Math League SCASD 2020- 21

Meet #5

Geometry

Self-study Packet

Problem Categories for this Meet (in addition to topics of earlier meets):

1. Mystery: Problem solving
2. **Geometry: Solid Geometry (Volume and Surface Area)**
3. Number Theory: Set Theory and Venn Diagrams
4. Arithmetic: Combinatorics and Probability
5. Algebra: Solving Quadratics with Rational Solutions, including word problems

Important Information you need to know about GEOMETRY:

Solid Geometry (Volume and Surface Area)

Know these formulas!

SHAPE	SURFACE AREA	VOLUME
Rect. prism	2(LW + LH + WH)	LWH
Any prism	sum of areas of all surfaces	H(Area of Base)
Cylinder	$2\pi R^2 + 2\pi RH$	$\pi R^2 H$
Pyramid	sum of areas of all surfaces	1/3 H(Base area)
Cone	$\pi R^2 + \pi RS$	$1/3\ \pi R^2 H$
Sphere	$4\ \pi R^2$	$4/3\ \pi R^3$

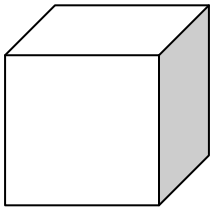
Surface Diagonal: any diagonal (NOT an edge) that connects two vertices of a solid while lying on the surface of that solid.

Space Diagonal: an imaginary line that connects any two vertices of a solid and passes through the interior of a solid (does not lie on the surface).

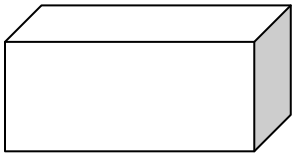
Category 2
Geometry
Meet #5 - April, 2019

Calculator Meet

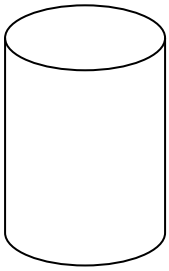
1) The total surface area of a cube is 150 square centimeters. How many cubic centimeters are in its volume?



2) Each edge of a rectangular solid is a prime number of inches long. The volume of the solid is 165 cubic inches. How many square inches are in its total surface area?



3) How many 1-gallon cans of paint must Vincent buy in order to paint the entire inside surface of a closed cylindrical tank, including the floor and ceiling, that is 67 feet high and 23 feet in diameter? One gallon of paint covers 432 square feet. Use $\pi \approx 3.14$.



Answers

1) _____ cu. cm

2) _____ sq. in.

3) _____ cans

Solutions to Category 2
Geometry
Meet #5 - April, 2019

Answers
1) 125
2) 206
3) 14

1) There are six congruent square faces on a cube. Divide the given total surface area (150) by 6 to get the area of one surface (25). Then take the square root of 25 to get the length of one edge (5). Finally, cube 5 to get the volume: 5x5x5 = 125 cubic centimeters.

2) Prime factor the volume of 165: 165 = 3 x 5 x 11.
The total surface area = 2lw + 2lh + 2wh
= 2(3)(5) + 2(3)(11) + 2(5)(11)
= 30 + 66 + 110
= 206 square inches.

3) Find the inside surface area, in square feet, of the cylinder. Then divide by the number of square feet that one can of paint will cover. Any remainder must be rounded *UP*:

Total surface area = the area of two circles plus the wraparound rectangle
= 2(pi)(radius)(radius) + (height)(circumference)
= 2(3.14)(11.5)(11.5) + (67)(2)(3.14)(11.5)
= 830.53 + 4838.74
= 5669.27 square feet

The number of paint cans = 5669.27 divided by 432
= 13.12331

Round up so that the number of cans is 14.

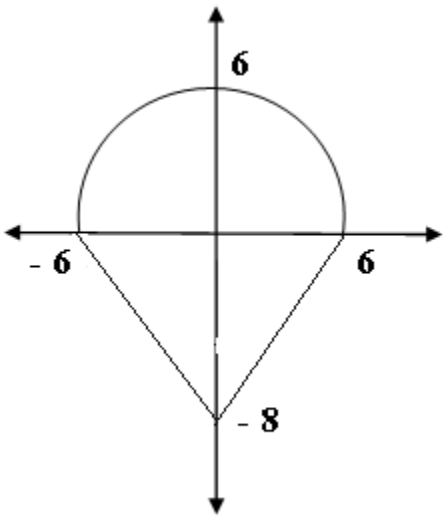
Category 2
Geometry
Meet #5 - April, 2017

Calculator Meet

1) A cube has a volume of 1331 cubic yards. How many square yards are in the total surface area?

2) Eight baseballs are packed tightly into a cubical box whole length is six inches. Each baseball is three inches in diameter. What percent of the box is empty space? Round your answer to the nearest whole percent. Use $\pi \approx 3.14.$

3) This plane figure, drawn on the x-y Cartesian coordinate plane, is rotated 180 degrees, using the y-axis as the axis of rotation. What is the volume, in cubic units, of the resulting 3-D object? The curve above the x-axis is a semi-circle. Use $\pi \approx 3.14.$ **Round your answer to the nearest ten cubic units.**



Answers

1) _____

2) _____ **%**

3) _____

Solutions to Category 2
Geometry
Meet #5 - April, 2017

Answers
1) 726
2) 48 (%)
3) 750

1) The cube root of 1331, or 11, is the length of one side of the cube. So, the total surface area is (6)(11)(11) = 726 square yards.

2) The empty space = (the volume of the cube) - the total volume of the eight baseballs

$= (\text{one side})^3 - (8)\left(\frac{4}{3}\pi r^3\right)$

$= (6)^3 - (8)\left(\frac{4}{3}(3.14)(1.5)^3\right)$

$= 216 - (8)((1.333)(3.14)(3.375))$

$= 216 - (8)(14.1265)$

$= 216 - (113.0.12)$

= 102.988

As a percent of the cube, the empty space is 102.988 / 216 = 0.4767 . . . or about 48% when rounded to the nearest whole percent.

3) When rotated 180 degrees, the plane figure looks like an ice cream cone with a hemisphere on top and a cone on the bottom. The volume is given by this formula:

Volume $= \left(\frac{1}{2}\right)\left(\frac{4}{3}\pi r^3\right) + \frac{1}{3}\left(\pi r^2 h\right)$

$= \left(\frac{1}{2}\right)\left(\frac{4}{3}\pi(6)^3\right) + \frac{1}{3}\left(\pi(6)^2(8)\right)$

$= \left(\frac{1}{2}\right)((1.333)(3.14)(216)) + \frac{1}{3}\left((3.14)(6)^2(8)\right)$

$= 452.0469... + 301.409...$

= 753.4559... or about 750 cubic units when rounded to the nearest ten cubic units.

Category 2
Geometry
Meet #5 - March, 2015
Calculator meet

1) The area of the top circular surface of a cylinder is 289π square feet. The height of the cylinder is 19 feet. How many cubic feet are in the volume of the cylinder? Express your answer in terms of π.

2) A cube and a rectangular solid have the same volume. The dimensions of the rectangular solid are 12 cm by 18 cm by 125 cm. How many square centimeters are in the area of one surface (face) of the cube?

3) A tennis ball has a diameter of 2.5 inches. Three tennis balls fit tightly inside a cylindrical can. What percent of the can is air that lies outside of the tennis balls? Use $\pi \approx 3.142$. Round your final answer to the nearest tenth of a percent.

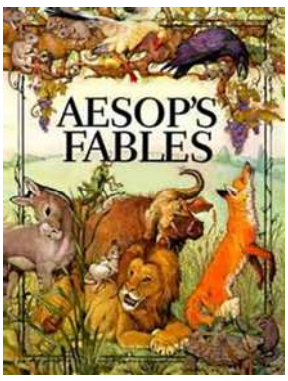
Answers

1) _____

2) _____

3) _____ **%**

On March 26, 1484, William Caxton printed his translation of Aesop's Fables.



Solutions to Category 2
Geometry
Meet #5 - March, 2015

Answers
1) 5491π
2) 900
3) 33.3

1) The solution is one simple step - that the volume of a cylinder is equal to the product of its height and the area of the top (or bottom) circular surface. (area of circular top)(height) = (289π)(19) = 5491π cubic feet.

2) The volume of the rectangular solid = (length)(width)(height) = (12)(18)(125) = 27,000 cc. The length of one side of the cube is equal to the cube root of 27,000, or 30 cm. The area of one surface of that cube is equal to the square of one side, or (30)(30) = 900 square cm.

3) The amount of air inside the can that lies outside the tennis balls is equal to the volume of the cylindrical, minus the volume of the tennis balls = $\pi r^2 h - 3\left(\frac{4}{3}\right)\pi r^3$. The radius of the top of the can is half the diameter of one of the tennis balls = (0.5)(2.5) = 1.25 inches. The height of the can is three times the diameter of one ball, = (3)(2.5) = 7.5 inches.

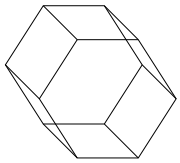
Air space = (3.142)(1.25)(1.25)(7.5) - (3)(4/3)(3.142)(1.25)(1.25)(1.25)
= (36.820312) - 24-546875
= 12.27343

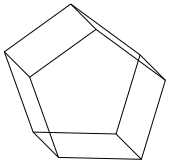
The percent of air inside the can that lies outside the three tennis balls is 12.27343 / 36.820312 = 0.3333 which, to the nearest tenth of a percent, is 33.3%.

Category 2
Geometry
Meet #5, March/April 2013

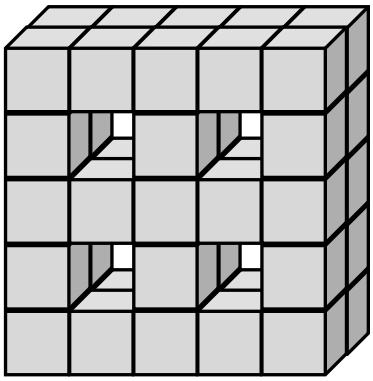
You may use a calculator.

1. How many more surface diagonals are there on a hexagonal prism than there are on a pentagonal prism?





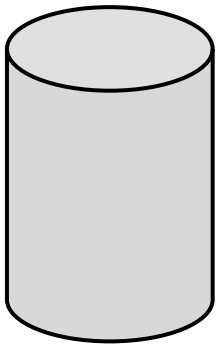
2. Some unit cubes have been removed from a 2-by-5-by-5 structure, leaving 4 holes in the structure as shown at right. How many square units are there in the remaining surface area of the entire structure?



3. Mr. Peterson has a rain barrel in the shape of a cylinder. The height of the rain barrel is 42 inches and the diameter of the rain barrel is 24 inches. One cubic foot of space can hold 7.48 gallons of water. How many gallons of water does Mr. Peterson's rain barrel hold when it is full? Express your answer to the nearest whole number of gallons.

Answers

1. _____ diagonals
2. _____ sq. units
3. _____ gallons



Solutions to Category 2
Geometry
Meet #5, March/April 2013

Answers
1. 10 diagonals
2. 114 sq. units
3. 82 gallons

1. On the hexagonal prism, there are 9 diagonals on each of the 2 hexagonal bases and 2 diagonals on each of the 6 rectangular sides. That's $9 \times 2 + 2 \times 6 = 30$ diagonals. On the pentagonal prism, there are 5 diagonals on each of the 2 pentagonal bases and 2 diagonals on each of the 5 rectangular sides. That's $5 \times 2 + 2 \times 5 = 20$ diagonals. The desired difference is $30 - 20 =$ **10 diagonals**.

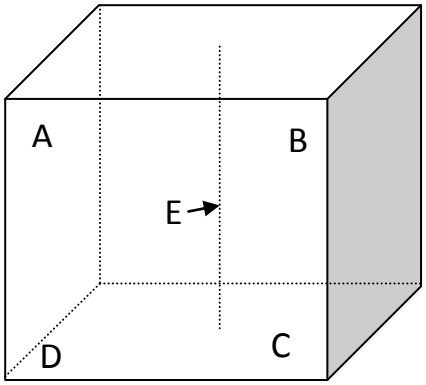
2. The surface area of a solid $2 \times 5 \times 5$ structure would be $2(2\times5+2\times5+5\times5)=2(10+10+25)=90$ square units. When each of the holes is created, we lose 2 square units of the original surface area but create 8 more square units inside the hole. Each hole adds $8 - 2 = 6$ square units, so we get $4 \times 6 = 24$ more square units. That brings the total surface area of the entire structure to $90 + 24 =$ **114 square units**.

3. The volume of a cylinder is the area of the base times the height. Mr. Peterson's rain barrel has a diameter of 24 inches, so the radius is 12 inches and the area of the base is $\pi \times 12^2 = 144\pi$ square inches. Multiplying this by the height and using an approximation of π, we get a volume of $144\pi \times 42 = 6048\pi \approx 19000.325$ cubic inches. To convert cubic inches to cubic feet, we need to divide by 12 *three times*—once for each dimension—or by $12^3 = 1728$. The result is about 10.996 cubic feet. Finally, each cubic foot is about 7.48 gallons, so Mr. Peterson's rain barrel can hold about $7.48 \times 10.996 = 82.25$ gallons, or **82 gallons** to the nearest whole number of gallons.

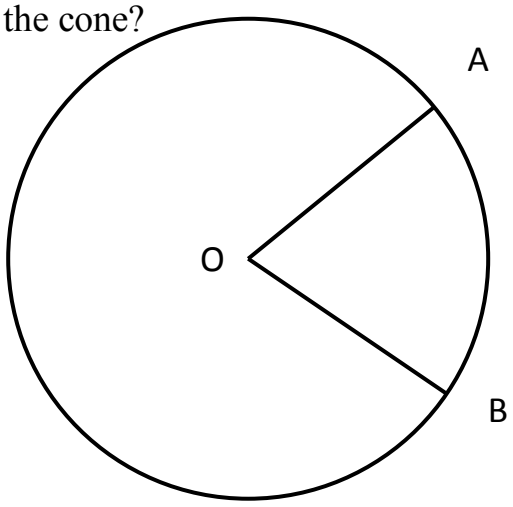
Category 2 – Geometry

1. What is the surface area of a cylinder with a radius of 3 feet and a height of 10 feet? *Include the top and bottom of the cylinder, use* $\pi = 3.14$*, and round your answer to the nearest whole square foot.*

2. If you take a cube whose sides all measure 1 inch, and connect all four corners (A, B, C, D) on one face of the cube to the cube's center, E, then how many cubic inches are there in the volume of the resulting pyramid $ABCDE$?
   Express your answer as a common fraction.



3. The radius OA of the circle below measures 10 inches.
   We cut off a sector AOB with a central angle $\measuredangle AOB = 72\ degrees$.
   We then roll the remaining (larger) shape into a cone.
   How many inches are there in the height of the cone?



Answers

1. _____

2. _____

3. _____

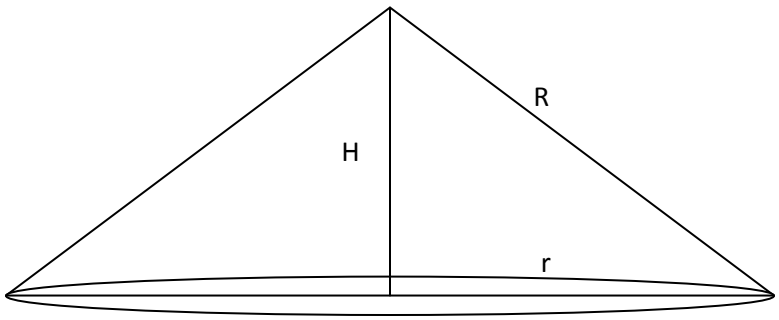
Solutions to Category 2 – Geometery

Answers
1. 245
2. $^1/_6$
3. 6

1. The area is the outside plus the top/bottom:
   $2 \cdot \pi \cdot R \cdot H + 2 \cdot \pi \cdot R^2 = 2 \cdot \pi \cdot R \cdot (H + R) =$
   $2 \cdot 3.14 \cdot 3 \cdot (10 + 3) = 244.92 \cong 245_{sq\ feet}$

2. Two ways to see this: The cube's center E is half the distance between the face $ABCD$ and the opposing face, so its distance from face $ABCD$ (the pyramid's height) is ½ inch. And the volume is the area of $ABCD$ multiplied by the height, divided by 3, so $\frac{1}{6}$. Thought of differently, the cube has 6 faces, so we can build 6 such pyramids, covering all of the cube's volume, so a pyramid's volume is one-sixth of the cube's volume (which is one cubic inch).

3. Since we cut off a fifth ($\frac{72}{360}$) or the circle, then the cone's base will obey:
   $2 \cdot \pi \cdot r = \frac{4}{5} \cdot 2 \cdot \pi \cdot R$ and so $r = \frac{4}{5} \cdot R = 8\ inches.$
   The cone's height can be calculated using Pythagoras:
   $H^2 = R^2 - r^2 = 10^2 - 8^2 = 36$, and so $H = 6\ inches$.
   The larger the angle cut, the taller the cone.



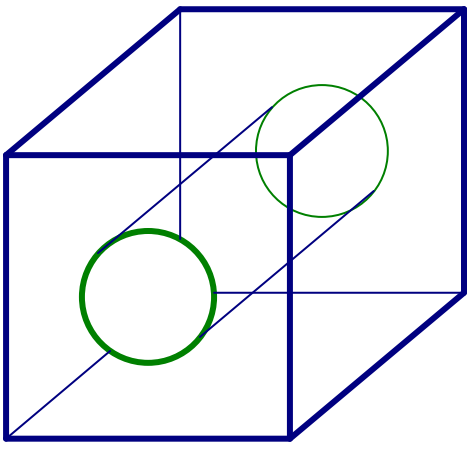
You may use a calculator today.

Category 2
Geometry
Meet #5, March 2009

1. What is the total surface area of a solid hemisphere with radius 12 cm? Use 3.14 as an estimation for π and express your answer as a decimal.

2. The radius of cone A is 6 times as long as the radius of cone B. The height of cone A is one-ninth the height of cone B. If the volume of cone A is 200 cm^3, how many cm are in the volume of cone B?

3. In the diagram below, an 8 inch by 8 inch by 8 inch cube has a cylinder with radius 2 in drilled through the center of one face all the way through the cube and out the opposite face. What is the surface area of the resulting figure? Use 3.14 as an estimation for π and express your answer as a decimal.



Answers

1. _____
2. _____
3. _____

Solutions to Category 2
Geometry
Meet #5, March 2009

Answers

1. 1356.48

2. 50

3. 459.36

1. The surface area of a sphere is found by the formula $SA = 4\pi r^2$, so a hemisphere would have a surface area of $2\pi r^2$. However, there is also a circular base to a hemisphere which has an area of πr^2, for a total surface area of $3\pi r^2$. Since the radius is 12 cm, the surface area is $SA = 3\pi(12)^2 = 3\pi(144) = 432\pi \approx 1356.48$

2. Volume of a cone is found by the formula $V = \frac{1}{3}\pi r^2 h$. If r is the radius of cone B, then $6r$ is the radius of cone A. If h is the height of cone B, then $\frac{h}{9}$ is the height of cone A. We can then write a formula for the volume of cone A as $V_A = \frac{1}{3}\pi(6r)^2\left(\frac{h}{9}\right)$, while the volume of cone B is $V_B = \frac{1}{3}\pi r^2 h$. Since we know the volume of cone A is 200, we can substitute 200 for V_A and simplify to get:

$V_A = \frac{1}{3}\pi(6r)^2\left(\frac{h}{9}\right) \rightarrow 200 = \frac{1}{3}\pi 36r^2\left(\frac{h}{9}\right) \rightarrow 200 = \frac{4}{3}\pi r^2 h \rightarrow 50 = \frac{1}{3}\pi r^2 h = V_B$

3. Describing the surface area in words, there are 6 squares, although 2 of them have circular holes cut out of two of them. There is also "tube" through the middle of the cube which is a cylinder without bases. So the surface area of the whole thing is:

$6(8)^2 - 2\pi(2)^2 + 2\pi(2)(8) =$
$384 - 8\pi + 32\pi =$
$384 + 24\pi \approx$
$384 + 24(3.14) =$
$384 + 75.36 =$
459.36