Math League SCASD

Meet #3

Arithmetic

2018 - Self-study Packet

Problem Categories for this Meet (in addition to topics of earlier meets):

1. Mystery: Problem solving
2. Geometry: Properties of Polygons, Pythagorean Theorem
3. Number Theory: Bases, Scientific Notation
4. **Arithmetic: Integral Powers (positive, negative, and zero), roots up to the sixth**
5. Algebra: Absolute Value, Inequalities in one variable including interpreting line graphs

Important information you need to know about ARITHMETIC
Integral Powers (Positive, Negative, and Zero), Roots up to the Sixth

- Anything to the power of zero is 1.
- A number to a negative power is the same thing as the reciprocal of that number to the opposite positive power.
- When multiplying two powers with the same base, keep the base and add the exponents.
- When dividing two powers with the same base, keep the base and subtract the exponents.

Laws of Exponents

$a^b \times a^c = a^{b+c}$ $\quad (ab)^c = a^c b^c$ $\quad (a^b)^c = a^{bc}$

$\frac{a^b}{a^c} = a^{b-c}$ $\quad \left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$ $\quad a^{-n} = \left(\frac{1}{a}\right)^n$

Common Powers worth Memorizing

(Focus on learning the squares and cubes and the higher powers of the numbers less than six)

x	x^2	x^3	x^4	x^5	x^6
1	**1**	**1**	**1**	**1**	**1**
2	**4**	**8**	**16**	**32**	**64**
3	**9**	**27**	**81**	**243**	**729**
4	**16**	**64**	**256**	**1024**	**4096**
5	**25**	**125**	**625**	**3125**	**15,625**
6	**36**	**216**	**1296**	**7776**	**46,656**
7	**49**	**343**	**2401**	**16,807**	**117,649**
8	**64**	**512**	**4096**	**32,768**	**262,144**
9	**81**	**729**	**6561**	**59,049**	**531,441**
10	**100**	**1000**	**10,000**	**100,000**	**1,000,000**

Squares of higher numbers:

11^2=121, 12^2=144, 13^2=169, 14^2=196, 15^2=225, 16^2=256, 17^2=289, 18^2=324, 19^2=361, 20^2=400

Category 4
Arithmetic
Meet #3 - January, 2019

1) If $3^N = 729$ **, then what is the value of N?**

2) Evaluate: $\sqrt{36} + 4\sqrt[3]{125} - 3\sqrt[4]{16} + 7\sqrt[5]{100{,}000}$

3) What is the value of

$$\sqrt[3]{8\sqrt{121} + 2\sqrt{\sqrt[3]{1331} + \sqrt[6]{512}} + \sqrt{625}}\ ?$$

ANSWERS
1) ________
2) ________
3) ________

Solutions to Category 4
Arithmetic
Meet #3 - January, 2019

Answers	
1)	6
2)	90
3)	5

1) 3 x 3 x 3 x 3 x 3 x 3 = 729, so N = 6.

2) $\sqrt{36} + 4\sqrt[3]{125} - 3\sqrt[4]{16} + 7\sqrt[5]{100{,}000}$

$= 6 + 4(5) - 3(2) + 7(10)$
$= 6 + 20 - 6 + 70$
$= 90$

3) $\sqrt[3]{8\sqrt{121} + 2\sqrt{\sqrt[3]{1331} + \sqrt[6]{5^{12}}} + \sqrt{625}}$

$= \sqrt[3]{8(11) + 2\sqrt{11 + 25} + 25}$

$= \sqrt[3]{88 + 2\sqrt{36} + 25}$

$= \sqrt[3]{88 + 2(6) + 25}$

$= \sqrt[3]{88 + 12 + 25}$

$= \sqrt[3]{125}$

$= 5$

Category 4
Arithmetic
Meet #3 - January, 2017

1) **Evaluate:** $7^0 + 6^1 + 5^2 + 4^3 + 3^4 + 2^5 + 1^6 + 0^7$

2) **If** $72 + \sqrt{N - 3} = 78$, **then what is the value of** $N^2 - 17$ **?**

3) **What is the value of** $\left(\sqrt{\left(\sqrt[3]{2^6}\right)} \cdot \sqrt{3^4} \cdot \sqrt[6]{1^4}\right)^3$ **?**

ANSWERS
1) ________
2) ________
3) ________

Solutions to Category 4
Arithmetic
Meet #3 - January, 2017

Answers
1) 210
2) 1504
3) 5832

1) $7^0 + 6^1 + 5^2 + 4^3 + 3^4 + 2^5 + 1^6 + 0^7$
= 1 + 6 + 25 + 64 + 81 + 32 + 1 + 0
= 210

2) $72 + \sqrt{N - 3} = 78$
$\sqrt{N - 3} = 6$
$N - 3 = 36$
$N = 39$
So, $N^2 - 17 = (39)^2 - 17 = 1521 - 17 = 1504.$

3) $\left(\sqrt{\left(\sqrt[3]{2^6}\right)} \cdot \sqrt{3^4} \cdot \sqrt[6]{1^4}\right)^3$

= $\left(\sqrt{2^{6/3}} \times 9 \times 1\right)^3$

= $\left(\sqrt{2^2} \times 9\right)^3$

= $(2x9)^3$

= $(18)^3$

= **5832**

Category 4
Arithmetic
Meet #3 - January, 2015

1) Find the value of $8^0 + 7^1 + 6^2 + 5^3 + 4^4$

2) Evaluate $\left(\left(\sqrt[3]{\sqrt{64}}\right)^{-1}\right)^{-4}$

3) $A^{\frac{B}{C}}$ **means** $\sqrt[C]{A^B}$ **or, equivalently,** . $\left(\sqrt[C]{A}\right)^B$

Find the value of $10{,}000^{\frac{3}{4}} - 64^{\frac{2}{3}} \cdot 32^{\frac{3}{5}}$

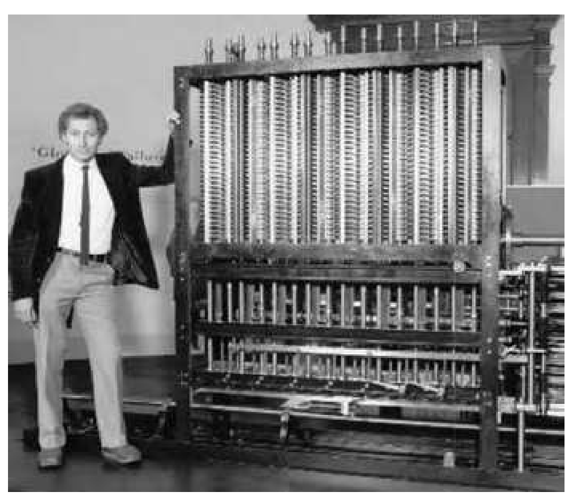
On this date, January 8, Charles Babbage patented the first computer in the year 1889.

ANSWERS

1) _______

2) _______

3) _______



Solutions to Category 4
Arithmetic
Meet #3 - January, 2015

Answers

1) 425

2) 16

3) 872

1) $8^0 + 7^1 + 6^2 + 5^3 + 4^4 = 1 + 7 + 36 + 125 + 256 = 425$

2) $\left(\left(\sqrt[3]{\sqrt{64}}\right)^{-1}\right)^{-4} = \left(\left(\sqrt[3]{8}\right)^{-1}\right)^{-4} = \left((2)^{-1}\right)^{-4} = \left(\frac{1}{2}\right)^{-4} = 2^4 = 16$

3) $10{,}000^{\frac{3}{4}} - 64^{\frac{2}{3}} \cdot 32^{\frac{3}{5}} = \left(\sqrt[4]{10{,}000}\right)^3 - \left(\sqrt[3]{64}\right)^2 \cdot \left(\sqrt[5]{32}\right)^3$

$= 10^3 - 4^2 \times 2^3$

$= 1000 - 16 \times 8$

$= 1000 - 128$ **(Multiply before subtracting!)**

$= 872$

Category 4
Arithmetic
Meet #3, January 2013

1. Evaluate the following expression. You should get a whole number.

$$\sqrt{3^2+3^3}+\sqrt{8^2+8^3}$$

2. Evaluate the expression below for a = 18 and b = 24. Express your answer as a mixed number in lowest terms.

$$\left(\frac{a^3}{b^2}\right)^0+\left(\frac{a^{-3}}{b^{-2}}\right)^{-1}$$

3. How many whole numbers are there between $\sqrt[4]{1000}$ and $\sqrt{1000}$?

Answers

1. _______________

2. _______________

3. ______ whole numbers

Solutions to Category 4
Arithmetic
Meet #3, January 2013

Answers

1. 30

2. $11\frac{1}{8}$

3. 26 whole numbers

1.
$$\sqrt{3^2+3^3}+\sqrt{8^2+8^3}=$$
$$\sqrt{9+27}+\sqrt{64+512}=$$
$$\sqrt{36}+\sqrt{576}=6+24=30$$

2. Notice that the first term in this expression is raised to the zero power, so it's value is 1. As for the second term, we substitute 18 for *a* and 24 for *b*, and evaluate as follows:

$$\left(\frac{a^{-3}}{b^{-2}}\right)^{-1}=\left(\frac{18^{-3}}{24^{-2}}\right)^{-1}=\frac{24^{-2}}{18^{-3}}=\frac{18^3}{24^2}=\frac{18\times18\times18}{24\times24}=\frac{3\times3\times9}{4\times2}=\frac{81}{8}=10\frac{1}{8}$$

Putting the two terms together, we get $\left(\frac{a^3}{b^2}\right)^0+\left(\frac{a^{-3}}{b^{-2}}\right)^{-1}=1+10\frac{1}{8}=\mathbf{11\frac{1}{8}}$.

3. We need to find a perfect fourth power that is slightly less than 1000 and a perfect square that is slightly greater than 1000. The two short lists below can help. From the list on the left, we see that $\sqrt[4]{1000}$ must be greater than 5, but less than 6. From the list on the right, we see that $\sqrt{1000}$ must be greater than 31, but less than 32. There are 31 whole numbers less than 32, but we need to exclude 5 of these, so there are 31 – 5 = **26 whole numbers** between 5 and 32.

$4^4=256$	$30^2=900$
$5^4=625$	$31^2=961$
$6^4=1296$	$32^2=1024$

Category 4 – Arithmetic

1. Evaluate the following expression:

$$(3^3 - 2^5)^2 + \sqrt[3]{2^6} + \sqrt{9 \cdot 4^2}$$

2. N is a natural number that makes the equation below true. Find the value of N.

$$N^2 \cdot \left(\frac{3}{5}\right)^2 = \left(\frac{5^{-1} \cdot \sqrt{144}}{\sqrt[3]{8}}\right)^2 \cdot \sqrt{81}$$

3. M is a natural number greater than 1 such that the difference between the 4th and 3rd powers of M is 4 times the difference between the 3rd and 2nd powers. What is the value of M?

Answers

1. ________________

2. ________________

3. ________________

Solutions to Category 4 - Arithmetic

Answers
1. 41
2. 6
3. 4

1. $(3^3 - 2^5)^2 + \sqrt[3]{2^6} + \sqrt{9 \cdot 4^2} =$

$(27 - 32)^2 + \sqrt[3]{64} + \sqrt{3^2 \cdot 4^2} =$

$(-5)^2 + 4 + 3 \cdot 4 = 25 + 4 + 12 = 41$

2. $N^2 \cdot \left(\frac{3}{5}\right)^2 = \left(\frac{5^{-1} \cdot \sqrt{144}}{\sqrt[3]{8}}\right)^2 \cdot \sqrt{81}$

$N^2 \cdot \frac{9}{25} = \left(\frac{12}{5 \cdot 2}\right)^2 \cdot 9$

$N^2 \cdot \frac{9}{25} = \left(\frac{6}{5}\right)^2 \cdot 9 = 6^2 \cdot \frac{9}{25}$

$N^2 = 6^2 \; so\ the\ natural\ solution\ is\ N = 6$

3. We know that $M^4 - M^3 = 4 \cdot (M^3 - M^2)$. Taking the common factors out in each side of the equation we get:

$M^3 \cdot (M - 1) = 4 \cdot M^2 \cdot (M - 1)$ so we can divide both sides by $M^2 \cdot (M - 1)$ to get $M = 4$.

Editor note: The original question did not have "greater than 1" and so M=1 was a debated but accepted answer since 0 = 4 times 0. Also accepted was 1,4.

Category 4
Arithmetic
Meet #3, January 2009

1. What is the largest possible integer value of a if $3^a < 2^{13}$?

2. What is the value of the expression below? Express your answer as a common fraction.

$$\left(3\frac{1}{8}\right)^5 \times \left(1\frac{1}{4}\right)^{-7}$$

3. There are exactly 38 whole numbers that are greater than $\sqrt[3]{400}$ and less than $\sqrt{n}$. If n is a whole number, what is the largest possible value of n?

Answers
1. _______________
2. _______________
3. _______________

Solutions to Category 4
Arithmetic
Meet #3, January 2009

Answers

1. 8

2. $\frac{125}{2}$

3. 2116

1. $2^{13} = 8192$. The largest power of 3 less than that is $3^8 = 6561$, so a = 8.

2. $\left(3\frac{1}{8}\right)^5 \times \left(1\frac{1}{4}\right)^{-7} = \left(\frac{25}{8}\right)^5 \times \left(\frac{5}{4}\right)^{-7} = \left(\frac{25}{8}\right)^5 \times \left(\frac{4}{5}\right)^7 =$

$$\frac{25^5 \times 4^7}{8^5 \times 5^7} = \frac{5^{10} \times 2^{14}}{2^{15} \times 5^7} = \frac{5^3}{2} = \frac{125}{2}$$

3. $7^3 = 343$ and $8^3 = 512$, so $\sqrt[3]{400}$ is between 7 and 8. The first whole number greater than $\sqrt[3]{400}$ then is 8. The 38^{th} whole number greater than $\sqrt[3]{400}$ is 45. Therefore $\sqrt{n}$ must be larger than 45, but not larger than 46 or there would be 39 numbers between $\sqrt[3]{400}$ and $\sqrt{n}$. $45^2 = 2025$ while $46^2 = 2116$. So the largest whole number value of n would be 2116 as 45 is less than $\sqrt{2116}$ while 46 is not.