Math League SCASD

Meet #3

Algebra

2018 - Self-study Packet

Problem Categories for this Meet (in addition to topics of earlier meets):

1. Mystery: Problem solving
2. Geometry: Properties of Polygons, Pythagorean Theorem
3. Number Theory: Bases, Scientific Notation
4. Arithmetic: Integral Powers (positive, negative, and zero), roots up to the sixth
5. **Algebra: Absolute Value, Inequalities in one variable including interpreting line graphs**

Important information you need to know regarding ALGEBRA

Absolute value; inequalities in one variable including interpreting line graphs

Absolute Value is the distance a number is from zero. Absolute value is never negative.

The symbol for absolute value is $|\ |$

$|-4| = 4$ and $|4| = 4$

Inequalities

- To solve an inequality, solve as if it were a regular equation. Remember to switch the direction of the inequality sign only if you multiply or divide by a negative!

Category 5
Algebra
Meet #3 - January, 2019

1) **Find the value of this absolute value expression:**

$$|7| + |-10| + |0|$$

2) **The graph below represents all values of X that make the accompanying absolute value inequality true. What is the value of A?**

$$|X - A| > 12$$

3) **For the inequality $-7 - 3(2N - 5) < 38$, the domain of N is {all negative integers}. In other words, only negative integers can be considered as replacements for N to make the inequality true. What is the sum of all possible values of N that make this inequality true?**

ANSWERS

1) _______

2) _______

3) _______

Solutions to Category 5
Algebra
Meet #3 - January, 2019

Answers
1) 17
2) 18
3) - 10

1) $|7| + |-10| + |0|$

$= 7 + 10 + 0$
$= 17$

2) $|X - A| > 12$

This absolute value inequality can be interpreted as follows: "The distance between a value of X and A is more than 12 units." A must be the midpoint of 6 and 30, namely, 18. So, the distance between 18 and 6 is 12 units and the distance between 18 and 30 is also 12 units. All values to the left of 6 or to the right of 30 are greater than 12 units from 18. Therefore, A is 18.

3) $-7 - 3(2N - 5) < 38$

$-7 - 6N + 15 < 38$	Distribute the 3
$8 - 6N < 38$	Combine like terms
$-6N < 30$	Subtracting 8 from both sides
$N > -5$	Dividing both sides by - 6 reverses the sense of the inequality (< becomes >).

Since the domain is {negative integers}, the solution includes the numbers - 4, - 3, -2, and - 1. The sum of these numbers is - 10.

Category 5
Algebra
Meet #3 - January, 2017

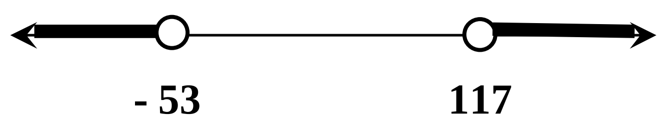
1) **What is the sum of the two different values of x that make this absolute value equation true?**

$$|2x + 6| = 28$$

2) **Solve this inequality:** $-4(3C - 5) < 92$

3) **The graph below represents all values of A that make the accompanying absolute value inequality true. What is the absolute value of the difference B - C ?**

$$|A - B| > C$$



ANSWERS

1) ________

2) ________

3) ________

Solutions to Category 5
Algebra
Meet #3 - January, 2017

Answers

1) - 6

2) C > - 6
or
- 6 < C

3) 53

1) Either $2x + 6 = 28$ or $2x + 6 = -28$
$2x = 22$ or $2x = -34$
$x = 11$ or $x = -17$
Then the sum is $11 + (-17) = -6$

2)

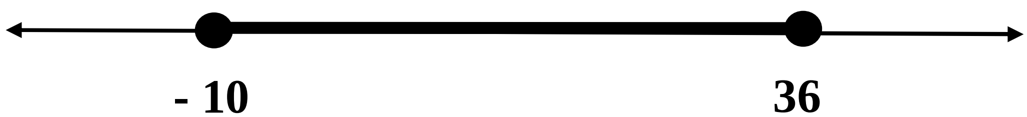
$-4(3C - 5) < 92$	**original inequality**
$-4(3C) - (-4)(5) < 92$	**Distribute the -4.**
$-12C + 20 < 92$	**having multiplied**
$-12C < 72$	**subtracting 20 from both members**
$C > -6$	**Dividing both members by - 12 reverses the sense of order.**

3) The midpoint of - 53 and 117 is (- 53 + 117) / 2, or 64 / 2, or 32. That midpoint is 85 units from 117 and is also 85 units from - 53. The absolute value inequality can be stated thusly: "The distance between any point, A, and 32, is greater than or equal to 85 units." In symbols, this inequality looks like this: $|A - 32| > 85$. So, B = 32 and C = 85. The absolute value of the difference B - C is 53 !

Category 5
Algebra
Meet #3 - January, 2015

1) The equation $|X-3|=5$ can be translated as, "The distance between X and 3 on the number line is exactly five units." What are the two values of X that make this equation true? You may list them in the answer box in any order.

2) The graph below is the set of all real values of W that make the following inequality true: $|W-A| \le C$



What are the values of A and C ?

3) Find the smallest integer value of N that makes the following inequality true:

$$2(3N+7) < 3(4N-5)-9$$

On this date in history - January 8 - The U. S. Mint at Carson City, Nevada, began issuing coins in 1870. Here, the Morgan silver dollar is shown, with the "CC" mint mark shown on the reverse of the coin.

ANSWERS

1) ___ and ___

2) A = ______

C = ______

3) ______

Solutions to Category 5
Arithmetic
Meet #3 - January, 2015

Answers

1) - 2 ; 8
(any order)

2) A = 13
C = 23

3) 7

1) Either $X - 3 = 5$ or $X - 3 = -5$. So, either $X = 8$ or $X = -2$.

2) Students may take a hint from the wording of #1 in order to translate this inequality as , "The distance between W and A is less than or equal to C." If the endpoints, - 10 and 36, are to be equidistant from A, then A is their midpoint, or 13. So, A = 13. The distance between A and either endpoint is 23 units. So, C = 23.

3)

$2(3N + 7) < 3(4N - 5) - 9$	original inequality
$6N + 14 < 12N - 15 - 9$	distribute
$6N - 12N < -14 - 15 - 9$	subtract 12N from both sides
$-6N < -38$	combine like terms
$N > 38/6$	divide both side by - 6 (which changes the sense of the inequality from $<$ to $>$)

The smallest integer value of N that is greater than 38/6 is 7.

Category 5
Algebra
Meet #3, January 2013

1. Solve the following inequality. Write your solution with the *x* on the left, the appropriate inequality sign in the middle, and a mixed number in lowest terms on the right.

$$7(5x + 19) - 2 > 40$$

2. For his birthday, Jarod received $54 from his Aunt Edna and three gift cards from his Uncle Joe. Although the gift cards are for different stores, each gift card has the same dollar value. If the absolute value of the difference between the money from Aunt Edna and the value of the three gift cards from Uncle Joe is $18, what is the absolute value of the difference between the two possible amounts that each gift card is worth?

3. How many integer values of *n* make the following inequality true?

$$2 < \left|\frac{20}{n}\right|$$

Answers

1. ____________________
2. $ __________________
3. ___________ integers

Solutions to Category 5
Algebra
Meet #3, January 2013

Answers
1. $x > -2\frac{3}{5}$
2. $12
3. 18 integers

1.

$$\begin{aligned} 7(5x+19)-2 &> 40 \\ 7(5x+19) &> 42 \\ 5x+19 &> 6 \\ 5x &> -13 \\ x &> \frac{-13}{5} \\ \boldsymbol{x} &\boldsymbol{> -2\frac{3}{5}} \end{aligned}$$

or

$$\begin{aligned} 7(5x+19)-2 &> 40 \\ 35x+133-2 &> 40 \\ 35x+131 &> 40 \\ 35x &> -91 \\ 5x &> -13 \\ \boldsymbol{x} &\boldsymbol{> -2\frac{3}{5}} \end{aligned}$$

2. We can capture this idea in the absolute value equation $|54-3x|=18$, where *x* is the unknown value of each gift card. To solve this, we consider two separate equations as shown above. The absolute value of the difference between $24 and $12 is $24 – $12 = **$12**.

$$\begin{aligned} 54-3x &= 18 \\ 54-18 &= 3x \\ 36 &= 3x \\ x &= 12 \end{aligned} \qquad \begin{aligned} 54-3x &= -18 \\ 54+18 &= 3x \\ 72 &= 3x \\ x &= 24 \end{aligned}$$

3. Since there is an absolute value sign in this inequality, we need to solve two separate inequalities as shown below.

$$\begin{aligned} 2 &< \frac{20}{n} \\ 2n &< 20 \\ n &< 10 \end{aligned} \qquad \textbf{\textit{and}} \qquad \begin{aligned} 2 &> -\frac{20}{n} \\ 2n &> -20 \\ n &> -10 \end{aligned}$$

We need to find values of *n* that satisfy both inequalities, so it helps to write the solution as the compound inequality $-10 < n < 10$. The **18 integers** in this range are *n* = -9, -8, -7, -6, -5, -4, -3, -2, -1, 1, 2, 3, 4, 5, 6, 7, 8, and 9. We have to skip *n* = 0, since we cannot divide by zero.

Category 5 – Algebra

1. How many integers satisfy the inequality below?

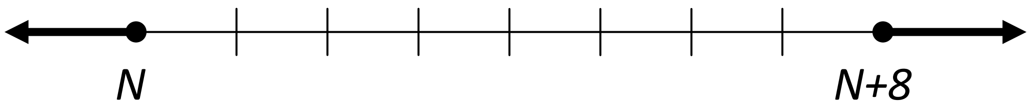
$$\left|\frac{N}{2}\right| - 12 < 4$$

2. The solution of the inequality $|x - M| \leq 2$ is given by: $3 \leq x \leq 7$.

   What is the value of the parameter M?

3. The solution to the inequality $|x + 1| \geq 4$ is given by the line graph below.

   What is the value of N?



Answers

1. ________________

2. ________________

3. ________________

Solutions to Category 5 - Algebra

Answers
1. 63
2. 5
3. -5

1. We can rewrite the inequality $\left|\frac{N}{2}\right| - 12 < 4$ as $\left|\frac{N}{2}\right| < 16$ or $|N| < 32$. This is true for the integers in the range $\{-31, -30, \dots., 0, \dots, 30, 31\}$ - a total of 63 integers.

2. When solving $|x - M| \leq 2$, if the argument inside the absolute value is positive we get $x \leq M + 2$, and if it is negative we get $x \geq M - 2$. Comparing this to the given solution $3 \leq x \leq 7$ we see that $M = 5$.

3. When solving $|x + 1| \geq 4$, if the argument is positive we get $x + 1 \geq 4\ or$ $x \geq 3$, and if it's negative we get $x + 1 \leq -4\ or\ x \leq -5$. Comparing these to the line graph we conclude that $N = -5$.

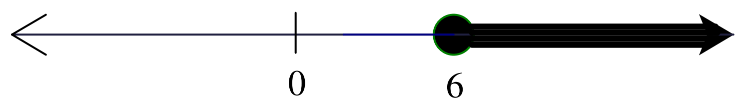
Category 5
Algebra
Meet #3, January 2009

1. What is the least possible solution for x in the inequality below?

$$|8 - 2x| \leq 16$$

2. For what value of K does the solution to the inequality below match the graph below?

$$x(K - 5) + 12 \leq K - 8$$



3. Mike chose 3 distinct numbers from the set below and took the absolute value of their sum. Sean chose 3 distinct numbers (not necessarily different from Mike's) from the set below and took the sum of their absolute values. What is the greatest possible absolute value of the difference between Mike's and Sean's result?

$$\{-12, -7, -2,\ 1,\ 3,\ 6,\ 11\}$$

Answers
1. ________________
2. ________________
3. ________________

Solutions to Category 5
Algebra
Meet #3, January 2009

Answers

1. -4

2. 2

3. 30

1. $|8-2x| \le 16$

$-16 \le 8-2x \le 16$

$-24 \le -2x \le 8$

$12 \ge x \ge -4$ So – 4 is the least integer solution for x.

2. $x(K-5)+12 \le K-8$

$x(K-5) \le K-20$

$x \le \frac{K-20}{K-5}$

Because we've divided by $(K-5)$ if $(K-5)$ is a negative number, we would have to reverse the $\le$ to $\ge$. According to the graph, the solution is $x \ge 6$ which means the inequality was reversed and $(K-5)$ must be negative. Therefore we actually have $x \ge \frac{K-20}{K-5}$ and since we know $x \ge 6$ the equation below finds K.

$$6 = \frac{K-20}{K-5}$$
$$6(K-5) = K-20$$
$$6K-30 = K-20$$
$$5K = 10$$
$$K = 2$$

3. To get the largest absolute value of a sum Mike wants to pick numbers that are all positive or all negative so that he is adding their values. The largest he could get would be $|(-12)+(-7)+(-2)| = |-21| = 21$. The smallest value Mike could get would be $|(-12)+1+11| = 0$ or $|(-7)+1+6| = 0$.

For Sean to get the largest sum of the absolute values he just needs the three numbers with the largest absolute values. In this case he wants $-12, 11$ and -7 for which the sum of their absolute values is $|-12|+|11|+|-7| = 12+11+7 = 30$. The smallest value he could get would be $|-2|+|1|+|3| = 6$.

The greatest possible difference between Mike's result and Sean's result would be by taking Sean's largest (30) and Mike's smallest (0). $30-0=30$.