

# Math League SCASD 2020

## Meet #4

# Geometry

## Self-study Packet

Problem Categories for this Meet (in addition to topics of earlier meets):

1. Mystery: Problem solving
- 2. Geometry: Properties of Circles**
3. Number Theory: Modular Arithmetic, Series and Sequences
4. Arithmetic: Percent Applications
5. Algebra: Word Problems (linear, including direct proportions or systems)

## ***Important information you need to know about GEOMETRY: Properties of Circles***

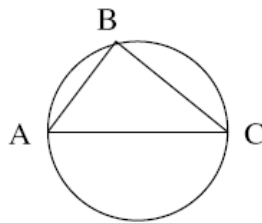
Area of a Circle:  $A = \pi r^2$

Circumference of a Circle:  $C = \pi d$  or  $C = 2\pi r$

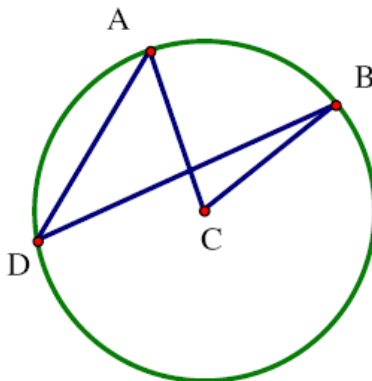
\*(Very Important!) *Be sure to use the test's given value for  $\pi$  and not the  $\pi$  button on your calculator!!!*

### **Other properties:**

- If AC is a diameter of a circle and point B is any other point on the circle, angle ABC will be a right angle.



- The measure of an inscribed angle is half the measure of the arc it "subtends." For example, if arc AB is  $70^\circ$ , then the measure of angle ADB is  $35^\circ$



**Category 2**

**Geometry**

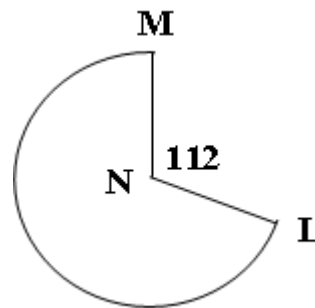
**Meet #4 - February, 2019**



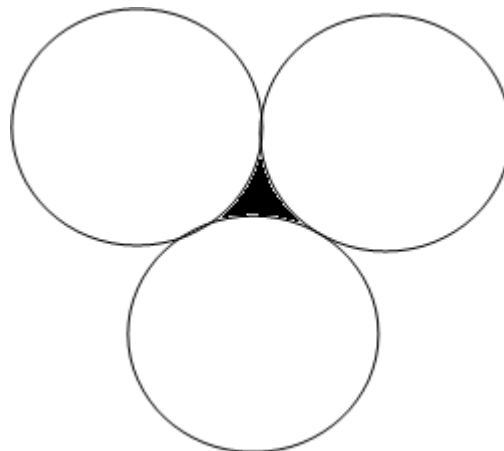
*Calculator Meet*

1) The diameter of a circle measures 16 centimeters. Its circumference is A centimeters and its area is B square centimeters. What is the value of B - A ? Use  $\pi \approx 3.14$ .

2) In the partial circle to the right, point N is the center. If NM and NL are radii and each radius measures 6.7 feet. How many square feet are in the area of this figure? Use  $\pi \approx 3.1416$ . Round your final answer to the nearest tenth of a square foot. The central angle measures 112 degrees.



3) Three congruent (identical) circles are tangent to one another, as shown. (Two circles are tangent if they intersect at only one point.) The radius of each circle is 26 inches. How many square inches are in the area of the black region? Use  $\pi \approx 3.1$ . Round your final answer to the nearest whole number of square inches.



**Answers**

1) \_\_\_\_\_

2) \_\_\_\_\_ sq. ft.

3) \_\_\_\_\_ sq. in.

**Solutions to Category 2**  
**Geometry**  
**Meet #4 - February, 2019**

<u>Answers</u>	
1)	150.72
2)	97.2
3)	123

1)  $A = \text{circumference} = (\pi) (\text{diameter})$   
 $= (3.14) (16)$   
 $= 50.24 \text{ cm.}$

$B = \text{area} = (3.14) (8) (8)$   
 $= 200.96 \text{ sq. cm.}$

$B - A = 200.96 - 50.24 = 150.72.$

2)  $\text{Area of figure} = (\text{fraction of circle}) (\text{area of circle})$   
 $= (248 / 360) (3.1416) (6.7) (6.7)$   
 $= 97.15152 \dots$   
 $= 97.2 \text{ square feet when rounded to the nearest tenth.}$

3) Draw radii through points of tangency to create an equilateral triangle, each of whose central angles measures 60 degrees. Find the area of the triangle, then subtract the areas of the three sectors to get the area of the black region.

$\text{area of triangle} - \text{area of three sectors}$   
 $= (\text{side squared divided by } 4) (\text{square root of } 3) - (3) (1/6) (3.1) (26) (26)$   
 $= [(52) (52) / 4] (1.73205 \dots) - 1047.8$   
 $= (1170.865 \dots) - 1047.8$   
 $= 123.06 \dots$   
 $= 123 \text{ when rounded to the nearest whole number of square inches.}$

**Category 2**  
**Geometry**  
**Meet #4 - March, 2017**

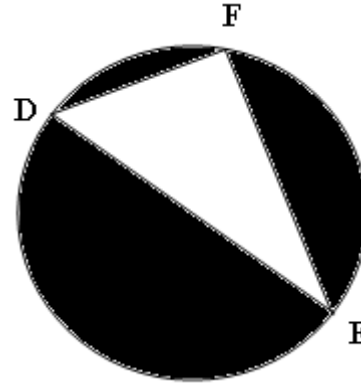


*Calculator Meet*

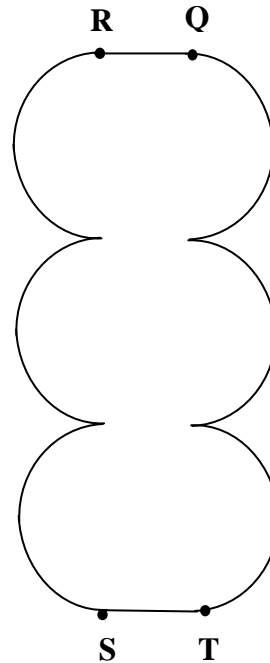
- 1) The diameter of this semicircle is 28 centimeters. How many square centimeters are in its area? Use  $\pi \approx 3.14$ . Round your final answer to the nearest tenth.



- 2) DE is the 25-inch diameter of this circle. FE = 20 inches. How many square inches are in the total area of the shaded regions? Use  $\pi \approx 3.142$ . Round your final answer to the nearest hundredth.



- 3) Six congruent semi-circles surround rectangle QRST. The area of the rectangle is an integer number of square meters between 160 and 168. The width of the rectangle, QR, and the length, QT, are odd prime numbers.  $RS > RQ$ . How many meters are in the perimeter of the figure? Use  $\pi \approx 3.1$ . Round your final answer to the nearest whole number. Note: The figure is not drawn to scale.



Answers

1) \_\_\_\_\_

2) \_\_\_\_\_

3) \_\_\_\_\_

**Solutions to Category 2  
Geometry  
Meet #4 - March, 2017**

<u>Answers</u>	
1)	307.7
2)	340.94
3)	85

1)  $A = \frac{1}{2} \pi r^2$

$$A \approx \frac{1}{2}(3.14)(14)^2$$

$$A \approx \frac{1}{2}(3.14)(196)$$

$$A \approx 307.72$$

$$A \approx 307.7 \quad \text{rounded to the nearest tenth.}$$

- 2) Angle DFE, inscribed in a semicircle, is a right angle, so that triangle DFE is a right triangle.

**Plan:** a) Find the area of the triangle, b) find the area of the circle,  
c) subtract the area of the triangle from the area of the circle to find the total area of the shaded regions.

a) Use the Pythagorean Theorem to find that  $DF = 15$ . Then the area of the triangle is  $(0.5)(15)(20) = 150$ .

b) Area of the circle is  $A = \pi r^2 \dots A \approx (3.142)(12.5)^2 \dots A \approx (3.142)(156.25) \dots A \approx 490.9375 \dots A \approx 490.94$  to the nearest hundredth.

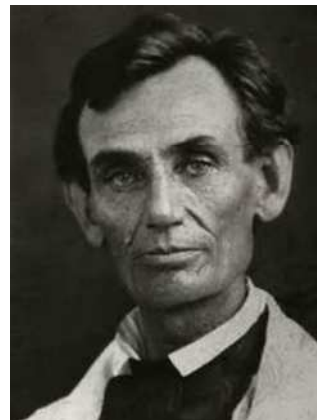
c)  $490.94 - 150 = 340.94$  square inches.

- 3) The only number between 160 and 168 that is the product of two primes is 161. Its factors are 23 and 7. So,  $RQ = 7$  and  $RS = 23$ . The radius of each semicircle is  $23/6$ .

The perimeter of the figure is  $2(RQ) + 6\pi r \dots \approx 2(7) + 6(3.1)\left(\frac{23}{6}\right)$

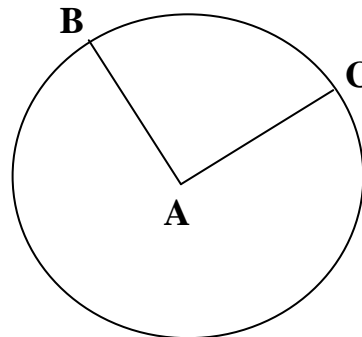
$\approx 14 + (3.1)(23) \dots \approx 14 + 71.3 \dots \approx 85.3 \approx 85$  meters (rounded to the nearest whole number).

**Category 2**  
**Geometry**  
**Meet #4 - February, 2015**  
*Calculator meet*



*"Whenever I hear anyone arguing for slavery, I feel a strong impulse to see it tried on him personally."  
... Abraham Lincoln, born February 12, 1809.*

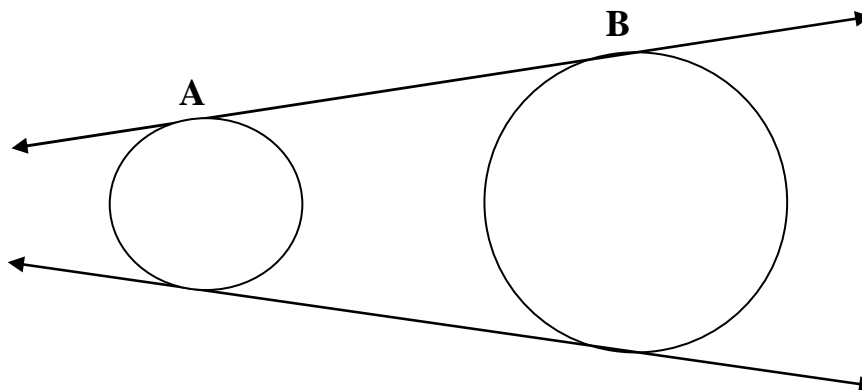
- 1) How many inches longer is the circumference of a circle of radius = 20 inches than a circle of radius = 10 inches? Use  $\pi \approx 3.14$ .
- 2) Given circle A with diameter = 42 feet. What is the perimeter, in feet, of sector ABC that includes radii AC and AB and arc BC? Angle A is a right angle. Use  $\pi \approx 3.14$ .



- 3) Two lines are tangent, externally, to two circles with diameters of 46 cm and 74 cm. AB = 48 cm. Points A and B are points of tangency. How many cm apart are the centers of the two circles? (Figure not to scale)

Answers

- 1) \_\_\_\_\_
- 2) \_\_\_\_\_
- 3) \_\_\_\_\_



**Solutions to Category 2  
Geometry  
Meet #4 - February, 2015**

1) Find the difference between the circumferences of the two circles =  $2(\pi)(R) - 2(\pi)(r)$   
 $= 2(3.14)(20) - 2(3.14)(10)$   
 $= 125.6 - 62.8 = 62.8$

2) Arc BC =  $\frac{1}{4}$  (circumference of circle)  
 $= (0.25)(2)(3.14)(21) = 32.97$ .  
 So, the perimeter of the sector = arc + 2 radii  
 $= 32.97 + (2)(21) = 32.97 + 42 = 74.97$  feet.

<u>Answers</u>	
1)	62.8
2)	74.97
3)	50

3) The radius drawn to a tangent from the center is perpendicular to the tangent.

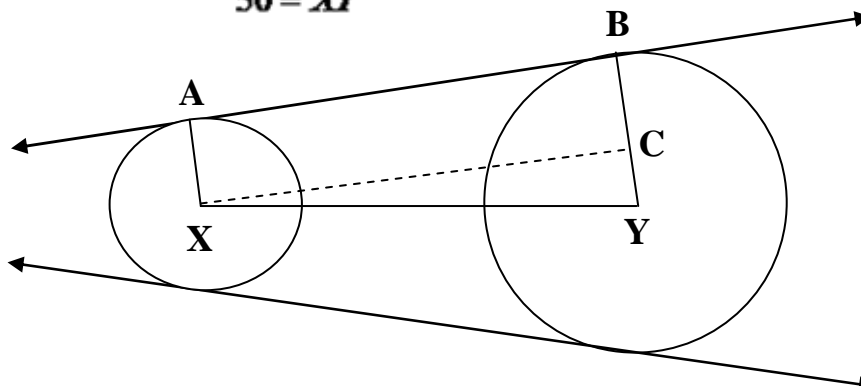
- \* Let XA and YB be the two radii.
- \* Draw XY to form a trapezoid with XA parallel to YB.
- \* Draw a segment from X, parallel to AB, to a point C on YB.
- \* We now have a rectangle and a right triangle.
- \*  $BY - BC = 37 - 23 = 14 = YC$ .
- \*  $AB = XC = 48$ .
- \* For right triangle XCY, use the Pythagorean Theorem to find the length of XY, the distance between the centers of the two circles:

$$14^2 + 48^2 = (XY)^2$$

$$196 + 2304 = (XY)^2$$

$$2500 = (XY)^2$$

$$50 = XY$$





## Category 2

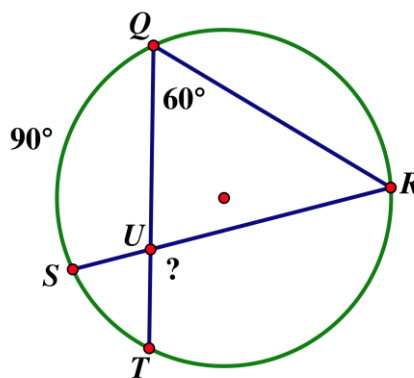
### Geometry

#### Meet #4, February 2013

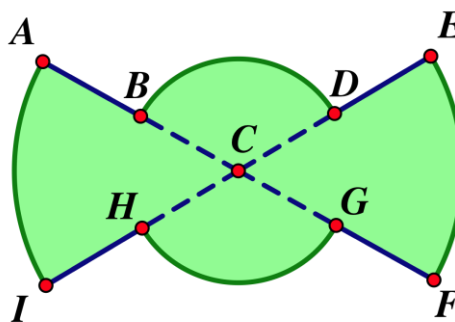
*You may use a calculator.*

1. A car turns 180 degrees on an arc of a circle. The tires on the inside of the turn are 20 feet from the center of the arc and the tires on the outside of the turn are 25 feet from the center of the arc. How many feet farther do the outside tires travel than the inside tires? Use 3.14 for  $\pi$  and round your answer to the nearest whole number of feet.

2. In circles, the measure of an inscribed angle is equal to half the measure of the intercepted arc. If the measure of arc QS is  $90^\circ$  and the measure of angle RQT is  $60^\circ$ , as shown in the figure, how many degrees are there in the measure of angle RUT?



3. The figure below was created from two concentric circles with centers at point C. Line segments AF and EI are bisected by point C, and each of the resulting segments is then bisected at points B, D, G, and H, as shown. If segment AB is 2 cm and angle BCD is 120 degrees, how many square centimeters are there in the entire figure? Use 3.14 for pi and express your answer to the nearest tenth of a square centimeter.



#### Answers

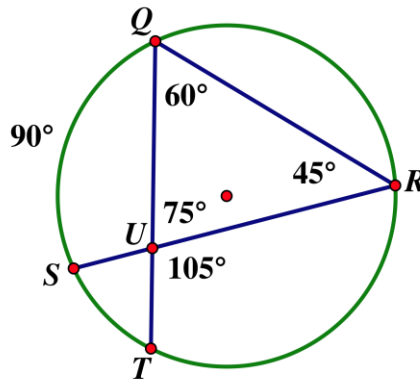
1. \_\_\_\_\_ feet
2. \_\_\_\_\_ degrees
3. \_\_\_\_\_ sq. cm

Solutions to Category 2  
Geometry  
Meet #4, February 2013

Answers
1. 16 feet
2. 105 degrees
3. 25.1 sq. cm.

1. The circumference of an entire circle is the diameter times  $\pi$ , so half that amount is the radius times  $\pi$ . The tires on the outside of the turn travel  $25\pi$  feet and the tires on the inside of the turn travel  $20\pi$  feet. The difference is  $25\pi - 20\pi = 5\pi$ , which is about 15.7 feet, or **16 feet** to the nearest foot.

2. Angle QRS intercepts arc QS, so it must be  $90 \div 2 = 45$  degrees. Since triangles have an angle sum of 180 degrees, the measure of angle QUR must be 75 degrees. Finally, angle RUT is supplementary to QUR, so it must measure  $180 - 75 = \mathbf{105}$  degrees.



3. The area of a circle is the square of the radius times  $\pi$ . Our picture can be thought of as  $\frac{2}{3}$  of a circle with radius 2 cm and  $\frac{2}{6} = \frac{1}{3}$  of a circle with radius 4 cm. The area is thus

$$\frac{2}{3} \times \pi \times 2^2 + \frac{1}{3} \times \pi \times 4^2 = \frac{8}{3} \pi + \frac{16}{3} \pi = \frac{24}{3} \pi = 8\pi \approx \mathbf{25.1}$$
 square centimeters.

Category 2 – Geometry

1. The Earth’s radius is 3,960 miles, and the Moon’s radius is 1,080 miles.

What is the difference (in miles) in the length of their equators?

*Assume both are perfect spheres, use  $\pi = 3.14$  and round to the nearest whole number.*

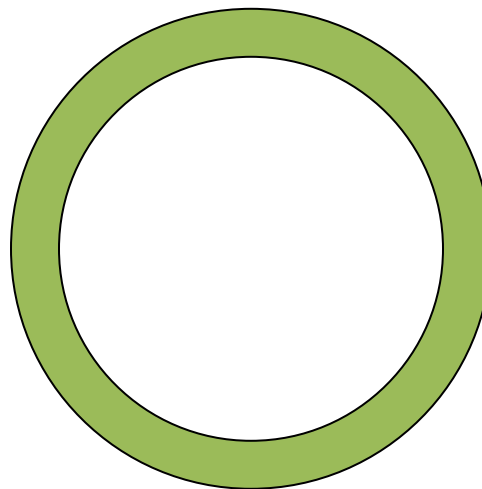
2. A square is inscribed inside a circle whose radius is 1 inch.

How many square inches are in the area of the square?

***Inscribed*** means that the square’s corners all touch the circle from the inside.

3. The shaded ring-shaped area measures 36% of the outer circle’s area.

If the radius of the outer circle is 10 inches, then how many inches are there in the radius of the inner circle?



Answers

1. \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_

Solutions to Category 2 – Geometry

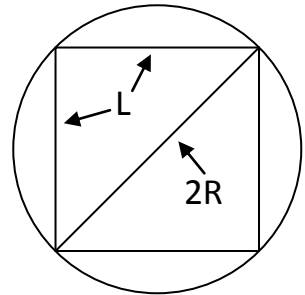
<u>Answers</u>	
1.	18,086
2.	2
3.	8

1. Since a circle's circumference (the equator) is given by the formula  $C = 2 \cdot \pi \cdot R$  then the difference is:

$$2 \cdot \pi \cdot (3,960 - 1,080) = 6.28 * 2,880 = 18,086.4$$

$$\cong 18,086 \text{ miles}$$

2. Calling the circle's radius  $R$  and the square's side  $L$ , we use Pythagoras to see that  $L^2 + L^2 = (2 \cdot R)^2 = 4$ , and the square's area is simply  $L^2 = 2_{square \text{ inch}}$



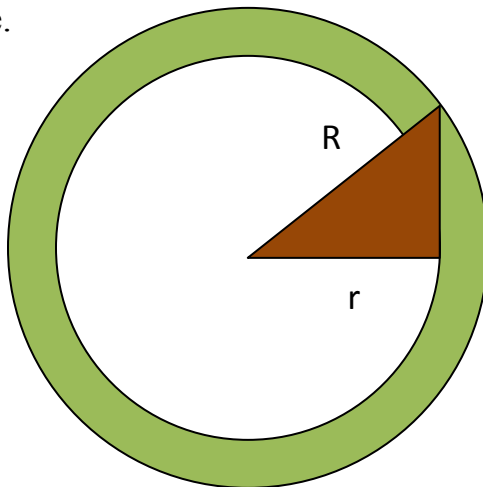
3. Let's call the radii of the large and small circles  $R$  and  $r$  respectively. The ring's area is the difference in the circles' area, so we know that:

$$\pi \cdot (R^2 - r^2) = 36\% \cdot \pi \cdot R^2 \text{ or in different words, } r^2 = 64\% \cdot R^2 = 64_{in \text{ sq}}$$

and so  $r = 8 \text{ inches}$ .

It's worth seeing the relationship to the Pythagorean theorem here:

The area of the ring is the same as the area of a circle whose radius is the other leg of the triangle.

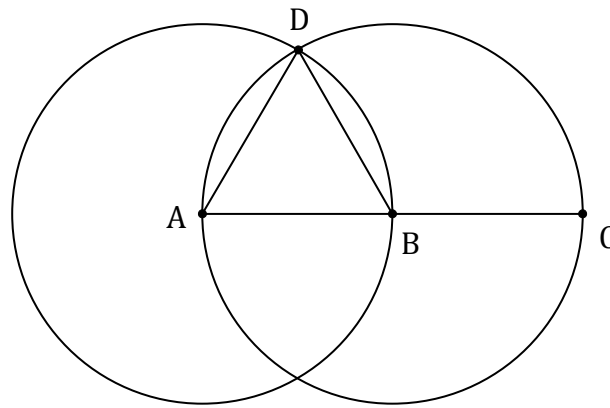


Category 2

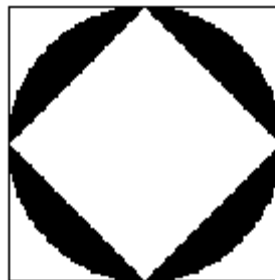
Geometry

Meet #4, February 2009

1. How many square centimeters are in the area of a circle which has a diameter of 12 cm? Express your answer to the nearest thousandth of a square centimeter. Use 3.141592 as an estimation for  $\pi$ .
2. In the figure below, two congruent circles pass through each other's center.  $\overline{AC}$  is the diameter of the circle with center B and A is the center of the other circle. Point D is one of the intersection points of the two circles. How many degrees are there in  $\angle DBC$ ?



3. In the diagram below, a circle is inscribed in a square with side lengths 14 cm. A smaller square is inscribed in the circle. The area inside the circle but outside the inner square is then shaded. What is the area of the white part of the diagram? Use 3.14159 for  $\pi$  and express your answer as a decimal to the nearest hundredth.



Answers	
1.	_____
2.	_____
3.	_____

Solutions to Category 2  
 Geometry  
 Meet #4, February 2009

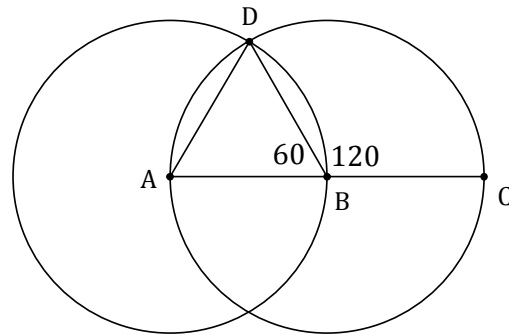
Answers

1. 113.097

1. The formula for the area of a circle is  $A = \pi r^2$ . If the diameter of the circle is 12 cm, then the radius is 6 cm. The area of the circle is then  $A = \pi 6^2 = 36\pi \approx 36(3.141592) \approx 113.0973 \approx 113.097$ .

2. 120

2. By drawing in the segment AD, we have an equilateral triangle ABD since all three sides are radii of two circles which are congruent. Since the triangle is equilateral,  $\angle ABD$  is  $60^\circ$ . We also know that  $\angle DBC$  is supplementary to  $\angle ABD$ , so  $\angle DBC = 180 - 60 = 120^\circ$



3. The area of the largest square is  $14^2 = 196 \text{ cm}^2$ . The diameter of the circle is 14, so its radius is 7 and its area is  $\pi 7^2 = 49\pi \approx 49(3.14159) \approx 153.937 \approx 153.94 \text{ cm}^2$ . The smaller square is exactly half of the larger square, as shown below, so its area is  $98 \text{ cm}^2$ . The area of the white region would then be  $196 - 153.94 + 98 = 140.06 \text{ cm}^2$ .

