

Math League SCASD

Study Packets

Meet #1

Algebra

Self-study Packet

2022-2023

Problem Categories for this Meet:

1. Mystery: Problem solving
2. Geometry: Angle measures in plane figures including supplements and complements
3. Number Theory: Divisibility rules, factors, primes, composites
4. Arithmetic: Order of operations; mean, median, mode; rounding; statistics
5. **Algebra: Simplifying and evaluating expressions; solving equations with 1 unknown including identities**

Important information you need to know about Algebra...

Simplifying and evaluating expressions; solving equations with 1 unknown including identities

Equations (The Basics)

- Goal: Get the Variable Alone
- Whatever you do to one side of the equation, you must do to the other!
- Undo equations using the *reverse* order of operations.
- Use inverse operations (opposites) to get the variable alone.
 - Multiplication and Division undo each other.
 - Adding and Subtracting undo each other.
 - Squaring and Taking the Square Root of a number undo each other.

Simplifying Expressions

Coefficient: a number multiplying a variable. The coefficient of $7a$ is 7. The coefficient of $-2xy$ is -2 . The coefficient of x is 1, because 1 multiplied by x is x .

Like terms: have exactly the same variable(s) raised to the same power

<i>Like Terms</i>	<i>Not Like Terms</i>
$7x$ and $9x$	$3x$ and $3y$
$4x^2$ and $-12x^2$	$3x^3$ and $4x^2$
$-ab^3c^2$ and $4ab^3c^2$	$5a^4b$ and $12ab^4$

To combine like terms, add their coefficients. Keep the variable and the exponent the same!

Distributive Property: To *distribute*, multiply what is directly outside of the parentheses by everything inside the parentheses.

For example: $a(b + c) = ab + ac$
 $7(x - 5) = 7x - 35$

Identity: an equation that is always true.


For example $7x + 5 = 7x + 2 + 3$ is an identity, because no matter what value you substitute for x , the equation will always be true.

Category 5

Algebra

Meet #1 - November, 2020

- 1) If $A = 7$ and $B = 3$ and $C = 6$, then evaluate $AB - BC + ABC$.

- 2) What value of  makes the following equation true?

$$3(2 \text{  - 4) + 4 \text{ } = 33 - 5(\text{ } - 3)$$

- 3) If $7 - M = -2$ and $0.75N = 15$ and $-5(7 - E) = -50$ then find the value of H such that the following equation is true:

$$2(M - E) - H = 6N$$

Answers

1) _____

2) _____

3) _____

Solutions to Category 5
Algebra
Meet #1 - November, 2020

$$\begin{aligned}
 1) \quad & AB - BC + ABC \\
 &= (7)(3) - (3)(6) + (7)(3)(6) \\
 &= 21 - 18 + 126 \\
 &= 3 + 126 \\
 &= 129
 \end{aligned}$$

2) Replacing the turkey with the letter W,
 for simplicity:

$$\begin{aligned}
 3(2W - 4) + 4W &= 33 - 5(W - 3) \\
 6W - 12 + 4W &= 33 - 5W + 15 \\
 10W - 12 &= 48 - 5W \\
 15W &= 60 \\
 W &= 4
 \end{aligned}$$

$$\begin{array}{lll}
 3) \quad 7 - M = -2 & 0.75N = 15 & -5(7 - E) = -50 \\
 -M = -9 & N = 15 / 0.75 & -35 + 5E = -50 \\
 M = 9 & N = 20 & 5E = -15 \\
 & & E = -3
 \end{array}$$

Now substitute the values of M, N, and E, from above, into this equation:

$$\begin{aligned}
 2(M - E) - H &= 6(20) \\
 2(9 - (-3)) - H &= 120 \\
 2(12) - H &= 120 \\
 24 - H &= 120 \\
 -H &= 96 \\
 H &= -96
 \end{aligned}$$

Answers

1) 129

2) 4

3) -96




Category 5




Algebra


Meet #1 - October, 2018





1) What value of W makes the following equation an identity?

$$2(5x + 3W) = 7x + 3(x + 8)$$

2) If  = 10, and  = 2, and  = 6, then what is

the value of 5  $+ 3$  $- 4$  ?

3) What value of  makes the following equation true?

$$3(2 \text{  - 5) - 2(4 \text{  - 7) = 3 \text{  - 25 - 11 \text{ 

Answers$$

1) _____

2) _____

3) _____

Solutions to Category 5
Algebra
Meet #1 - October, 2018

$$\begin{aligned} 1) \quad 2(5x + 3W) &= 7x + 3(x + 8) \\ 10x + 6W &= 7x + 3x + 24 \\ 6W &= 24 \\ W &= 4 \end{aligned}$$

$$\begin{aligned} 2) \quad 5(10) + 3(2) - 4(6) \\ &= 50 + 6 - 24 \\ &= 56 - 24 \\ &= 32 \end{aligned}$$

3) To simplify, substitute a letter, say T, for the ghost:

$$\begin{aligned} 3(2T - 5) - 2(4T - 7) &= 3T - 25 - 11T \\ 6T - 15 - 8T + 14 &= -8T - 25 \\ -2T - 1 &= -8T - 25 \\ 24 &= -6T \\ -4 &= T \end{aligned}$$

Answers

1) 4

2) 32

3) -4

Category 5

Algebra

Meet #1 - October, 2016

1) If $3N - 15 = 24$ and $6(2M + 7) = 18$, then what is the value of $N - M$?

2) An identity is an equation that is true for all values of the variable. What value of C makes the following equation an identity in the variable H ?

$$5(H + 2) + 11 = 3(8 + H) + 2H + C$$

3) The symbol " ¢ " is used in the following equation as a sign of multiplication. What value of E makes the equation true?

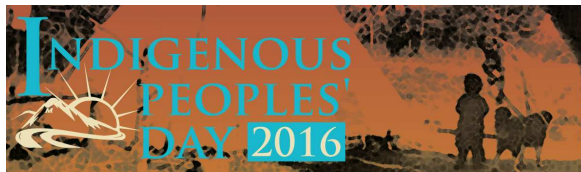
$$8 \text{ ¢ } 15 \text{ ¢ } 21 \text{ ¢ } 100 \text{ ¢ } E = 4 \text{ ¢ } 30 \text{ ¢ } 63 \text{ ¢ } 1000$$

Answers

1) _____

2) _____

3) _____



The UN declared August 9 as Indigenous Peoples' Day," celebrating the lives of Native Peoples around the globe. In the USA, many states now celebrate this holiday concurrently with Columbus Day.

Solutions to Category 5
Algebra
Meet #1 - October, 2016

$$\begin{array}{ll}
 1) \quad 3N - 15 = 24 & \text{and} \quad 6(2M + 7) = 18 \\
 \quad \quad 3N = 39 & \quad \quad 12M + 42 = 18 \\
 \quad \quad N = 13 & \quad \quad 12M = -24 \\
 & \quad \quad M = -2
 \end{array}$$

Therefore, $N - M = 13 - (-2) = 15$.

$$\begin{array}{l}
 2) \quad 5(H + 2) + 11 = 3(8 + H) + 2H + C \\
 \quad \quad 5H + 10 + 11 = 24 + 3H + 2H + C \\
 \quad \quad 5H + 21 = 5H + 24 + C \\
 \quad \quad 21 = 24 + C \\
 \quad \quad -3 = C
 \end{array}$$

3) Although there is a lot of arithmetic involved here, there is a more simplified approach utilizing factorization. A number on one side of the equation may be expressed as a factorization of a number on the other side. For example, the 8 on the left side can be expressed as 2×4 , where the number 4 is on the right side. The entire equation could be expressed as such:

$$\begin{array}{l}
 8 \cancel{\text{¢}} 15 \cancel{\text{¢}} 21 \cancel{\text{¢}} 100 \cancel{\text{¢}} E = 4 \cancel{\text{¢}} 30 \cancel{\text{¢}} 63 \cancel{\text{¢}} 1000 \\
 (2 \cancel{\text{¢}} 4) \cancel{\text{¢}} 15 \cancel{\text{¢}} 21 \cancel{\text{¢}} 100 \cancel{\text{¢}} E = 4 \cancel{\text{¢}} (2 \cancel{\text{¢}} 15) \cancel{\text{¢}} (3 \cancel{\text{¢}} 21) \cancel{\text{¢}} (10 \cancel{\text{¢}} 100)
 \end{array}$$

Now divide both sides by the product of the common factors of $2 \cancel{\text{¢}} 4 \cancel{\text{¢}} 15 \cancel{\text{¢}} 21 \cancel{\text{¢}} 100$, leaving us with the following equation:

$$\begin{array}{l}
 E = 3 \cancel{\text{¢}} 10 \\
 E = 30
 \end{array}$$

Answers

1) 15

2) -3

3) 30

Category 5
Algebra
Meet #1 - October, 2014



- 1) If $3W + 7 = 34$ and $5A - 9 = -39$, then what is the value of $-2WA$?
- 2) The letters of the alphabet, $\{A, B, C, \dots, Z\}$, correspond to the consecutive integers $\{-8, -7, -6, \dots, 17\}$. For example, $A = -8$, $B = -7$, $N = 5$, and so on. The number, zero, is included among the integers. Find the sum of the numbers corresponding to the letters in the following word:

HALLOWEEN

- 3) Find the value of N if

$$2(N + 1) + 3(4N - 5) - 6(7N + 8) - 9(10N - 11) = 12(13 - 14N) + 32$$

Answers

1) _____

2) _____

3) _____

Solutions to Category 5
Algebra
Meet #1 - October, 2014

$$\begin{array}{ll} 1) & 3W + 7 = 34 & 5A - 9 = -39 \\ & 3W = 27 & 5A = -30 \\ & W = 9 & A = -6 \end{array}$$

$$\text{So, } -2WA = (-2)(9)(-6) = 108$$

Answers

1) 108

2) 14

3) 3

$$\begin{array}{cccccccccccccccccccc} 2) & A & B & C & D & E & F & G & H & I & J & K & L & M & N & O & P & Q & R & S & T \\ & -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \end{array}$$

$$\begin{array}{cccccc} U & V & W & X & Y & Z \\ 12 & 13 & 14 & 15 & 16 & 17 \end{array}$$

$$\begin{aligned} \text{HALLOWEEN} &= (-1) + (-8) + (3) + (3) + (6) + (14) + (-4) + (-4) + (5) \\ &= 14 \end{aligned}$$

$$\begin{aligned} 3) \quad 2(N + 1) + 3(4N - 5) - 6(7N + 8) - 9(10N - 11) &= 12(13 - 14N) + 32 \\ 2N + 2 + 12N - 15 - 42N - 48 - 90N + 99 &= 156 - 168N + 32 \\ -118N + 38 &= 188 - 168N \\ 50N &= 150 \\ N &= 3 \end{aligned}$$

Category 5

Algebra

Meet #1, October 2012

1. Evaluate the expression below if $a = \frac{1}{2}$, $b = \frac{2}{3}$, and $c = \frac{3}{4}$.
Express your answer as a mixed number in lowest terms.

$$\frac{ab + ac + bc}{abc}$$

2. Find the value of M that makes the equation below an identity.

$$11(4x + 9) - 19x + M = 76x - 17(3x - 5)$$

3. On the second day of a three-day math challenge, Julie completed 6 more than twice as many problems as she completed on the first day. On the third day, she completed 1 less than two thirds as many problems as she completed on the second day. If Julie completed 74 problems in all, how many problems did she complete on the first day?

Answers

1. _____

2. _____

3. _____

Solutions to Category 5
Algebra
Meet #1, October 2012

Answers

1. $4\frac{5}{6}$
2. **-14**
3. **15**

1. The correct evaluation is shown below.

$$\frac{\frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{3}{4} + \frac{2}{3} \times \frac{3}{4}}{\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}} = \frac{\frac{1}{3} + \frac{3}{8} + \frac{1}{2}}{\frac{1}{4}} = \frac{\left(\frac{8}{24} + \frac{9}{24} + \frac{12}{24}\right)}{\frac{1}{4}} = \frac{4}{1} \times \frac{29}{24} = \frac{29}{6} = 4\frac{5}{6}$$

2. On the third line of the algebra below, we see $25x$ on each side. Since 25 times any real number is equal to 25 times the same number, the solution to the equation depends only on M . If M is **-14**, then x can be any real number, making the original equation an identity. If M is any other number, then there would be no solution to the original equation.

$$\begin{aligned} 11(4x + 9) - 19x + M &= 76x - 17(3x - 5) \\ 44x + 99 - 19x + M &= 76x - 51x + 85 \\ 25x + 99 + M &= 25x + 85 \\ M &= 85 - 99 \\ M &= \mathbf{-14} \end{aligned}$$

3. Let x be the number of problems Julie completed on the first day. Then we can write and solve the following equation.

$$\begin{aligned} x + (2x + 6) + \left(\frac{2}{3}(2x + 6) - 1\right) &= 74 \\ x + 2x + 6 + \frac{4x}{3} + 4 - 1 &= 74 \\ \frac{13x}{3} + 9 &= 74 \\ \frac{13x}{3} &= 65 \\ x &= \mathbf{15} \end{aligned}$$

Category 5 – Algebra

1. Find the value of x that solves this equation:

$$\frac{(x-3)}{3} + \frac{x}{4} + \frac{(x-2)}{5} = x - 4$$

2. Tom and Jerry are both at home. Tom starts driving in a straight line at 60 miles-per-hour (mph), and 40 minutes later Jerry starts chasing him at 90 mph. How many miles from home will they be when Jerry catches up with Tom?

3. The solution to the equation below is $x = 3$. Find the value of M .

$$(M + 1) \cdot x + 2 \cdot x + 5 = (x + 3) \cdot M - 1$$

Answers
1. _____
2. _____
3. _____

Solutions to Category 5 - Algebra

Answers

- | | |
|----|-----|
| 1. | 12 |
| 2. | 120 |
| 3. | 5 |

1. If we multiply the original equation by the common denominator 60 we'd get:

$$20 \cdot (x - 3) + 15 \cdot x + 12 \cdot (x - 2) = 60 \cdot (x - 4)$$

whic we can aggregate to:

$$20 \cdot x - 60 + 15 \cdot x + 12 \cdot x - 24 = 47 \cdot x - 84 = 60 \cdot x - 240, \text{ or}$$

$$13 \cdot x = 156 \text{ and so } x = \frac{156}{13} = 12$$

2. When Jerry catches up with Tom, they both covered the same Distance (D), but Jerry did so 40 minutes ($\frac{2}{3}$ of an hour) more quickly.

If it took Tom a time T , then we can write the equation:

$$D = 60 * T = 90 * (T - \frac{2}{3}) \text{ [Distance = Speed * Time]}.$$

Solving for T we get $30 * T = 60$ or $T = 2 \text{ hours}$.

So the distance covered by Tom in 2 hours is 120 miles.

3. Since we know that $x = 3$ is the solution, we can substitute that value for x and then solve for M :

the equation $(M + 1) \cdot x + 2 \cdot x + 5 = (x + 3) \cdot M - 1$ will translate to:

$$(M + 1) \cdot 3 + 2 \cdot 3 + 5 = (3 + 3) \cdot M - 1$$

$$3 \cdot M + 14 = 6 \cdot M - 1$$

$$3 \cdot M = 15 \text{ or } M = 5$$