Math League SCASD

Meet #2

Geometry

Self-study Packet

Problem Categories for this Meet (in addition to topics of earlier meets):

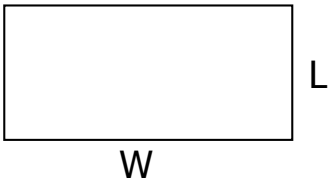
1. Mystery: Problem solving
2. **Geometry: Area and perimeter of polygons**
3. Number Theory: Divisibility GCF, LCM, prime factorization
4. Arithmetic: Fractions, terminating and repeating decimals, percents
5. Algebra: Word problems with 1 unknown; working with formulas; reasoning in number sentences

Important Information you need to know for Meet 2, Category 2…

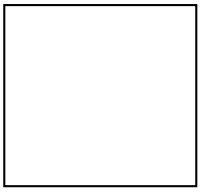
GEOMETRY: Area and Perimeter of Polygons

Shape	**Perimeter**	**Area**
Rectangle	2L + 2W	LW
Square	4s	s^2
Triangle	A + B + C	½ Bh
Parallelogram	2A + 2B	Bh
Trapezoid	A + C + B + b	½h(B + b)

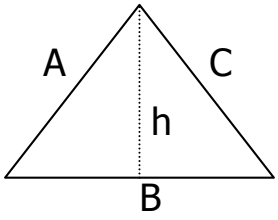
Rectangle



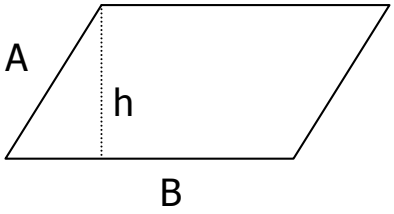
Square



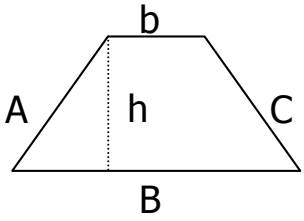
Triangle



Parallelogram



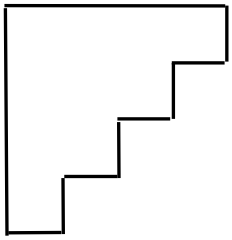
Trapezoid



To find the area of a more complex polygon, break the area into smaller parts and find the area of each part. Then add the areas together. If you memorize the formula for area of a rectangle and a triangle, you can find the area of virtually any polygon!

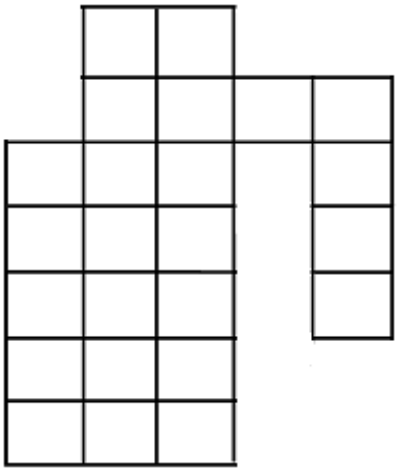
Category 2
Geometry
Meet #2 - December, 2018

1) In this polygon, all angles are right angles. Each of the short segments is three inches long. How many inches are in the perimeter of the polygon?



2) The rectangle to the right is 24 centimeters (cm) long and 16 centimeters wide. How many square centimeters are in the total area of the black triangles?

3) The diagram below consists of identical squares "glued" together at the edges. The total area of the resulting polygon is 216 square feet. How many feet are in its perimeter?



Answers

1) _____ inches

2) _____ sq cm

3) _____ feet

Solutions to Category 2
Geometry
Meet #2 - December, 2018

Answers
1) 48
2) 192
3) 90

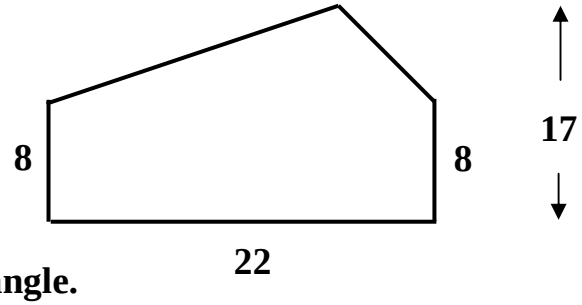
1) The sum of the 3" vertical lengths is (4)(3), or 12". The vertical side opposite those 3" lengths is also 12" as is the total lengths of the four horizontal 3" lengths and the long horizontal side opposite those 3" lengths. So, the perimeter is (4)(12), or 48".

2) The bases of the black triangles have a total base length that is equal to the base of the rectangle, or 24 cm. Their combined areas are the same as that of one triangle with a base of 24 cm and an altitude equal to the width of the rectangle, or 16 cm. Therefore, the total area of the black triangles is (1/2)(base)(altitude) = (1/2)(24)(16) = 192 square cm.

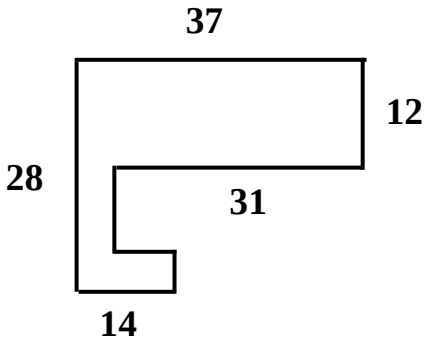
3) There are 24 of the small squares. The area of one such square is equal to the total area of the squares divided by the number of squares, or 216 / 24, or 9 square feet. Each side of one square is the square root of 9, or 3 feet. The perimeter of the polygon consists of 30 of these 3' segments and is equal to (30)(3), or 90 feet.

Category 2
Geometry
Meet #2 - December, 2016

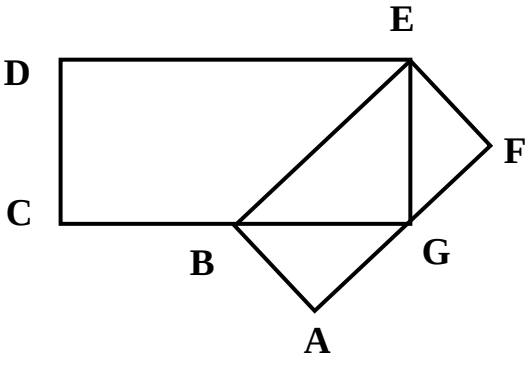
1) How many square inches are in the area of this pentagon? All measurements are in inches. Each of the base angles is a right angle.



2) How many centimeters are in the perimeter of this polygon? All angles are right angles and all segment lengths are in centimeters.



3) Rectangles CGED and ABEF intersect at points B, G, and E. B is the midpoint of CG. How many square inches are in the area of quadrilateral BCDE ? The area of triangle ABG is 24 square feet and the area of triangle GFE is 38 square feet.



Answers

1) _____

2) _____

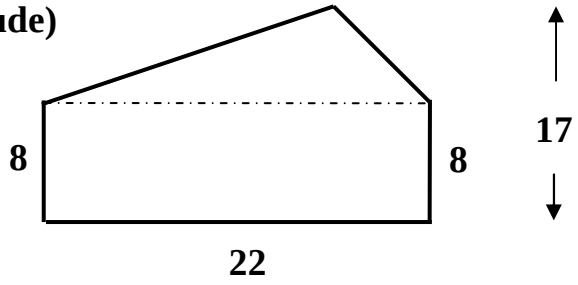
3) _____

Solutions to Category 2
Geometry
Meet #2 - December, 2016

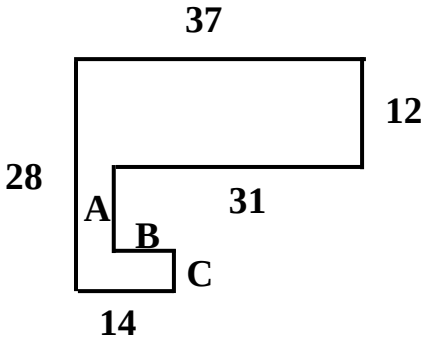
Answers	
1)	275
2)	146
3)	26,784

1) Divide the pentagon into a triangle and a rectangle with a horizontal line parallel to the base, making a 22 x 8 rectangle and a triangle whose base is 22 and whose altitude is (17 - 8), or 9. The total area is

rectangle + triangle
= (base)(height) + 1/2 (base)(altitude)
= (22)(8) + 1/2 (22)(9)
= 176 + 99
= 275 square inches.



2) There are three unlabelled segments. A + C + 12 is the same length as the segment marked 28. The width of the skinny column on the left = 37 - 31, or 6. 14 - 6 = B = 8. So, the perimeter = (14 + 28 + 37 + 31) + B + (A + C + 12) = 110 + 8 + (28) = 146 centimeters.

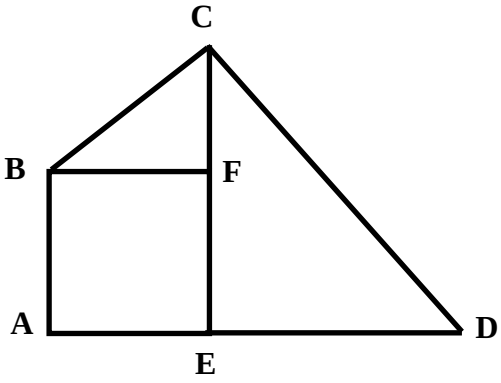


3) Triangle BGE is half of rectangle ABEF. The sum of triangles ABG and GFE is also half of rectangle ABEF. So, the area of triangle BGE = 24 + 38 = 62 square feet. With B as the midpoint of CG, triangle BGE is 1/4 of rectangle CGED. So, quadrilateral BCDE is the other 3/4 of rectangle CGED . . . or, another way, rectangle CGED is 3 (area of triangle BGE) = 3(62) = 186 square feet.
The question asks, "How many square INCHES are in the area of quadrilateral BCDE," so we must convert 186 square feet to square inches: 186 square feet x 144 square inches per square foot = 26,784 square inches.

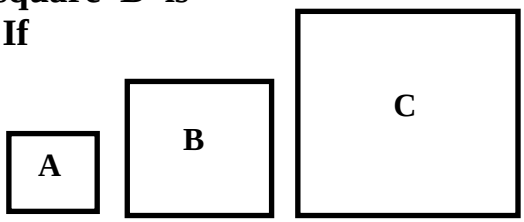
Category 2
Geometry
Meet #2 - November, 2014

Figures are not necessarily drawn to scale.

1) The area of square ABFE is 25 square feet. The area of triangle BFC is 10 square feet. The measure of angle D in triangle CDE is 45 degrees. How many feet are in the length of segment AD ?



2) The perimeter of square A is 2/3 of the perimeter of square B, and the perimeter of square B is 2/3 of the perimeter of square C. If one side of square A is 4 centimeters, then how many square centimeters are in the area of square C ?



3) Triangle XYZ has a perimeter of 10 inches. The lengths of its sides are all whole numbers. If n is the length of side YZ, then what is the difference between the largest and smallest possible values of n ?

Answers

1) _____

2) _____

3) _____

Solutions to Category 2
Geometry
Meet #2 - November, 2014

Answers
1) 14
2) 81
3) 2

1) Start with the square. If the area is 25, then one side is the square root of 25, or 5. Then the base of of the triangle, BF, is also 5. If the area of triangle BFC is 10, then its altitude, CF, is 4. That makes CE = 5 + 4 = 9. Because angle D is 45 degrees, right triangle CED is isosceles. So, ED is also 9. Finally, AD = AE + ED = 5 + 9 = 14.

2) Since one side of A is 4, so its perimeter is 4(4), or 16. Then this 16 is 2/3 of the perimeter of B, so the perimeter of B is 16(3/2), or 24. Then 24 is 2/3 of the perimeter of C, so the perimeter of C is 24(3/2), or 36. So, one side of C is 36/4, or 9. The area of C is 9(9), or 81.

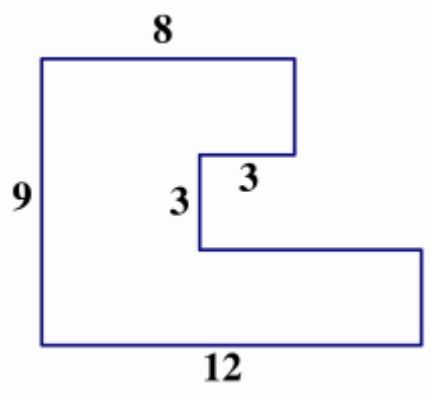
3) This problem utilizes the notion that the sum of any two sides of a triangle must be larger than the third side. Since the perimeter is a fixed value of 10, this chart shows the possibilities for the lengths of the sides (and the ones that fail):

side 1	side 2	side 3	possible?
1	1	8	no
1	2	7	no
1	3	6	no
1	4	5	no
2	2	6	no
2	3	5	no
2	4	4	yes
3	3	4	yes

The first six options fail because the sum of the first two sides does not exceed the third side. The bottom two succeed because the sum of any two of the sides does exceed the third side.
So, the smallest possible value of n is 2 and the largest possible value is 4, and their difference is 2.

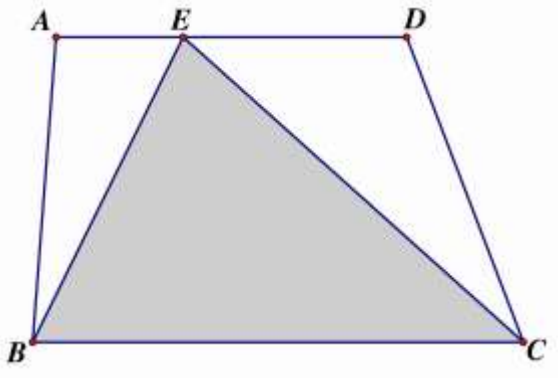
Category 2
Geometry
Meet #2, November/December 2012

1. Find the number of inches in the perimeter of the figure at right. All angles are right angles and all lengths are in inches.



2. Five squares of gold all have the same thickness, but they have edge lengths of 1 cm, 5 cm, 7 cm, 7 cm, and 11 cm. If the gold is melted down and recast with the same thickness as before into five identical squares, how many centimeters are there in the edge length of each square?

3. In trapezoid ABCD, side AD, which is 15 cm, is parallel to side BC, which is 21 cm. The area of trapezoid ABCD is 234 square centimeters. If point E is on side AD, how many square centimeters are there in the area of triangle BEC? Express your answer to the nearest tenth of a square centimeter.



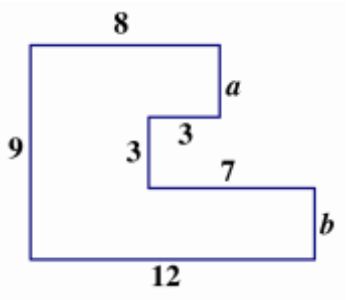
Answers

1. _____ inches
2. _____ cm
3. _____ sq. cm

Solutions to Category 2
Geometry
Meet #2, November/December 2012

Answers
1. **48 inches**
2. **7 cm**
3. **136.5 sq. cm**

1. The total vertical rise on the right side of the figure must be equal to the 9 inches we see on the left side of the figure, so $a + 3 + b = 9$. Similarly, the total of the horizontal lengths would equal the 12 inches we see on the bottom, but there is an extra 3 + 3 = 6 inches because the figure turns into itself for 3 inches and then must come back 3 inches. The total perimeter is thus 9 + 12 + 9 + 12 + 6 = **48** inches.



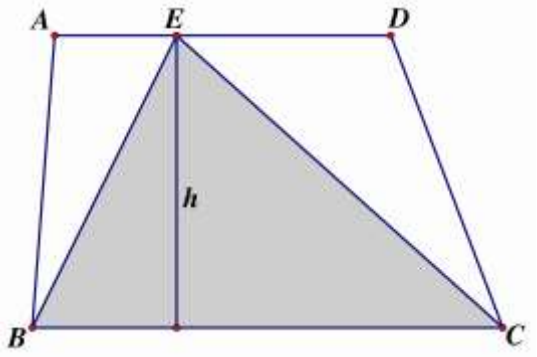
2. The total surface area of the 5 squares is $1^2 + 5^2 + 7^2 + 7^2 + 11^2 = 1 + 25 + 49 + 49 + 121 = 245$ square cm. If the gold is to be recast in 5 equal squares of the same thickness as before, they must each have a surface area of 245 ÷ 5 = 49 square cm. The side length of each square would be $\sqrt{49}$ = **7 cm**. Notice that we squared the side lengths, then averaged these squared numbers, and finally took the square root of this average. This square root of the mean of the squares is called the "root mean square" or the "quadratic mean."

3. Using the area formula for a trapezoid $A = \frac{1}{2}h(b_1 + b_2)$, we substitute in the known values and solve for the unknown height *h* as follows:

$$234 = \frac{1}{2}h(21 + 15) \Rightarrow 234 = 17h \Rightarrow h = 13.$$

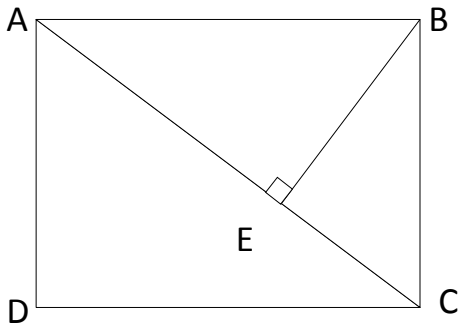
Triangle BEC has the same heigth, so it's area is $\frac{1}{2} \times 21 \times 13 = 10.5 \times 13$

= **136.5 square centimeters**.

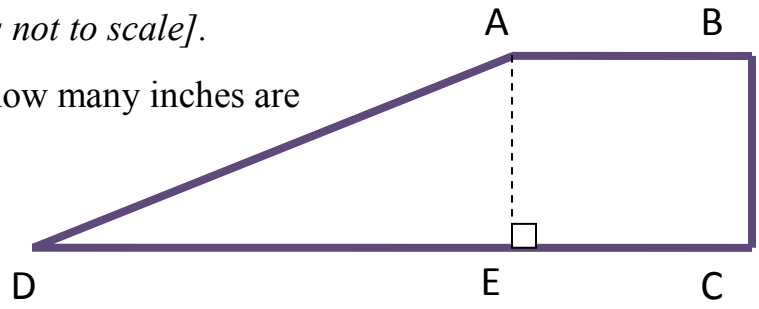


Category 2 – Geometry

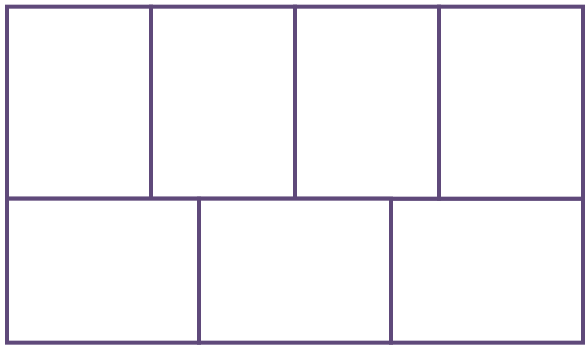
1. The perimeter of rectangle $ABCD$ measures 70 cm.
   $\overline{AD} = 15\ cm$, $\overline{AC} = 25\ cm$
   How many centimeters in the measure of $\overline{BE}$?



2. In the drawing below, the area of trapezoid $ABCD$ is four times the area of rectangle $ABCE$. *[The drawing is not to scale]*.
   If $\overline{DC}$ measures 70 inches, then how many inches are there in the measure of $\overline{EC}$?



3. The rectangle below is divided into 7 congruent (identical) rectangles.
   The total area of all is 2,100 square inches.
   How many inches in the perimeter of each one?



Answers

1. _____

2. _____

3. _____

Solutions to Category 2 – Geometery

Answers	
1.	12
2.	10
3.	70

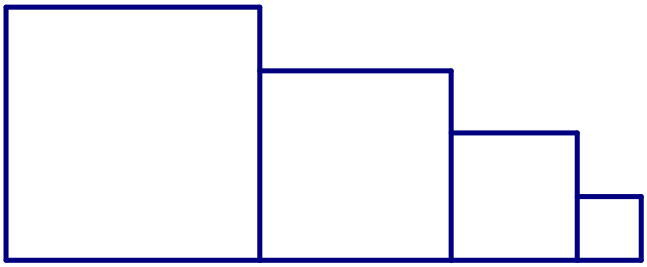
1. For the perimeter to measure 70 cm, $\overline{DC}$ must measure 20 cm. The area of triangle ADC then is $\frac{1}{2} \cdot \overline{AD} * \overline{DC}$ and this must equal the area of triangle ABC which can be expressed as $\frac{1}{2} \cdot \overline{AC} * \overline{BE}$. When we plug in the known values we get $\overline{BE} = 12\ cm$. [Using similar triangles, you can also observe that $\frac{\overline{AC}}{\overline{AD}} = \frac{\overline{AB}}{\overline{BE}}$].

2. If we call the trapezoid's height $\overline{AE} = \overline{BC} \equiv H$, then we know that the rectangle's area is $\overline{EC} \times H$, and the trapezoid's area is larger by the triangle's area $\frac{1}{2} \times \overline{DC} \times H$. Knowing that $\overline{EC} + \overline{DC} = 70$, and naming $\overline{EC} \equiv L$, we can write: $\frac{1}{2} \times (70 - L) = 3 \times L$ [Expressing the fact that the triangle's area is 3 times the rectangle's, and cancelling out H]. Solving, we get $L = 10\ inches$.

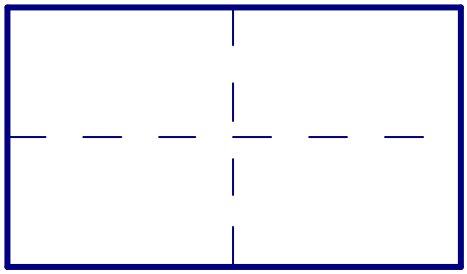
3. Let's call a rectangle's width W, and its height H.
   Each reactangle's area is $W * H = 300\ square\ inches$.
   In the drawing we see 4 rectangles in the top row and 3 in the bottom row, so we can conclude that $4 \cdot W = 3 \cdot H$ or $H = \frac{4 \cdot W}{3}$. When we subtitute this in the first equation we get $\frac{4}{3} \cdot W^2 = 300$ or $W^2 = \frac{900}{4} = 225$.
   So we get $W = 15\ inches\ and\ H = 20\ inches$, and the perimeter is:
   $2 \cdot (W + H) = 70\ inches$.
   [Another way to solve is to notice that the whole area is $3 \cdot H * (H + W) = 2{,}100$. Combined with $W * H = 300$ this leads to $H^2 = 400$].

Category 2
Geometry
Meet #2, December 2008

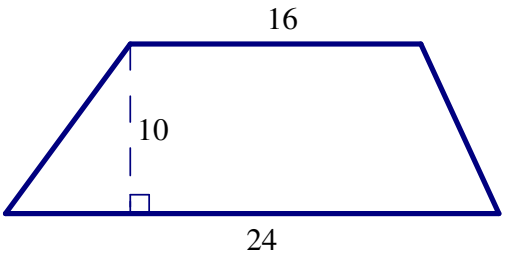
1. The four squares below have areas of 16, 9, 4, and 1 respectively. The squares are lined up one next to the other as shown below. What is the perimeter of the overall shape below?

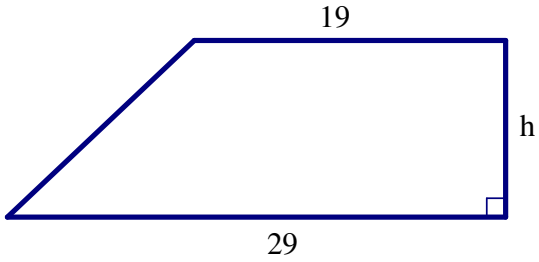


2. A rectangle with a perimeter of 60 inches is cut into 4 smaller congruent rectangles by cutting the rectangle in half both horizontally and vertically as shown to the right. How many inches are in the sum of the perimeters of the four new rectangles?



3. The trapezoid on the right has an area that is 16 more than the area of the trapezoid on the left. What is the value of h (the height) in the trapezoid on the right?





Answers

1. _____
2. _____
3. _____

Solutions to Category 2
Geometry
Meet #2, December 2008

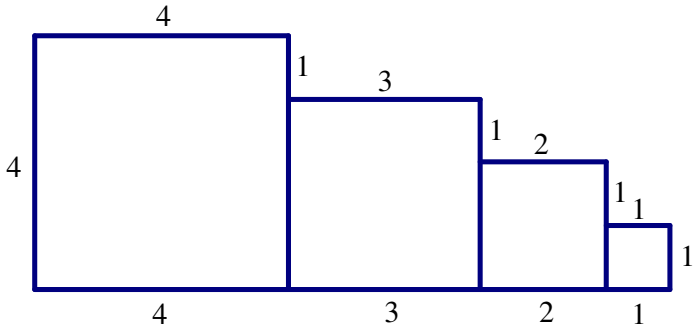
Answers

1. 28

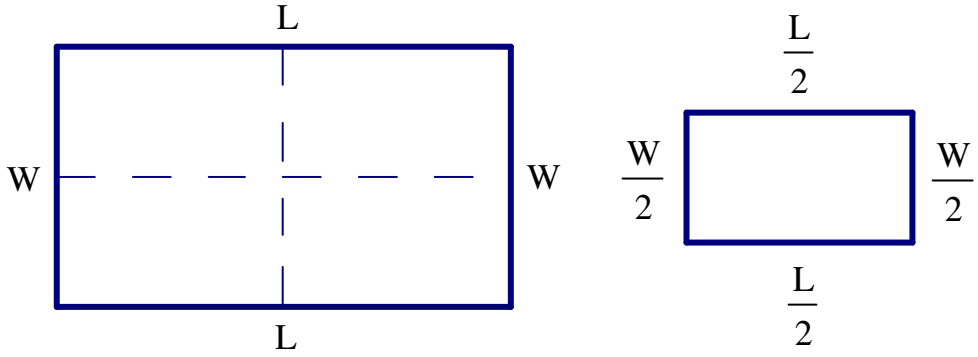
2. 120

3. 9

1. The total perimeter of the figure is:
$4 + 1 + 3 + 1 + 2 + 1 + 1 + 1 + 1 + 2 + 3 + 4 + 4 = 28$



2. Labeling the rectangles' length and width with L and W we can call the perimeter of the original rectangle 2W + 2L. Each of the smaller rectangles would have a length of $\frac{L}{2}$ and a width of $\frac{W}{2}$ giving them each a perimeter of W + L and the four of them combined would be 4W + 4L. That's exactly twice the perimeter of the original, so the combined perimeter of the four rectangles must be 2(60) = 120



3. The area of the trapezoid on the left is $A_{Trapezoid} = \frac{(b_1+b_2)\cdot h}{2} = \frac{(16+24)\cdot 10}{2} = \frac{400}{2} = 200$. So the area of the trapezoid on the right is 200+16 = 216. Using the formula for area of a trapezoid we can find the missing height given the area.

$$216 = \frac{(19+29)\cdot h}{2}$$
$$216 = \frac{48h}{2}$$
$$216 = 24h$$
$$9 = h$$