

# Math League SCASD 2020

## Meet #4

# Algebra

## Self-study Packet

Problem Categories for this Meet (in addition to topics of earlier meets):

1. Mystery: Problem solving
2. Geometry: Properties of Circles
3. Number Theory: Modular Arithmetic, Series and Sequences
4. Arithmetic: Percent Applications
5. **Algebra: Word Problems (linear, including direct proportions or systems)**

## Important information you need to know about ALGEBRA: Word Problems

- Formula to know: distance = rate • time
- Set up equations by trying to write everything in terms of one variable.

### Example

*Melinda has 4 more green M&Ms than Logan. Adrianna has 1 fewer than double the number of green M&Ms Logan has. Between the 3 of them, they have a total of 43 green M&Ms. How many green M&Ms does each person have?*

### Solution

*(Everyone is being compared to Logan, so make Logan your standard variable).*

$$\text{Logan} = x$$

$$\text{Melinda} = x + 4$$

$$\text{Adrianna} = 2x - 1$$

*Logan + Melinda + Adrianna = 43 green M&Ms, so...*

$$x + (x + 4) + (2x - 1) = 43$$

$$4x + 3 = 43$$

$$4x = 40$$

$$x = 10$$

*x represents Logan, so Logan has 10, Melinda has 14, Adrianna has 19.*

*Double check—is  $10 + 14 + 19$  equal to 43? Yes!*

- If all else fails, guess and check!

### Double check your work!

- ✓ Are answers rounded properly, as instructed?
- ✓ Is your answer in the requested form? (Mixed number, decimal, improper fraction, whole number)
- ✓ Does your answer make sense?

## Category 5

### Algebra

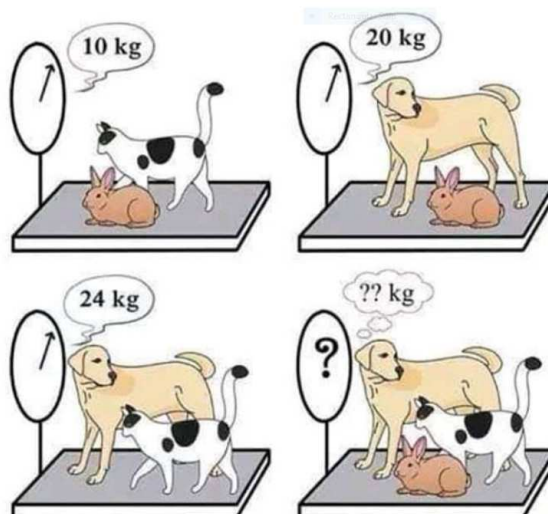
Meet #4 - February, 2019

### Calculator Meet

- 1) There are three consecutive positive integers, such that the largest is 44 less than twice the smallest. What is the value of the smallest of these three consecutive integers?
- 2) Michael ran 10 miles in 80 minutes. Brady ran 16 miles in 80 minutes. How many more miles did Brady run than Michael in 2.5 hours if they each maintained their same rate of speed? Express your answer as a decimal.
- 3) The picture below was discovered in the Internet. Together, the cat and the rabbits weigh a total of 10 kg (kilograms), the dog and rabbit weigh a total of 20 kg, and the dog and cat weigh a total of 24 kg. In all, how much do the dog, cat, and rabbit weigh?

#### ANSWERS

- 1) \_\_\_\_\_
- 2) \_\_\_\_\_ miles
- 3) \_\_\_\_\_ kg



**Solutions to Category 5**  
**Algebra**  
**Meet #4 - February, 2019**

**Answers**

- 1) Let  $X$ ,  $X + 1$ , and  $X + 2$  represent three consecutive positive integers.

$$X + 2 = 2X - 44$$

$$46 = X$$

So, the smallest integer is 46.

1) 46

2) 11.25

3) 27

- 2) Michael's rate is 10 miles per 80 minutes, or 1 mile per 8 minutes. In 2.5 hours, or 150 minutes, he can run  $150 / 8$ , or 18.75 miles. Brady's rate is 16 miles per 80 minutes, or 1 mile per 5 minutes. In 150 minutes, Brady can run  $150 / 5$ , or 30 miles. So, in 2.5 hours, Brady can run  $30 - 18.75$ , or 11.25 miles farther.
- 3) Although the answer to this problem can be deciphered without writing formal equations, the following algebraic solution involves solving a system of equations in three variables.

Let  $D$  = the weight of the dog in kg

$C$  = the weight of the cat in kg

$R$  = the weight of the rabbit in kg

Then  $C + R = 10$

$$D + R = 20$$

$$D + C = 24$$

Adding the three equations produces  $2C + 2D + 2R = 54$

Dividing both sides by 2 yields  $C + D + R = 27$

Therefore, the three animals weigh a total of 27 kg.

## Category 5

### Algebra

Meet #4 - March, 2017

### *Calculator Meet*

- 1) Three burgers and a drink cost \$18.55. Seven burgers and a drink cost \$40.95. What is the cost of one drink?
  
  
  
  
  
  
  
  
  
  
- 2) Jason can run an average of 100 yards in 18 seconds. If he can maintain that pace, how many minutes would it take for him to run two miles? There are 1760 yards in a mile.
  
  
  
  
  
  
  
  
  
  
- 3) There are 2 tulips for every 5 crocuses in Alice's garden of 84 plants. Crimson's garden has 4 tulips for every 3 crocuses. If the two gardens were combined, there would be an equal number of crocuses and tulips. How many tulips are in Crimson's garden?

#### ANSWERS

1) \$ \_\_\_\_\_

2) \_\_\_\_\_

3) \_\_\_\_\_

**Solutions to Category 5**  
**Arithmetic**  
**Meet #4 - March, 2017**

<u>Answers</u>	
1)	1.75
2)	10.56
3)	144

- 1) Let  $B$  = the number of burgers and  
 $D$  = the number of drinks

$$3B + D = 18.55$$

$7B + D = 40.95$  Since the number of drinks is the same for each order, the difference in the number of burgers is 4 and the difference in the costs is \$22.40. So, the cost of one burger is  $22.4 / 4$ , or \$5.60 and the cost of one drink is \$1.75.

- 2) Maintaining a pace implies that the distance versus time is proportional:

$$\frac{\text{distance in yards}}{\text{time in seconds}} = \frac{100}{18} = \frac{(2)(1760)}{X}$$

Cross products are equal, so

$$100X = (18)(2)(1760)$$

$$100X = 63,360$$

$$X = 633.6$$

Convert 633.6 to minutes by dividing by 60, yielding an answer of 10.56 minutes.

- 3) Using the fact that there are 2 tulips for every 5 crocuses in Alice's garden, we can scale the total (7) by a factor of 12 for each type to make a total of 84 plants. Then Alice has (2)(12), or 24 tulips and (5)(12), or 60 crocuses, or a difference of 36 for the two types of flowers. Crimson has 4 tulips for every 3 crocuses, or a difference of 1 for the two types. Scaling that difference by a factor of 36 would give Crimson (4)(36), or 144 tulips and (3)(36), or 108 crocuses. Combining the two gardens yields a total of  $24 + 144$ , or 168 tulips and  $60 + 108$ , or 168 crocuses. So, Crimson has 144 tulips.

**Category 5**  
**Algebra**  
**Meet #4 - February, 2015**  
***Calculator meet***

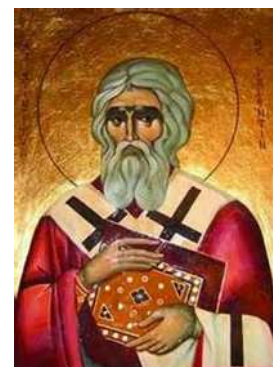


- 1) Tom completes three passes of every five that he throws. In an average football season, Tom throws 465 passes. How many passes does he *fail* to complete?
  
- 2) Four burgers and three drinks cost \$20.05. Seven burgers and four drinks cost \$32.85. How much money would it cost to provide a classroom of 23 students so that each student receives two burgers and one drink?
  
- 3) On the Isle of Sodor, a set of parallel train tracks is 400 miles long. On one end of one track, Train Thomas leaves the station at 5:00 PM. On the opposite end of the other track, Train Diesel leaves the station at 6:30PM. If Train Thomas travels 40 mph and Train Diesel travels 60 mph, then at what time will they meet?

.....

Father Frank O'Gara of Whitefriars Street Church in Dublin, Ireland, tells the real story of Saint Valentine, who was a Roman priest. At that time, 269 A.D., young people were forbidden by the Church to be married. Valentine broke the

law and, eventually, was caught, imprisoned, and tortured for performing marriage ceremonies against the command of Claudius II. Valentine was sentenced to a three-part execution of a beating, stoning, and finally decapitation, all because of his stand for Christian marriage. The last words he wrote were in a note to justice Asterius' daughter: "from your Valentine."



**ANSWERS**

- 1) \_\_\_\_\_
- 2) \$ \_\_\_\_\_
- 3) \_\_\_\_\_ P.M.

**Solutions to Category 5**  
**Algebra**  
**Meet #4 - February, 2015**

**Answers**

1) 186

2) 209.99

3) 9:54

- 1) Let  $P$  = the number of passes that Tom completes. Then write and solve a proportion.

$$\frac{\text{\# of passes completed}}{\text{\# of passes thrown}} = \frac{3}{5} = \frac{P}{465}$$

Cross products are equal:  $5P = (3)(465)$

$$5P = 1395 \text{ and } P = 279.$$

Tom completes 279 passes, so he fails to complete  $465 - 279$ , or 186 passes.

- 2) Let  $B$  = the number of burgers

$D$  = the number of drinks.

$$4B + 3D = 20.05 \text{ and } 7B + 4D = 32.85.$$

To eliminate  $D$ , multiply both sides of the first equation by 4 and both sides of the second equation by  $-3$ , then add the two equations:

$$16B + 12D = 80.2 \text{ and } -21B - 12D = -98.55$$

Adding yields  $-5B = -18.35$ , so  $B = 3.67$  and then  $D = 1.79$ .

So, a burger costs \$3.67 and a drink costs \$1.79.

So that members of the class of 23 students each has two burgers and one drink, the total cost will be  $(23)[(2)(3.67) + 1.79] = (23)[7.34 + 1.79]$   
 $= (23)(9.13) = \$209.99$ .

- 3) Let  $T$  = the number of hours that Diesel rides.

$T + 1.5$  = the number of hours that Thomas rides.

$60T$  = the number of miles that Diesel rides.

$40(T + 1.5)$  = the number of miles that Thomas rides.

Their total distance = 400 miles, so  $60T + 40(T + 1.5) = 400$

$$60T + 40T + 60 = 400$$

$$100T + 60 = 400$$

$$100T = 340$$

$$T = 3.4$$

So, 3.4 hours after 6:30 P.M. is 9:54 P.M.



## Category 5

*You may use a calculator.*

### Algebra

#### Meet #4, February 2013

1. Abby weighs 9 pounds less than twice as much as her younger sister Bridget. If they weigh 102 pounds together, how many pounds does Abby weigh?
2. Benny and Lenny live 1 mile from their school. Benny walks to school at an average speed of 3 mph and his brother Lenny rides his bike at an average speed of 10 mph. If Lenny wants to arrive at school 2 minutes before Benny, how many minutes after Benny should he leave the house?
3. If 7 thneed-makers can make 444 thneeds in 6 hours and 10 minutes, how many thneeds can 10 thneed-makers make in 4 hours and 40 minutes?

#### Answers

1. \_\_\_\_\_ pounds
2. \_\_\_\_\_ minutes
3. \_\_\_\_\_ thneeds

Solutions to Category 5  
Algebra  
Meet #4, February 2013

Answers

1. 65 pounds
2. 12 minutes
3. 480 thneeds

1. We will let  $A$  be Abby's weight and  $B$  be Bridget's weight. Translating the English to algebra, we get  $A = 2B - 9$  and  $A + B = 102$ . We can substitute the value of  $A$  from the first equation into the second equation and solve for  $B$  as shown below.

$$2B - 9 + B = 102$$

$$3B - 9 = 102$$

$$3B = 111$$

$$B = 37$$

The value of  $A$  must be  $102 - 37 = \mathbf{65 \text{ pounds}}$ . We can also check that  $2 \times 37 - 9 = 74 - 9 = 65$  is the same, which it is.

2. Since Benny can walk 3 miles in 1 hour, it will only take him  $1/3$  of an hour, or 20 minutes, to walk the 1 mile to school. Similarly, it will take Lenny  $1/10$  of an hour, or 6 minutes, to ride the 1 miles to school. If Lenny wanted to arrive at the same time as Benny, he could leave  $20 - 6 = 14$  minutes later. Since he wants to arrive at school 2 minutes before Benny, Lenny should leave  $14 - 2 = \mathbf{12 \text{ minutes}}$  after Benny.

3. First let's convert the times to minutes: 6 hrs. 10 min. = 370 min. and 4 hrs. 40 min. = 280 min. For now, let's just stick with 7 thneed-makers and see how many thneeds they could make in 280 minutes. We could

set up a proportion:  $\frac{444 \text{ thneeds}}{370 \text{ minutes}} = \frac{x \text{ thneeds}}{280 \text{ minutes}}$ . Solving for  $x$ , we get

$$x = \frac{444 \times 280}{370} = 336 \text{ thneeds.}$$

If we divide this by 7, we find that each

thneed-maker must make  $336 \div 7 = 48$  thneeds in 280 minutes.

Therefore, 10 thneed-makers would make  $10 \times 48 = \mathbf{480 \text{ thneeds}}$  in 280 minutes.

Category 5 – Algebra

1. If I add 9 to my secret number, I'll get a number that is 90% more than the number I'll get if I subtract 9 from my secret number.

What is my secret number?

2. Donald and Mickey both collect stamps.

Donald figured he needs 30% more stamps in order to have half as many as Mickey does.

Mickey figured that 12 more stamps would make his collection 3 times the size of Donald's. How many stamps do they have altogether?

3. You want to fill an empty tank with 50 gallons of water and 50 gallons of oil.

At your disposal are two barrels with different mixtures that you may use to fill the empty barrel:

- Barrel *A* contains a mixture of 80% water and 20% oil.
- Barrel *B* contains a mixture of 40% water and 60% oil.

How many gallons of mixture from barrel *A* will you use?

Answers
1. _____
2. _____
3. _____

Solutions to Category 5 - Algebra

1. Calling my secret number  $N$ , we know that:

$N + 9 = 190\% \cdot (N - 9)$ , which we can simplify into:

$$9 \cdot (1 + 1.9) = N \cdot (1.9 - 1) \text{ or } 0.9 \cdot N = 9 \cdot 2.9$$

$$\text{So } N = \frac{9 \cdot 2.9}{0.9} = 29$$

Answers

1. 29
2. 108
3. 25

2. If Donald has  $D$  stamps, and Mickey has  $M$  stamps, we know that:

$$D + 30\% \cdot D = 1.3 \cdot D = \frac{1}{2} \cdot M$$

$$M + 12 = 3 \cdot D$$

So we can write the second equation as  $2.6 \cdot D + 12 = 3 \cdot D$ , so  $0.4 \cdot D = 12$ , or  $D = 12 \cdot 2.5 = 30$  and  $M = 3 \cdot D - 12 = 78$ . So together  $D + M = 108$ .

3. If we are to use  $G$  gallons from barrel  $A$ , then it will contains  $80\% \cdot G$  gallons of water. We will use  $(100 - G)$  total gallons from barrel  $B$ , to bring the total volume to 100 gallons, and these will contain  $40\% \cdot (100 - G)$  gallons of water, and so we require that  $80\% \cdot G + 40\% \cdot (100 - G) = 50$  gallons, and the solution for this is  $G = 25$  gallons.

*Check:* the 25 gallons from  $A$  will be 20 gallons water and 5 gallons oil, and the 75 gallons from  $B$  will be 30 gallons water and 45 gallons oil.

It is insightful to summarize the given information in this problem like this:

	Barrel A	Barrel B
Water %	80	40
Oil %	20	60

## Category 5

### Algebra

#### Meet #4, February 2009

1. It took Joey 6 days to assemble the 32 ginger bread houses he planned to sell at the local fair. If he assembles gingerbread houses at the same rate as he did for the local fair, how many days will it take him to assemble the 112 ginger bread houses he plans to sell at the state fair?
2. Billy's teacher has told her students that the value of a word can be found by assigning a fixed number of points for each consonant and a different fixed value for each vowel and finding the sums of the values for the whole word (all consonants are worth the same as each other and all vowels are worth the same as each other, but consonants and vowels are worth a different amount from each other). If Billy finds that the value of the word "math" is 35 points and the value of the word "league" is 60 points, then what is the value of the word "Intermediate"? (A, E, I, O, U are the vowels, all other letters here are consonants.)
3. Jan has 2 animals, Pat the cat and Snog the dog. If both animals gained 20 pounds, Snog would be 25% heavier than Pat. If they both lost 20 pounds, Snog would be 50% heavier than Pat. What is the combined weight, in pounds, of Pat and Snog?

Answers	
1.	_____
2.	_____
3.	_____

## Solutions to Category 5

### Algebra

#### Meet #4, February 2009

Answers

1. 21

2. 114

3. 140

1. Since it took Joey 6 days to build 32 houses that gives us the ratio  $\frac{6 \text{ days}}{32 \text{ houses}}$ . He needs to build 112 houses and we want to know the number of days, call that  $d$ . We can write a second ratio and form a proportion now:  $\frac{6}{32} = \frac{d}{112} \rightarrow \frac{3}{16} = \frac{d}{112} \rightarrow \frac{3 \times 7}{16 \times 7} = \frac{21}{112} = \frac{d}{112}$   
So  $d = 21$  days.

2. "Math" has 3 consonants and 1 vowel, while "league" has 2 consonants and 4 vowels. Using  $c$  as the value of a consonant, and  $v$  as the value of a vowel we can write these two equations:

$$3c + 1v = 35$$

$$2c + 4v = 60$$

Multiplying the first equation by 4 results in these two equations:

$$12c + 4v = 140$$

$$2c + 4v = 60$$

Subtracting the two equations we get:

$10c = 80$  which tells us that a consonant is worth 8 points. Using the original equation for "math" we have  $3(8) + v = 35 \rightarrow v = 11$  so vowels are worth 11 points. "Intermediate" has 6 consonants and 6 vowels for a value of  $6(8) + 6(11) = 48 + 66 = 114$  points

3. Using  $P$  for Pat's weight and  $S$  for Snog's weight we get the equations:

$$1.25(P + 20) = S + 20 \quad \text{and} \quad 1.5(P - 20) = S - 20$$

$$1.25P + 25 - 20 = S \quad \text{and} \quad 1.5P - 30 + 20 = S$$

$$1.25P + 5 = S \quad \text{and} \quad 1.5P - 10 = S$$

Substituting for  $S$  we get:

$$1.25P + 5 = 1.5P - 10$$

$$15 = .25P$$

$$60 = P$$

Plugging 60 in for  $P$  in the first equation we get

$$1.25(60 + 20) = S + 20$$

$$1.25(80) = S + 20$$

$$100 = S + 20 \rightarrow 80 = S$$

So the combined weight is  $60 + 80 = 140$  lbs