

Math League SCASD

2020

Meet #4

Arithmetic

Self-study Packet

Problem Categories for this Meet (in addition to topics of earlier meets):

1. Mystery: Problem solving
2. Geometry: Properties of Circles
3. Number Theory: Modular Arithmetic, Series and Sequences
- 4. Arithmetic: Percent Applications**
5. Algebra: Word Problems (linear, including direct proportions or systems)

Important Information you need to know about ARITHMETIC: Percent Applications

- Change a percent to a decimal by moving the decimal two places to the left.
- Change a decimal to a percent by moving the decimal two places to the right and adding the percent sign.
- *Of* means multiply
- *Is* means equals

Example: Find $\frac{1}{2}\%$ of 2000.

$$\frac{1}{2} = .5 \quad \frac{1}{2}\% = .005$$
$$.005 \bullet 2000 = 10$$

- To calculate how much money you will have if you deposit x dollars at an interest rate of $y\%$ after t years, first change the interest rate to a decimal, then multiply the original deposit by the sum of one plus the interest rate raised to the power of t .

Example: How much money will you have after 8 years if you deposit \$10,000 at an interest rate of 6% a year?

$$\$10,000 \bullet 1.06^8 \approx \$15,938.48$$

**Be careful! Annual interest rate is per year. Monthly interest rate is per month. If you are told that the annual interest is 6% but that interest is compounded monthly, you need to find the monthly interest rate by dividing by twelve before you can compute.*

Category 4

Arithmetic

Meet #4 - February, 2019



Calculator Meet

1) Carolyn treated her friends to lunch. The bill came to \$ 28.60. There was a 5% meals tax added to the bill. Carolyn also left a 20% tip (based on the \$ 28.60 cost of the meal). How much money did this lunch cost Carolyn in all, including meals, tax, and tip?

2) Pele was injured in a soccer game. During many months of recovery, he lost 20% of his body weight. Once he began to train again to get into shape to play soccer again, he regained all the weight that he had lost. By what percent did his weight increase during the time Pele was training to get back into shape?

3) The equation to the right calculates the value A , that is the result of investing an initial amount of money, M , at an annual (yearly) interest rate of R for T years when the interest is compounded W times annually. Alexandria wanted to invest \$80,000 at her bank at an annual interest rate of 3.7%, compounded annually (yearly), for 12 years. Kamala insisted that she would earn more interest at her bank at an annual rate of 3.2%, compounded three times monthly, for 12 years. Who was correct? Also, how much more interest was earned with the more profitable deal? Round your answer to the nearest hundred dollars. Note: *You must get both parts correct to earn credit for this problem.*

$$A = M \left(1 + \frac{R}{W} \right)^{WT}$$

ANSWERS

1) \$ _____

2) _____ %

3) who? _____

\$ _____

Solutions to Category 4
Arithmetic
Meet #4 - February, 2019

<u>Answers</u>	
1)	35.75
2)	25 (%)
3)	Alexandria AND 6,300

- 1) The total of the meals, tax, and tip is equal to 125% of 28.60, or $(1.25)(28.6) = 35.75$.
- 2) For a student who has not yet studied algebra, an effective strategy is to start with a number like 100 pounds. Losing 20% brings Pele's weight down to 80 pounds. Then 25% of 80 added to 80 yields $20 + 80$, or 100 pounds.

3)

$$A = M \left(1 + \frac{R}{W} \right)^{WT}$$

$$A = 80,000 \left(1 + \frac{0.032}{36} \right)^{(36)(12)}$$

$$A = 80,000(1.0008888\dots)^{432}$$

$$A = 80,000(1.46789504\dots)$$

$$A = 117,431.6038$$

This is how much money Kamala would have after 12 years.

Now for Alexandria:

$$A = M(1+R)^T \quad \text{This is the simpler version, as } W = 1.$$

$$A = 80,000(1+0.037)^{12}$$

$$A = 80,000(1.037)^{12}$$

$$A = 80,000(1.546482738\dots)$$

$$A = 123,718.619$$

This is how much money Alexandria would have after 12 years.

The difference would be $123,718.619 - 117,431.6038$, or \$6,287.0152, with Alexandria having earned more. Rounding the final answer to the nearest hundred yields \$6,300. To earn two points credit, students must have both answers correct - Alexandria AND 6,300.

Category 4

Arithmetic

Meet #4 - March, 2017

Calculator Meet

1) A 12-ounce sponge was soaked in water. Its weight was increased by 460%. The sponge was then wrung out (squeezed) so that its weight decreased by 80%. How many ounces does the sponge weigh now?

2) Candace counted the M&Ms in a full box as follows:

A = 19 red

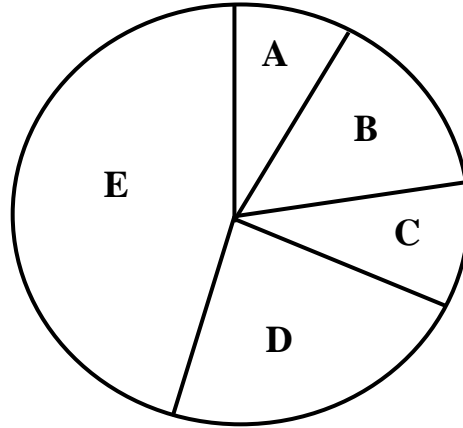
B = 36 green

C = 24 blue

D = 77 orange

E = 129 yellow

What percent of all the M&Ms is green?
Round your answer to the nearest whole percent.



3) The equation to the right calculates the value A, that is the result of investing an initial amount of money, M, at an annual (yearly) rate of R for T years when the interest is compounded W

$$A = M \left(1 + \frac{R}{W} \right)^{WT}$$

times annually. Emily would like to be a millionaire some day. How much money must Emily invest at an annual rate of 5.4% that compounds interest three times a month for 28 years? Round your answer either up to the next hundred dollars to assure that Emily will have at least \$1,000,000 after 28 years.

ANSWERS

1) _____

2) _____ %

3) \$ _____

Solutions to Category 4

Arithmetic

Meet #4 - March, 2017

Answers

1) 13.44

2) 13 (%)

3) 220,800

$$\begin{aligned} 1) \quad & 12 + (4.6)(12) - (2.4)[12 + (4.6)(12)] \\ & = 12 + 55.2 - (0.8)[12 + 55.2] \\ & = 67.2 - (0.8)(67.2) \\ & = 67.2 - 53.76 \\ & = 13.44 \text{ ounces} \end{aligned}$$

2) Divide the number of green M&Ms by the total number of M&Ms, then express the decimal answer as a percent.

$$\begin{aligned} & 36 / (19 + 36 + 24 + 77 + 129) \\ & = 36 / 285 \\ & = 0.1263157... \\ & = 13\% \text{ when rounded to the nearest whole percent.} \end{aligned}$$

$$3) \quad A = M \left(1 + \frac{R}{W} \right)^{WT}$$

original formula

$$1,000,000 = M \left(1 + \frac{0.054}{(3)(12)} \right)^{(3)(12)(28)}$$

substituting values, including the number $T=28$ for the number of years and $W = 3(12)$ for the number of times per year the interest is compounded.

$$1,000,000 = M \left(1 + \frac{0.054}{36} \right)^{1008}$$

evaluate

$$1,000,000 = M(1 + 0.0015)^{1008}$$

evaluate

$$1,000,000 = M(1.0015)^{1008}$$

evaluate

$$1,000,000 = M(4.530657773)$$

evaluate

$$M = \frac{1,000,000}{4.530657773}$$

solve for M

$$M = 220,718.5027$$

divide

This amount, \$220,718.5027, must be rounded up to \$220,800 in order to assure that Emily will have at least \$1,000,000 after 28 years.

Category 4
Arithmetic
Meet #4 - February, 2015
Calculator meet



- 1) The "Rule of 72" is often used to approximate the number of years, Y , it would take to double the value of an investment when given an annual rate of R percent, as follows: $(R)(Y) = 72$. For example, if \$50 were invested at an annual rate 6%, then it would take $72 / 6$, or 12 years, for it to double in value to \$100. If that same \$50 had been invested at 4% instead, then how many more years would it have taken for the \$50 to double in value?

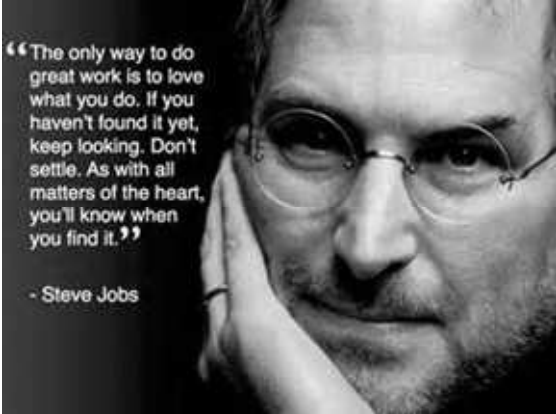
- 2) Gronk took his 74 teammates out for a steak dinner. The average price for each dinner was \$76.80. A 5% tax was then added onto the cost of the meal. He left the wait staff a generous 35% tip, based on the price of the meal only. What was the total amount, including meal, tax, and tip, that Gronk paid so that he and his teammates could have dinner?

- 3) Early in the race, Jen and Berry were running at the same speed. Berry then increased her speed by 20% while Jen increased her speed by only half as much. Berry is now running at 60 spans per second. How many spans per second is Jen running now?

.....

Steve Jobs, born in February of 1955, was an American entrepreneur, marketer, and inventor who was the co-founder, chairman, and CEO of Apple Inc. He is widely recognized as a pioneer of the personal computer revolution, transforming one industry after another, from computers and smart phones (iPhone) to music (iTunes) and movies (Pixar).

<u>ANSWERS</u>	
1)	_____
2)	_____
3)	_____



Solutions to Category 4
Arithmetic
Meet #4 - February, 2015

Answers

1) 6

2) 8064

3) 55

1) Using the rule of 72, $72 / 4 = 18$ years, which is 6 years more than 12.

2) $(75)(\$76.80) = \5760 , the total cost of the food only.

$(\$5760)(0.05) = \288 , the amount of tax.

$(\$5760)(0.35) = \2016 , the amount of the tip.

Meal + tax + tip = $\$5760 + \$288 + \$2016 = \8064 .

3) Working backwards, Berry's speed early in the race was $60 / 1.2 = 50$ spans per second.

Jen's speed now: $(50)(1.1) = 55$ spans per second.

Category 4

You may use a calculator.

Arithmetic

Meet #4, February 2013

1. This year, an all-day adult lift ticket at Ski Mountain costs \$78, which is 4% more than last year. How much did an all-day adult lift ticket cost last year?

2. The original price of a winter coat was \$60 before it was marked down by 30% for a sale. Mary bought the coat during the sale but she also used a 10% off coupon. How much did she pay for the coat? Express your answer in dollars to the nearest hundredth of a dollar.

3. Allan deposited \$10,000 into a savings account that paid 3% annual interest compounded monthly. Izak deposited \$10,000 into a savings account that paid 4% annual interest compounded monthly. Neither of them withdrew any money from his account. After one year, how much more money did Izak have in his account than Allan had in his account? Remember that, each month, interest will be paid on the balance at $\frac{1}{12}$ of the annual rate. Express your answer in dollars to the nearest hundredth of a dollar.

Answers

1. \$ _____

2. \$ _____

3. \$ _____

Solutions to Category 4
 Arithmetic
 Meet #4, February 2013

Answers	
1.	\$75 or \$75.00
2.	\$37.80
3.	\$103.26

1. The price of \$78 represents 104% of the previous year's price. To find the previous year's price, we simply divide 78 by 1.04, which is **\$75**.

2. When a price is marked down by 30%, it means the new price is $100 - 30 = 70\%$ of the old price, so the sale price of the coat was $60 \times 0.7 = \$42$. Mary also used a 10% off coupon, so she paid $100 - 10 = 90\%$ of the sale price, which was $42 \times 0.9 = \mathbf{\$37.80}$.

3. We can calculate the year-end balances directly as follows:

$$10000 \times \left(1 + \frac{0.03}{12}\right)^{12} \approx 10304.16 \text{ and}$$

$$10000 \times \left(1 + \frac{0.04}{12}\right)^{12} \approx 10407.42$$

After one year, Allan has $10,407.42 - 10,304.16 = \mathbf{\$103.26}$ more than Izak. Alternatively, we could keep track of the balances at the end of each month, as shown in the table below.

Month	Allan's Balance	Izak's Balance
0	\$10,000.00	\$10,000.00
1	\$10,025.00	\$10,033.33
2	\$10,050.06	\$10,066.78
3	\$10,075.19	\$10,100.33
4	\$10,100.38	\$10,134.00
5	\$10,125.63	\$10,167.78
6	\$10,150.94	\$10,201.67
7	\$10,176.32	\$10,235.68
8	\$10,201.76	\$10,269.80
9	\$10,227.26	\$10,304.03
10	\$10,252.83	\$10,338.38
11	\$10,278.46	\$10,372.84
12	\$10,304.16	\$10,407.42

Category 4 – Arithmetic

1. The tab for your dinner party at a restaurant came to \$160.

You want to leave a 20% tip, and also need to add 6.25% tax on the price of dinner. How much does the dinner cost altogether? (There is no tax on the tip, and assume you do not pay a tip on the tax either.)

2. If the price of a gallon of milk increases by 25%, and the price of a gallon of oil decreases by 15%, then both prices will be the same.

What percent of the price of (a gallon of) oil is the price of (a gallon of) milk?

3. Your company, *IMLEM Lemonade Inc.*, has purchased \$700 worth of supplies (lemons, sugar, paper cups). This is enough to make 3,000 cups of lemonade.

- You sold 80% of that number, for \$0.50 per cup.
- From the money you have left (the amount you received from customers minus the cost of your supplies), you pay 40% in salaries.
- From the amount left, you pay 20% sidewalk-tax to your parents.

How much money do you have left?

Answers	
1. \$	_____
2.	_____ %
3. \$	_____

Solutions to Category 4 - Arithmetic

1. The tax amount is $\$160 * 6.25\% = \10

The tip amount is $\$160 * 20\% = \32

Overall: $\$160 + \$10 + \$32 = \202

Editor's note: The original problem did not say there was no tip on the tax. This would give \$204, which was not allowed as an answer.

Answers

1. \$202 (\$202.00)

2. 68%

3. \$240

2. If we name the respective prices *A* and *B*, then we know that:

$A + 25\% \cdot A = B - 15\% \cdot B$ or: $125\% \cdot A = 85\% \cdot B$ and so:

$$A = \frac{85}{125} \cdot B = 68\% \cdot B$$

3. Our revenue (the amount we got from customers) is:

$$\$0.50_{per\ cup} * 80\% * 3,000_{cups} = \$1,200$$

Minus the cost of supplies, you have \$500 left.

After paying salaries, you have left $60\% \cdot \$500 = \300

After paying tax, you have left $80\% \cdot \$300 = \240

Category 4

Arithmetic

Meet #4, February 2009

1. The Euclidean Middle School math team had a score of 120 at the 1st IMLEM meet of the year and a score of 165 at the 2nd meet of the year. What was the percent increase in the math team's score from the 1st meet to the 2nd meet? Express your answer as a mixed number.
2. Akeelah bought a dictionary that was on sale for 27% off. The sale price was \$35.04. What was the original price of the dictionary in dollars?
3. Frank went to the bank to deposit \$6,000 in a new savings account. The bank had two offers for him. He could either earn 6% annual interest compounded monthly, or 7% annual interest compounded annually. At the end of 3 years, how much more interest will he have earned if he chooses the better offer than if he had chosen the worse offer? Assume no withdrawals or additional deposits were made during these three years. Round your answer to the nearest whole dollar.

Answers	
1.	_____
2.	_____
3.	_____

Solutions to Category 4
Arithmetic
Meet #4, February 2009

Answers

1. $37\frac{1}{2}$

1. The team's score increased by 45 points which is compared to the original score of 120 to find the percent increase.

$$\frac{45}{120} = \frac{3}{8} = 37\frac{1}{2}\% \text{ increase}$$

2. \$48

3. \$170

2. If Akeelah received 27% off, that means she still had to pay for 73% of the original price. Since she paid \$35.04 we can use the following equation to find the original price, x :

$$.73x = 35.40$$

$$x = 48$$

3. If Frank chooses to compound the interest monthly he would use this equation to find the value after 3 years:

$$V = 6000 \times \left(1 + \frac{.06}{12}\right)^{3 \times 12} = 6000(1.005)^{36} \approx 7180.083$$

If Frank decides to compound the interest annually he would use this equation to find the value after 3 years:

$$V = 6000 \times (1 + .07)^3 = 6000(1.07)^3 = 7350.258$$

Using the second option he would have earned $7350.258 - 7180.083 = 170.175 \approx 170$ more.