ______

◆Skill U Finding the coordinates of a point P on a circle

Recall If $P(x, y)$ lies at the intersection of the terminal side of θ in standard position and a circle centered at the origin with radius r, then $P(x, y) = P(r\cos\theta, r\sin\theta)$.

◆ **Example**

Find the coordinates of point P shown in the figure at right.



◆ **Solution**

$\cos 210° = -\cos 30° = -\frac{\sqrt{3}}{2}$ and

$\sin 210° = -\sin 30° = -\frac{1}{2}$

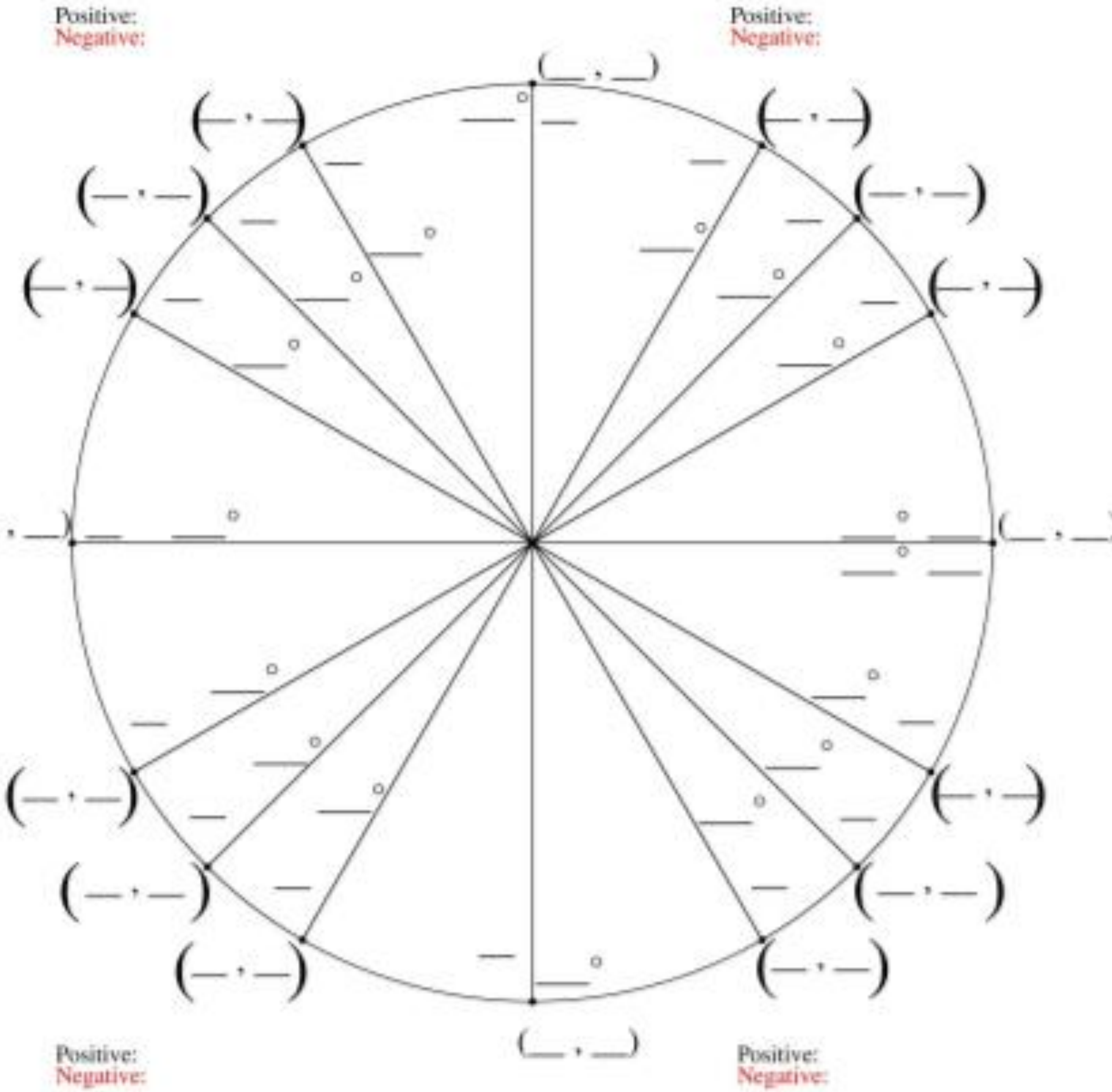
$r\cos\theta = 4\left(-\frac{\sqrt{3}}{2}\right)$ and $r\sin\theta = 4\left(-\frac{1}{2}\right)$

The coordinates of point P are $(-2\sqrt{3}, -2)$.

Point P is located at the intersection of a circle centered at the origin with a radius of r and the terminal side of angle θ in standard position. Find the exact coordinates of point P.

1. $\theta = 135°, r = 6$ ______ **2.** $\theta = 30°, r = 10$ ______ **3.** $\theta = 300°, r = 12$ ______

Fill in The Unit Circle



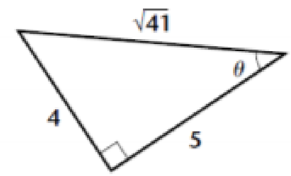
Be able to do this from memory!

◆ **Skill Y** Finding the trigonometric functions of an acute angle

Recall The hypotenuse is the longest side in a right triangle and is opposite the right angle.

◆ **Example**

Refer to the triangle shown at right and give values for $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, $\sec\theta$, and $\csc\theta$.



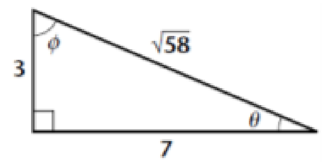
◆ **Solution**

The hypotenuse (hyp.) has a length of $\sqrt{41}$.
The leg opposite (opp.) θ has a length of 4.
The leg adjacent (adj.) to θ has a length of 5.

$\sin\theta = \frac{\text{opp.}}{\text{hyp.}} = \frac{4}{\sqrt{41}}$ $\quad\csc\theta = \frac{\text{hyp.}}{\text{opp.}} = \frac{\sqrt{41}}{4}$ $\quad\cos\theta = \frac{\text{adj.}}{\text{hyp.}} = \frac{5}{\sqrt{41}}$

$\sec\theta = \frac{\text{hyp.}}{\text{adj.}} = \frac{\sqrt{41}}{5}$ $\quad\tan\theta = \frac{\text{opp.}}{\text{adj.}} = \frac{4}{5}$ $\quad\cot\theta = \frac{\text{adj.}}{\text{opp.}} = \frac{5}{4}$

Refer to the triangle at right to find each value. Give exact answers.



1. $\sin\theta$ ________ **2.** $\cos\theta$ ________

3. $\tan\theta$ ________ **4.** $\csc\theta$ ________

5. $\sec\theta$ ________ **6.** $\cot\theta$ ________ **7.** $\sin\phi$ ________

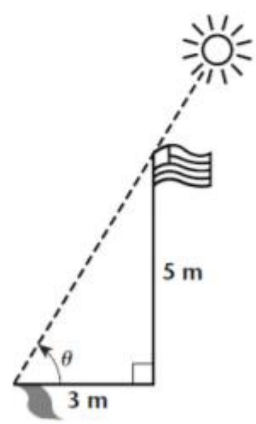
8. $\cos\phi$ ________ **9.** $\tan\phi$ ________ **10.** $\csc\phi$ ________

◆ **Skill CC** Applying inverse trigonometric functions

Recall $\sin\theta = \frac{\text{opp.}}{\text{hyp.}}$ $\quad\cos\theta = \frac{\text{adj.}}{\text{hyp.}}$ $\quad\tan\theta = \frac{\text{opp.}}{\text{adj.}}$

◆ **Example**

At a certain time of the day, the 5 meter flagpole shown at right casts a shadow that is 3 meters long. What is the angle of elevation of the sun at this time?



◆ **Solution**

Since 3 meters is the length of the side **adjacent** to θ and 5 meters is the length of the side **opposite** θ, use the tangent function.

$\tan\theta = \frac{5}{3}$

$\theta = \tan^{-1}\left(\frac{5}{3}\right)$

This last equation states that θ is the angle that has a tangent of $\frac{5}{3}$.

$\theta \approx 59°$ *Use calculator in **degree** mode.*

Find the measure of each angle to the nearest whole degree.

1. Find the measure of the smallest angle in a right triangle with sides of 3, 4, and 5 centimeters. ____________________

2. What is the angle between the bottom of the ladder and the ground as shown at right?

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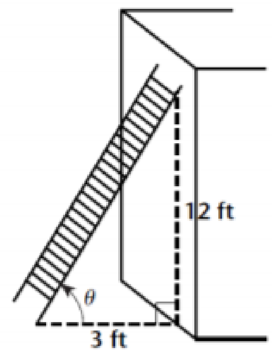

3. Find the angle at the peak of the roof as shown at right.

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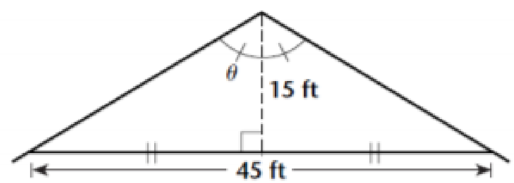

4. The hypotenuse of a right triangle is 3 times as long as the shorter leg. Find the measure of the angle between the shorter leg and the hypotenuse.

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◆Skill V Finding exact values for the trigonometric functions of an angle measured in radians

◆ **Example**

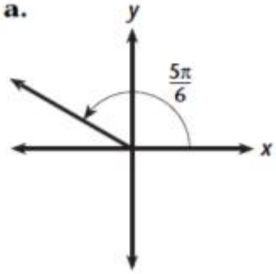
Give the exact value of each expression where the angle measures are in radians.

a. $\cos\frac{5\pi}{6}$ **b.** $\tan\frac{4\pi}{3}$ **c.** $\sin\left(-\frac{3\pi}{2}\right)$

◆ **Solution**

Use reference angles.

a.



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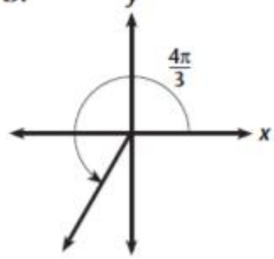
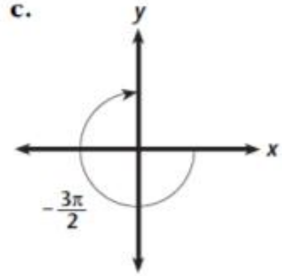


c.



$\frac{5\pi}{6}$ radians $= 150°$

$\cos 150° = -\cos 30° = -\frac{\sqrt{3}}{2}$

$\frac{4\pi}{3}$ radians $= 240°$

$\tan 240° = \tan 60° = \sqrt{3}$

$-\frac{3\pi}{2}$ radians $= -270°$

$\sin(-270°) = \sin 90° = 1$

Evaluate each expression. Give exact answers.

1. $\sin\frac{3\pi}{4}$ ________________ **2.** $\cos\frac{2\pi}{3}$ ________________ **3.** $\tan\frac{5\pi}{6}$ ________________

4. $\cos\left(-\frac{7\pi}{6}\right)$ ________________ **5.** $\tan\left(-\frac{\pi}{4}\right)$ ________________ **6.** $\sin\pi$ ________________

◆**Skill DD** Graphing functions of the form $y = a \sin b\theta$, $y = a \cos b\theta$, and $y = a \tan b\theta$

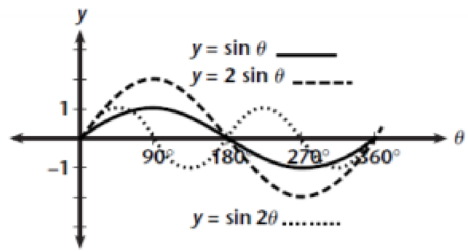
Recall The sine and cosine are periodic functions with a period of 360° or 2π radians. The tangent function has a period of 180° or π radians.

◆ **Example 1**

Graph $y = \sin \theta$, $y = 2 \sin \theta$, and $y = \sin 2\theta$ on the same set of axes.

◆ **Solution**

The graph of $y = a \sin b\theta$ has an amplitude (height above x-axis) of $|a|$ and period of $\frac{360°}{b}$.



function	amplitude	period
$y = \sin \theta$	1	360°
$y = 2 \sin \theta$	2	360°
$y = \sin 2\theta$	1	180°

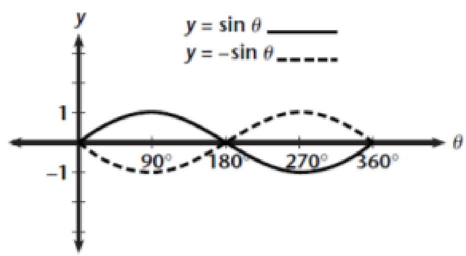
Check your results with a graphics calculator.

◆ **Example 2**

Graph $y = \sin \theta$ and $y = -\sin \theta$ on the same set of axes.

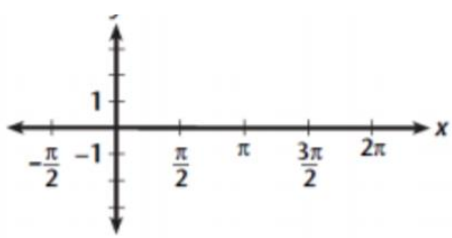
◆ **Solution**

Notice that the graph of $y = -\sin \theta$ is the reflection of $y = \sin \theta$ across the (horizontal) θ-axis.

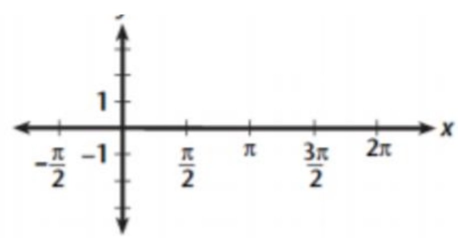


Sketch each pair of functions on the same set of axes. Use $0° \leq \theta \leq 360°$.

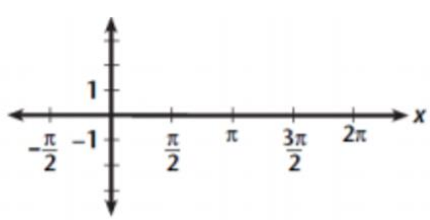
1. $y = \cos \theta$, $y = \frac{1}{2}\cos \theta$

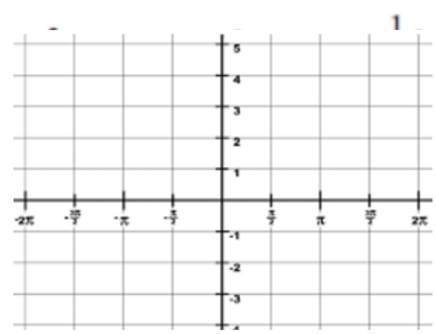


2. $y = \cos \theta$, $y = \cos 3\theta$



3. $y = \tan \theta$, $y = -\tan \theta$



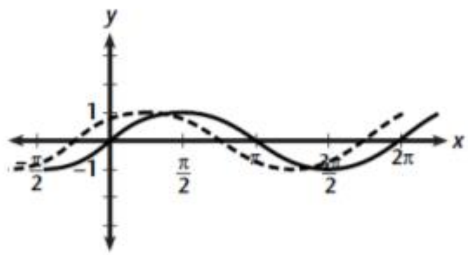


◆ **Skill EE** Graphing functions of the form $y = \sin(x - c) + d$, $y = \cos(x - c) + d$, and $y = \tan(x - c) + d$

Recall The graph of $y = \sin(x - c)$ is a **phase shift** (horizontal translation) of the graph of $y = \sin x$ to the **right** c units.
The graph of $y = \sin x + d$ is a **vertical shift** of the graph of $y = \sin x$ up d units.

◆ **Example 1**
Graph $y = \sin x$ and $y = \sin\left(x + \frac{\pi}{4}\right)$ on the same set of axes.

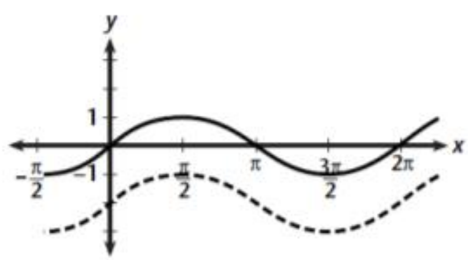
◆ **Solution**



phase shift of $\frac{\pi}{4}$ units to the left

◆ **Example 2**
Graph $y = \sin x$ and $y = \sin x - 2$ on the same set of axes.

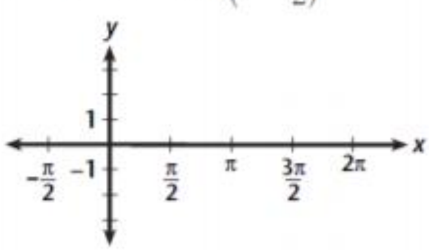
◆ **Solution**



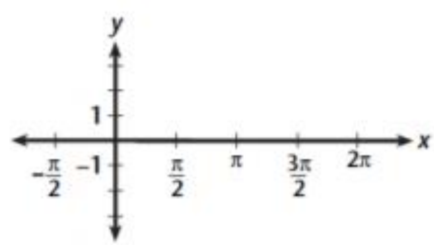
vertical shift of 2 units down

Sketch each pair of functions on the same set of axes. Use $-\frac{\pi}{2} \le x \le 2\pi$.

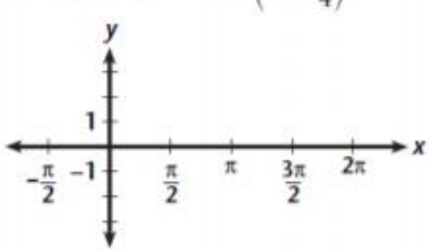
1. $y = \cos x$, $y = \cos\left(x - \frac{\pi}{2}\right)$



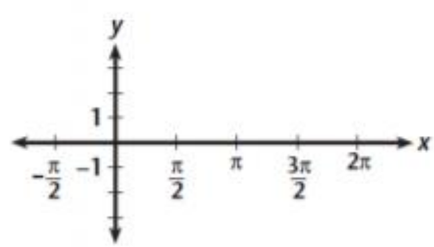
2. $y = \cos x$, $y = \cos x + 1$



3. $y = \tan x$, $y = -\tan\left(x + \frac{\pi}{4}\right)$



4. $y = \tan x$, $y = \tan x - 1$



◆**Skill GG** Simplifying expressions by using basic trigonometric identities

Recall Since $\sin^2\theta + \cos^2\theta = 1$, then $\sin^2\theta = 1 - \cos^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$.

◆ **Example 1**

Simplify $\csc\theta\tan\theta$ to $\sec\theta$.

◆ **Solution**

$\csc\theta\tan\theta = \frac{1}{\sin\theta}\cdot\frac{\sin\theta}{\cos\theta}$ *fundamental identities*

$= \frac{1}{\cos\theta}$ $\frac{\sin\theta}{\sin\theta} = 1$

$= \sec\theta$ *fundamental identity*

◆ **Example 2**

Simplify $(\sec\theta - 1)(\sec\theta + 1)$ to $\tan^2\theta$.

◆ **Solution**

$(\sec\theta - 1)(\sec\theta + 1) = \sec^2\theta - 1$ $(a - b)(a + b) = a^2 - b^2$

$= \tan^2\theta + 1 - 1$ *fundamental identity*

$= \tan^2\theta$

For exercises 5–10, show on your own paper how the first expression simplifies to the second expression.

1. $\sin x\cot x$ to $\cos x$

2. $\sin x\sec x\cot x$ to 1

3. $\cos^2 x - \sin^2 x$ to $1 - 2\sin^2 x$

4. $(1 + \sin x)(1 - \sin x)$ to $\cos^2 x$

5. $\tan x + \cot x$ to $\sec x\csc x$

6. $(\cos x - \sin x)^2$ to $1 - 2\cos x\sin x$