Math League SCASD

Meet #2 Arithmetic

Self-study Packet

Problem Categories for this Meet:

- 1. Mystery: Problem solving
- 2. Geometry: Angle measures in plane figures including supplements and complements
- 3. Number Theory: Divisibility rules, factors, primes, composites
- 4. Arithmetic: Order of operations; mean, median, mode; rounding; statistics
- 5. Algebra: Simplifying and evaluating expressions; solving equations with 1 unknown including identities

Meet #2 – Arithmetic

Ideas you should know:

Multiplying fractions:

$$\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}$$
$$\frac{2}{3} \times \frac{3}{7} = \frac{2}{3} \times \frac{3}{7} = \frac{2}{7}$$
 Cancel before x for speed



$$A \div \frac{B}{C} = A \times \frac{C}{B}$$

Writing fractions of fractions

Reciprocal: Multiplicative Inverse.

Reciprocal of 3 = 1/3. Reciprocal of 2/7 = 7/2. Divide by Y = Multiply by reciprocal of Y

Adding Fractions – common denominator

$$\frac{\frac{1}{2} + \frac{1}{6} = \frac{3}{3} \cdot \frac{1}{2} + \frac{1}{6} = \frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}}{\frac{1}{7} + \frac{1}{5} = \frac{5}{5} \cdot \frac{1}{7} + \frac{7}{7} \cdot \frac{1}{5} = \frac{5}{35} + \frac{7}{35} = \frac{12}{35}}$$

What do you mean by ***Of**"? $\frac{2}{3} \text{ of } 24 = \frac{2}{3} \times 24 = \frac{2}{3} \times \frac{24}{1} = \frac{48}{3} = 16$ $\frac{24}{32} = \frac{3 \times 8}{4 \times 8} = \frac{3}{4} \textcircled{O}$ Repeating decimal equivalent: $\frac{1}{3} = 0.333333333... = 0.\overline{3}$ $\frac{1}{9} = 0.111111... = 0.\overline{1} \qquad \frac{1}{99} = 0.01010101... = 0.\overline{01} \qquad \frac{1}{999} = 0.\overline{001}$ $\frac{2}{9} = 0.222222... = 0.\overline{2} \qquad \frac{2}{99} = 0.02020202.... = 0.\overline{02} \qquad \frac{2}{999} = 0.\overline{002}$ $103/999 = ? \qquad 0.\overline{103} \qquad 0.17171717... = ? \qquad 0.\overline{17}$ $0.51111... = 0.5\overline{1} = ? \qquad 1/9 = 0.11111... \qquad 1/90 = 0.011111... \qquad 0.51111.... = 1/2 + 1/90 = 45/90 + 1/90 = 46/90 = 23/45$

 $\underline{\text{Improper Fraction}} \frac{10}{3} \quad \frac{3}{2} \qquad \underline{\text{Mixed Numeral}} \quad 2\frac{1}{2}$

<u>= Mike is 50% taller than Bob</u>: This means he's 1.5 times as tall, not $\frac{1}{2}$ as tall!



- "I ate 50% as much as you" = half as much.
- "I ate 50% more than you" = 1.5 times as much
- "I ate 100% as much as you" = same
- "I ate 100% more than you" = twice as much
- "I ate 200% more than you" = 3 times as much
- "I ate 50% of you" = well, nevermind.

<u>= The price is 1/3 higher</u>: The price is 1+1/3 as high. If the original price was \$30, then 1/3 higher means it's \$40.

"What fraction is this repeating decimal?"

Another way to figure it out:

$$X = 0.12$$
 times
 100:

 $100X = 12.12$
 subtract X:

 $99X = 12$
 $\therefore X = 12/99 = 4/33$

If digits before the repeating pattern:

$$X = 0.1\overline{23}$$

$$10X = 1.\overline{23}$$

$$1000X = 123.\overline{23} \quad subtract:$$

$$990X = 122 \quad so, X = \frac{122}{990} = \frac{61}{495}$$

"15th digit in the decimal expansion of" problems

What is the 15^{th} digit of the decimal expansion of 1/7? $1/7 = 0.\overline{142857}$ You could just write it out and count digits. Another way is to say digit 3 is 2, and every 6^{th} digit after that is also a 2, and 15=3+6x2, so it's also 2.

What is the 601^{st} digit of the decimal expansion of $2/7 = 0.\overline{285714}$? Answer: It's 600 digits past the 1st, so it's the same as the 1st, or 2.

"What is 2/3 of 25% of 3/7 of 4/9 of 81" problems

These are simply multiplication – with a lot of cancellation usually.

 $\frac{2}{3} \cdot \frac{25}{100} \cdot \frac{3}{7} \cdot \frac{4}{9} \cdot 81$ Rewrite 25/100 as 1/4, cancel 3's and 4's: $\frac{2}{3} \cdot \frac{1}{4} \cdot \frac{3}{7} \cdot \frac{4}{9} \cdot 81$ or $\frac{2}{1} \cdot \frac{1}{1} \cdot \frac{1}{7} \cdot \frac{1}{9} \cdot 81$ and also cancel 9s from 1/9 and 81, and so we get 2x9/7 or 18/7 or 2 4/7.

Adding or subtracting repeating decimals

If you have 0.33333... plus 0.11111... you get 0.44444... which makes sense if you look at them as fractions: 3/9 + 1/9 = 4/9. It's tricky if the two repeating patterns have a different length:

From the 1999 meet: What is $0.\overline{51}+0.\overline{2}$? Answer: Write 0.2... as 0.22... and then it's 51/99+22/99 = 73/99 or 0.737373...

Dividing repeating decimals

This seems harder, but you can often do it in your head using fractions:

What is $0.\overline{21} \div 0.\overline{60}$? Answer: $\frac{21}{99} \div \frac{60}{99} = \frac{21}{99} \times \frac{99}{60} = \frac{21}{60} = \frac{7 \cdot 3}{20 \cdot 3} = \frac{7}{20}$

Category 4 Arithmetic Meet #2 - December, 2018



1) When 48% is written as a fraction and then reduced to lowest terms, what is the value of the new denominator?

2) For how many whole number values of J is $\frac{7}{J}$ greater than $\frac{1}{4}$ but less than $\frac{1}{3}$?

3) C \bigcirc D is defined to be equal to $\frac{1}{C} - \frac{1}{D} = \frac{1}{D}$. Find the value of 8 \bigcirc 12.

Express your answer as a fraction in lowest terms.



Solutions to Category 4 Arithmetic Meet #2 - December, 2018

- 1) 48% = 48/100 = 24/50 = 12/25. Denominator is 25.
- 2) 1/4 = 7/28. Also, 1/3 = 7/21. With 7 as a numerator, the fractions whose values lie between 1/4 and 1/3 have a denominator of 22, 23, 24, 25, 26, or 27. There are six such values for the new denominator.

3) 8 (1) 12 =
$$\frac{\frac{1}{8} - \frac{1}{12}}{\frac{1}{8} + \frac{1}{12}} = \frac{\frac{3}{24} - \frac{2}{24}}{\frac{3}{24} + \frac{2}{24}} = \frac{\frac{1}{24}}{\frac{5}{24}} = \frac{1}{5}$$

Ans	swers
1)	25
2)	6
3)	$\frac{1}{5}$

Category 4 Arithmetic Meet #2 - December, 2016

- 1) If $\frac{N+3}{28} = \frac{5}{7}$ then what is the value of N?
- 2) What whole number is 60% of $\frac{5}{9}$ of 0.6 of 780 ?

3) If the repeating decimal 3.81111... were increased by a value of T, the total would be the mixed numeral $4\frac{2}{3}$. Express T as a common fraction.



Solutions to Category 4 Arithmetic Meet #2 - December, 2016

1)	$\frac{N+3}{28} = \frac{5}{7}$ Scaling 5/7 by a factor of 4/4 yields
	4(5) = N + 3
	$20 = \mathbf{N} + 3$
	17 = N
2)	60% of $\frac{5}{9}$ of 0.6 of 780
	$= 0.6 \times \frac{5}{9} \times 0.6 \times 780$
	$=\frac{6}{10} \times \frac{5}{9} \times \frac{6}{10} \times 780$

Answers		
1)	17	
2)	156	
3)	77 90	

After doing some "cancelling" to reduce the product to lowest terms, the final answer is 156.

3) 3.811111... + T = $4\frac{2}{3}$ T = $4\frac{2}{3}$ - 3.81111... = 4.6666666 ... - 3.811111... = 0.855555 ...

If X = 0.855555... and 10X = 8.55555..., then the difference is

9X = 7.7, and X = 7.7 / 9 or 77 / 90.

Category 4 Arithmetic Meet #2 - November, 2014

- Last year, the number of World War II veterans marching in parades in Massachusetts was about 720, while the number in Pennsylvania was 640. In anticipation of the 70th anniversary of the end of World War II, those numbers are expected to increase by 20% next year. How many more Massachusetts veterans than Pennsylvania veterans are expected to march in next year's parades? (Note: The image at the bottom of this page is the World War II monument in Washington, D.C. which was first open to the public in 2004.)
- 2) Connor paid \$ 28.14 for a science kit, including a 5% sales tax. How many dollars was the tax? Express your answer as a decimal.
- 3) Evan spent 2/9 of his year's allowance on gifts for his family and friends and 1/5 on items for himself. He put the rest into his savings account at the bank. If he saved \$468, then how many dollars was his yearly allowance?



Solutions to Category 4 Arithmetic	
Meet #2 - November, 2014	Answers
1) Calculate as follows: (120% of 720) - (120% of 640) = (1.2 x 720) - (1.2 x 640) = 864 - 768 = 96	 1) 96 2) 1.34 3) 810

- 2) 105% of the cost of the science kit is \$28.14.
 So, 28.14 divided by 1.05 gives the cost of the science kit = \$26.80.
 The tax is 5% of the cost of the kit = 0.05 x 26.80 = \$1.34.
- 3) The fraction of Evan's allowance that is set aside for savings is 1 (2/9 + 1/5)
 - = 1 (10/45 + 9/45)
 - = 1 (19/45)
 - = 26/45
 - \$468 is 26/45 of his yearly allowance, so his yearly allowance is 468 divided by 26/45
 - $= 468 \times 45/26$
 - **= 810.**

Category 4 Arithmetic Meet #2, November/December 2012

1. What number is 20% less than $9\frac{2}{7}$? Express your answer as a mixed number in lowest terms.

2. Simplify the expression below to a common fraction.

$$\frac{\frac{3}{11} + 0.\overline{24}}{\frac{4}{9} + 0.\overline{41}}$$

3. What is the 202nd digit to the right of the decimal point in the decimal equivalent of $\frac{25}{202}$?



2. We can simplify the expression as follows:

$3 + 0\overline{24}$	3 24	27 + 24	51		
-+0.24 11		99	99	_ 51 _	3
$4 - \frac{-}{41}$	4 41	44+41	85	85	5
$\frac{-}{9}$ + 0.41	$\frac{-}{9}$ $\frac{-}{99}$	99	99		

3. We have to do long division to find the repeating decimal pattern for 25/202, as shown at right. The first digit to the right of the decimal point is not part of the repeating pattern. Then we get a repeating pattern of four digits. We know it repeats because we get a repeat remainder of 48. The 202^{nd} digit to the right of the decimal point is the 201^{st} digit in the pattern. Since 201 is 1 more than a multiple of 4, we get the first digit in the pattern, which is **2**.

 $\begin{array}{r}
 0.1\overline{2376} \\
 202) 25.00000 \\
 -202 \\
 480 \\
 -404 \\
 760 \\
 -606 \\
 1540 \\
 -1414 \\
 1260 \\
 -1212 \\
 48$

<u>Category 4 – Arithmetic</u>

1. Express the decimal 0.425 as a common fraction. [A fraction of the form $\frac{m}{n}$ which cannot be simplified].

2. Express the fraction $\frac{7}{48}$ as a decimal.

Use bar notation to note repeating digits.

3. Tim put all his savings in the Miracle bank.
After one year, his account's balance grew by 20%.
After the second year, his balance grew by an additional 25%, and was now \$60 more than his original deposit.
How much money does Tim have now in his account?



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So	lutions to Category 4 - Arithmetic		
<u>50</u>	Intions to Eulegory 4 Partitinetic		Answers
1.	$0.425 = \frac{425}{1000} = \frac{85}{200} = \frac{17}{40}$	1	17
	1000 200 40	1.	40
2.	0.14583	2	0.14583
	7 48	2.	0111000
	<u>0</u>	3. \$	180 (or 180)
	70		
	<u>48</u>		
	220		
	<u>192</u>		
	280		
	240		
	400		
	384		
	160		
	144		

160 ...

3. If we call his original deposit's amount D, then we can

write the information as follows:

 $D + 20\% \cdot D + 25\% \cdot (D + 20\% \cdot D) = D + \60

Replacing percents with numbers and aggregating:

 $D \cdot (1 + 0.2 + 0.25 * 1.2) = D +$ \$60 which we can aggregate further into:

1.5 * D = D + \$60 or D = \$120.

The balance now is 1.5 * D = \$180

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Solutions to Category 4 - Arithmetic	Answers
1. $0.425 = \frac{425}{1000} = \frac{85}{200} = \frac{17}{40}$	1. $\frac{17}{40}$
2. 0.14583	$2 0.1458\overline{3}$
7 48	2. 0.11505
<u>0</u>	3. \$180 (or 180)
70	
<u>48</u>	
220	
<u>192</u>	
280	
240	
400	
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Category 4 Arithmetic Meet #2, December 2008

- 1. What is the fraction that is equivalent to the decimal .3125? Express your answer as a common fraction. (A "common fraction" is written in the form $\frac{M}{N}$ where $\frac{M}{N}$ cannot be simplified. A "mixed number" is not a common fraction.)
- 2. How much larger is $37\frac{1}{2}\%$ of 96 than $\frac{5}{12}$ of 84?

3. What is the 80th digit to the right of the decimal point in the repeating decimal equivalent of $\frac{3}{70}$?



Solutions to Category 4 Arithmetic Meet #2, December 2008

Ansv	wers	1.	$.3125 = \frac{3125}{10000} = \frac{625}{2000} = \frac{125}{400} = \frac{25}{80} = \frac{5}{16}$		
1.	5 16				
2.	1	2	$37\frac{1}{2}\%$ of $96-\frac{3}{2} \times 96 = 36$		
3.	4	2.	57_{2}^{-} , 50_{8}^{-} , 50_{8}^{-} , 50_{8}^{-}		
			$\frac{3}{12}$ of $84 = \frac{3}{12} \times 84 = 35$		
			36 is 1 larger than 35		

3. $\frac{3}{70} = 0.0\overline{428571}$. The repeating part of the decimal has 6 digits, but due to the 0 at the beginning, each set of 6 repeating digits would end at the 7th, 13th, 19th,67th, 73rd, and 79th place after the decimal. So the 79th place after the decimal is 1 and the 80th place after the decimal is 4.