Math League SCASD
Study Packets

Meet #1
Arithmetic

Self-study Packet
2022-2023

Problem Categories for this Meet:

1. Mystery: Problem solving
2. Geometry: Angle measures in plane figures including supplements and complements
3. Number Theory: Divisibility rules, factors, primes, composites
4. **Arithmetic: Order of operations; mean, median, mode; rounding; statistics**
5. Algebra: Simplifying and evaluating expressions; solving equations with 1 unknown including identities

Important Information you need to know about Arithmetic....

Order of operations; mean, median, mode; rounding; statistics

Order of Operations:

1. **P**arentheses—Do everything inside grouping symbols first
2. **E**xponents—evaluate powers (both exponents and roots)
3. **M**ultiply and **D**ivide—in order from *Left* to *Right*
4. **A**dd and **S**ubtract—in order from *Left* to *Right*

***Many people remember the Order of Operations by memorizing the phrase "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally," or the word Pemdas.*

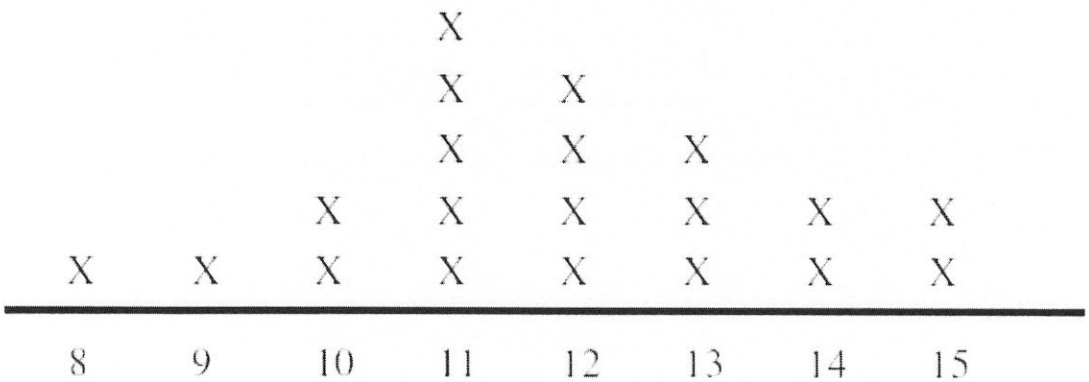
Mean: the *average* of a set of numbers (Add the numbers and divide by how many numbers there are).

Median: the *middle* number in a set when the set is ordered from least to greatest. If there are two middle numbers, the median is the average of the two.

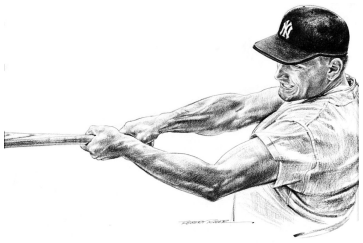
Mode: the **M**ost **O**ften **D**isplayed **E**ntry (the number that occurs the *most* in a set of numbers).

Range: the difference between the largest number and the smallest number in a set of numbers.

Line Plot: a graph that consists of a horizontal number line, on which each value of a set is denoted by an x over the corresponding value on the number line. The number of x's above each score indicates how many times each number occurred.



Category 4
Arithmetic
Meet #1 - November, 2020



1) What is the value of 17 + 5 (25 - 3 x 4) where the symbol x is a multiplication sign?

2) Babe Ruth broke the all-time Major League Baseball homerun record in 1927, hitting 60 homeruns. The record stood for 34 years until a virtually unknown player with a sub-par career hit "the shot heard 'round the world" on the final day of the 1961 season when Roger Maris, then with the New York Yankees, hit homerun #61. The stem and leaf plot below indicates how many homeruns were hit by Roger Maris in each of his 13 major league seasons. The number in the left-hand column is the tens digit. The number in the right-hand column is the units, or ones digit. What is the mean (average) number of homeruns that Maris hit per season? Round your answer to the nearest whole number.

0	5, 8, 9, 9
1	3, 4, 6, 9
2	3, 6
3	3, 9
6	1

3) The mean (average) of nine different positive whole numbers is 17. Each number is greater than 5. Six of the numbers are 7, 10, 16, 21, 28, and 36. What is the largest possible value that one of the remaining three numbers could be?

Answers

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #1 - November, 2020

1) $17 + 5(25 - 3 \times 4)$
$= 17 + 5(25 - 12)$
$= 17 + 5(13)$
$= 17 + 65$
$= 82$

Answers	
1)	82
2)	21
3)	22

2) To compute the average, add the 13 numbers, then divide by 13. Finally, round to the nearest whole number:

$(5 + 8 + 9 + 9 + 13 + 14 + 16 + 19 + 23 + 26 + 33 + 39 + 61) / 13$
$= (275) / 13$
$= 21.15 \ldots$

Rounded to the nearest whole number produces an answer of 21.

3) If the average of nine numbers is 17, then their sum is (9)(17), or 153. The sum of the given six numbers is $7 + 10 + 16 + 21 + 28 + 35$, or 117. Compute the difference between 153 and 117 to get the sum of the remaining three numbers, or 36. To maximize the value of one of these three numbers, and to make them all different AND satisfy that they be greater than 5, make two of the numbers as small as possible while satisfying those conditions. Those two smallest numbers are 6 and 8 (not 7, because 7 was already given). Subtract the sum $6 + 8$, or 14, from 36 to get the largest possible remaining number, or 22.

Category 4
Arithmetic
Meet #1 - October, 2018

1) What is the value of $142 - 3(18 + 5)$ **?**

2) As of July 9, the following Boston Red Sox baseball players had the given number of base hits (BH), doubles (D), and runs (R):

	BH	D	R
Mookie Betts	94	24	71
J.D. Martinez	108	21	64
Brock Holt	48	12	18
Andrew Benintendi	98	20	65
Xander Bogaerts	79	25	44
Rafael Devers	82	24	42
Mitch Moreland	68	15	41

Let A = the arithmetic mean (average) of the base hits
B = the mode of the doubles
C = the median of the runs

What is the sum of A + B + C ? Round your answer to the nearest whole number.

3) The arithmetic mean, or average, of six numbers in a set containing fifteen numbers is 28. The arithmetic mean of five other numbers in this set is 32. What is the arithmetic mean of the remaining numbers in the set if the sum of all fifteen numbers is 407? Round your answer to the nearest tenth.

Answers

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #1 - October, 2018

Answers
1) 73
2) 150
3) 19.8

1) $142 - 3(18 + 5)$
$= 142 - 3(23)$
$= 142 - 69$
$= 73$

2) **A = (the sum of the number of hits) divided by the total number of players**
= 577 / 7, or about 82.4 . . .
= 82 when rounded to the nearest whole number.

B = 24, the only repeated number of doubles

C = 44 , as 44 is the middle number when the seven numbers of runs are arranged in order: 18, 41, 42, 44, 64, 65, 71.

The sum of A + B + C = 82 + 24 + 44 = 150.

3) If the mean of six numbers in the set is 28, then their sum is (6)(28), or 168. If the mean of five other numbers is 32, then their sum is (5)(32), or 160. If the total of all fifteen numbers is 407, Then the total of the remaining four numbers is 407 - 168 - 160, or 79. The mean of those four numbers is 79 / 4, or 19.75. Rounding to the nearest tenth yields 19.8.

Category 4
Arithmetic
Meet #1 - October, 2016

1) Using the standard order of operations, find the value of $3^4 + 4^3$ **.**

2) Evaluate: $\left(17 - \left[9 \cdot (68-49)^2 - 4\left(2^6 - 6^2\right)^2\right]\right)^2$

3) Consider the set H = { 5, 8, 10, 3, 10, 5, 17, 10, 7 }.
Let A = the mean of set H
B = the median of set H
C = the mode of set H.
What is the mean of A, B, and C ? Round your answer to the nearest tenth.

Answers

1) _____

2) _____

3) _____

Columbus Day, celebrating the landing in the Americas of the Spanish explorer, Christopher Columbus, falls on the second Monday of October.

Solutions to Category 4
Arithmetic
Meet #1 - October, 2015

Answers
1) 145
2) 9216
3) 8.8

1) $3^4 + 4^3$

= (3)(3)(3)(3) + (4)(4)(4)

= 81 + 64

= 145

2) $\left(17-\left[9\cdot(68-49)^2-4\left(2^6-6^2\right)^2\right]\right)^2$

= $\left(17-\left[9\cdot(19)^2-4(64-36)^2\right]\right)^2$

= $\left(17-\left[9\cdot 361-4(28)^2\right]\right)^2$

= $\left(17-\left[3249-4(784)\right]\right)^2$

= $\left(17-\left[3249-3136\right]\right)^2$

= $\left(17-\left[113\right]\right)^2$

= $(-96)^2$

= 9216

3) Arranging the nine numbers in order, from smallest to largest:

H = { 3, 5, 5, 7, 8, 10, 10, 10, 17 }.

Then A = the mean of H = (3+5+5+7+8+10+10+10+10+17) / 9 = 75 / 9 = $8.\overline{3}$ **.**
B = the median of H = the middle number = 8.
C = the mode of H = the most frequent value = 10.

The mean of A, B, and C = $(8.\overline{3}$ **+ 8 + 10) / 3 =** $26.\overline{3}$ **/ 3 = 8.77 . . .** $\approx$ **8.8.**

Category 4
Arithmetic
Meet #1 - October, 2014

1) Using the standard order of operations, find the value of

$$2\left[2\left(2+2\times 2\right)\right]^2$$

2) The arithmetic mean (average) of 17, 26, 42, and 59 is equal to the arithmetic mean of 19 and N. What is the value of N ?

3) In preparation for handing out candy at Halloween, Frank bought forty 30-cent candy bars, X 50-cent candy bars, and fifteen 75-cent candy bars. If the median cost of all the candy bars is 50 cents, and X is a whole number, then what is the smallest possible value of X ?

Answers

1) _____

2) _____

3) _____

Solutions to Category 4
Arithmetic
Meet #1 - October, 2014

1)

$$\begin{aligned} & 2[2(2+2\times 2)]^2 \\ = \; & 2[2(2+4)]^2 \\ = \; & 2[2(6)]^2 \\ = \; & 2[12]^2 \\ = \; & 2[144] \\ = \; & 288 \end{aligned}$$

Answers

1) 288

2) 53

3) 26

2) The average of 17, 26, 42, and 59 is their sum divided by 4. The sum is 144. Dividing 144 by 4 yields 36. Then 36 is the average of 19 and N. 19 is 17 less than 36, so N is 17 more than 36. So, N = 53.

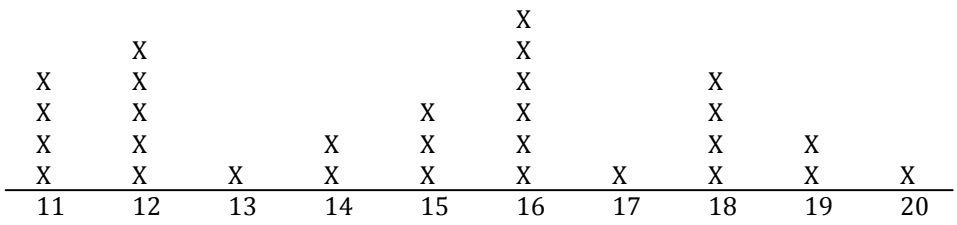
3) Since there are 40 of the least expensive candy bars, and only 15 of the most expensive, then we have to compensate by having at least 26 of the mid-priced candy bars in order for the 50-cent bar to be the median price.

Category 4
Arithmetic
Meet #1, October 2012

1. Evaluate the following expression according to the order of operations.

$$\frac{2+5\times 2^5-2\times 5^2+5}{2\times 5-2+5}$$

2. The line plot below shows the number of song tracks on a collection of CD's. Each X represents one CD. What is the positive difference between the median and the mode of the data?



Song Tracks on CD's

3. All 13 students who took a 12-point quiz received a positive integer score. The median was 10, the mean was 10, and the mode was 9. What is the maximum number of students who could have received perfect scores?

Answers

1. _____
2. _____
3. _____

Solutions to Category 4
Arithmetic
Meet #1, October 2012

Answers	
1.	**9**
2.	**1**
3.	**4**

1. The correct evaluation is shown below.

$$\frac{2+5\times 2^5-2\times 5^2+5}{2\times 5-2+5}=\frac{2+5\times 32-2\times 25+5}{10-2+5}=\frac{2+160-50+5}{13}=\frac{117}{13}=9$$

2. There are 29 X's on the line plot, so the median is the 15th number from the left, which is 15. The mode is 16, which can be seen as the tallest column of X's. The positive difference between these two is 16 – 15 = **1**.

3. Let's make a row of 13 blank spaces. We can place a 10 in the middle, since 10 is the median. We can start to place 12's at the far right of the line, but we must have one more 9 than 12's so that 9 is still the mode. Finally, since the mean is 10, we have to make sure that the sum of the numbers is 13 × 10 = 130. One possible set of numbers is shown below. The numbers that are circled could be changed, but the maximum number of perfect scores is **4**.

Category 4 – Arithmetic

1. Find the value of the expression:

   $[1 + 2 \div 3 \times 4] \times [9 \times (8 - 7) \div (6 + 5)]$

2. x represents the *mean* value of numbers in the list $\{2, 4, 6, 8\}$.

   y represents the *median* of the numbers in the list $\{0, 2, 4, 6\}$.

   z represents the *mode* of the numbers in the list $\{1, 2, 2, 3, 3, 3, 4, 4, 4, 4\}$.

   Find the value of the expression: $(x + y) \div z + (x + z) \div y$

3. Express $\frac{(4+4+\sqrt{4})^{\sqrt{4}}}{4\times4-\sqrt{4}}$ as a decimal rounded to the nearest hundredth.

Answers
1. _____
2. _____
3. _____

Solutions to Category 4 - Arithmetic

Answers	
1.	3
2.	5
3.	7.14

1. $[1 + 2 \div 3 \times 4] \times [9 \times (8 - 7) \div (6 + 5)] =$

$$\left[\frac{3}{3} + \frac{2 \times 4}{3}\right] \times \left[\frac{9 \times 1}{(6 + 5)}\right] = \frac{11}{3} \times \frac{9}{11} = \frac{9}{3} = 3$$

2. $x = \frac{2+4+6+8}{4} = 5$.

$y = 3$, the mean of the two middle values (2 *and* 4).

$z = 4$ as this is the most common value in the set.

Therefore: $(x + y) \div z + (x + z) \div y = (5 + 3) \div 4 + (5 + 4) \div 3 =$

$= 8 \div 4 + 9 \div 3 = 2 + 3 = 5$

3. $\frac{(4+4+\sqrt{4})^{\sqrt{4}}}{4\times4-\sqrt{4}} = \frac{(8+2)^2}{16-2} = \frac{100}{14} = 7.\overline{142857} \cong 7.14$