# Math League SCASD
# Study Packets

## Meet #1

# Number Theory

## Self-study Packet

# 2022 -2023

Problem Categories for this Meet:

1. Mystery: Problem solving
2. Geometry: Angle measures in plane figures including supplements and complements
3. **Number Theory: Divisibility rules, factors, primes, composites**
4. Arithmetic: Order of operations; mean, median, mode; rounding; statistics
5. Algebra: Simplifying and evaluating expressions; solving equations with 1 unknown including identities

# *Information you need to know about NUMBER THEORY...*

## Divisibility Rules

*A number is divisible by:*

- 2 if its ones digit is even (0, 2, 4, 6, 8)
- 3 if the sum of its digits is divisible by 3 (for example, to check if 364 is divisible by 3, add 3 + 6 + 4. You get 13. 13 is not divisible by 3, so 364 is not either).
- 4 if the number formed by its last two digits is divisible by 4 (for example, to check if 2,320 is divisible by 4, look at the number formed by the last two digits; in this case, 20. 20 is divisible by 4, so 2,320 is as well).
- 5 if its ones digit is 0 or 5
- 6 if it divisible by 2 *and* 3
- 8 if the number formed by its last *three* digits is divisible by 8 (similar to the rule for 4)
- 9 if the sum of its digits is divisible by 9 (similar to the rule for 3)
- 10 if its ones digit is 0

## Factoring

To find the factors of a number, it is useful to find the *pairs* of numbers that multiply to give you that number. Using the divisibility tricks, it is much easier to narrow this down. I recommend starting with 1. You can stop when you pass the square root of the number, because you have found all the factors.

For example, list all the factors of 120.

| | | | | |
|---|---|---|---|---|
| 1 x 120 | 2 x 60 | 3 x 40 | 4 x 30 | 5 x 24 |
| 6 x 20 | 8 x 15 | 10 x 12 | | |

The factors of 120 are 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, and 120.

## Primes and Composites

Prime Number: any number with **exactly** two factors

Composite Number: any number with **more than** two factors

** 0 and 1 are neither prime nor composite!

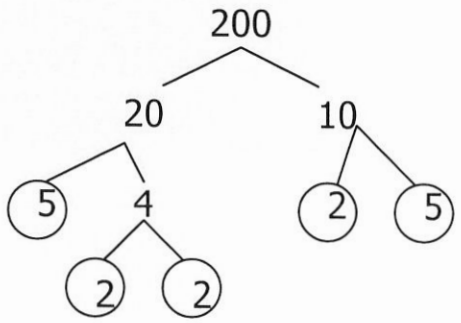
*It would be VERY useful to memorize all the prime numbers under 100 (Questions where you need to know them occur over and over again all season). They are:*

| | | | | | | |
|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 7 | 11 | 13 | 17 |
| 19 | 23 | 29 | 31 | 37 | 41 | 43 |
| 47 | 53 | 59 | 61 | 67 | 71 | 73 |
| 79 | 83 | 89 | 97 | | | |

**Prime Factoring**: Two common ways to find the prime factorization of a number are to use a factor tree or to use the ladder method. An example of each follows:

*Factor Tree:* Find the prime factorization of 200.

1. Start with your original number.
2. Find a pair of numbers that multiply to give you your original number.
3. Find a pair of numbers that multiply to give you each of the factors.
4. Continue until you only have prime numbers.
5. The prime factorization is written as a product of all the primes.




$200 = 2^3 \times 5^2$

*Ladder Method:* Find the prime factorization of 60.

1. Start with the lowest prime number (2) and check if your number is divisible by it.
2. If it is, divide your number by that. If not, try 3, then 5, then 7, etc, until you find a number that is a factor of your original number. Divide by it.
3. Repeat by dividing your new number by its smallest prime factor.
4. Continue until you are left with 1.
5. The prime factorization is written as a product of all its primes, which are conveniently ordered on the left of your ladder.

| | |
|---|---|
| 2 | 60 |
| 2 | 30 |
| 3 | 15 |
| 5 | 5 |
| | 1 |

$60 = 2^2 \bullet 3 \bullet 5$

**Category 3**
**Number Theory**
**Meet #1 - November, 2020**

**1) What is the smallest positive whole number that is divisible by 2, 3, 4, and 5 ?**

**2) If A = the sum of all prime numbers between 10 and 30 and B = the sum of all composite numbers between 31 and 47 then what is the value of B - A ?**

**3) There is only one whole number between 100 and 200, not including 100 or 200, that has exactly nine factors, including the number 1 and the number itself. What is that number?**

**Answers**

**1)** _____

**2)** _____

**3)** _____

**Solutions to Category 3**
**Number Theory**
**Meet #1 - November, 2020**

**1) Find the lowest common multiple of 2, 3, 4, and 5. That number is 60.**

**2) A = 11 + 13 + 17 + 19 + 23 + 29**
**= 112**
**B = 32 + 33 + 34 + 35 + 36 + 38 + 39 + 40 + 42 + 44 + 45 + 46**
**= 464**
**Then B - A = 464 - 112 = 352.**

**3) Positive whole numbers with an ODD number of factors are perfect squares or, simply, squares. The squares between 100 and 200 are as follows: 121, 144, 169, and 196. With 121 and 169 being squares of primes, those can be quickly eliminated. The number 144 is abundant and has waaaaaay too many factors. The factors of 196 are 1, 2, 4, 7, 14, 28, 49. 98, and 196 - exactly nine factors.**

**Answers**

**1) 60**

**2) 352**

**3) 196**

**Category 3**
**Number Theory**
**Meet #1 - October, 2018**

**1) The number 18 has six factors. Three of these are 1, 2, and 6. What is the sum of the other three factors?**

**2) The four digit number $3NN1$ is divisible by 9. What is the value of N?**

**3) The sum of the composite numbers between 140 and 160 is how much greater than the sum of the prime numbers between 140 and 160? "Between" 140 and 160" does not include the numbers 140 and 160.**

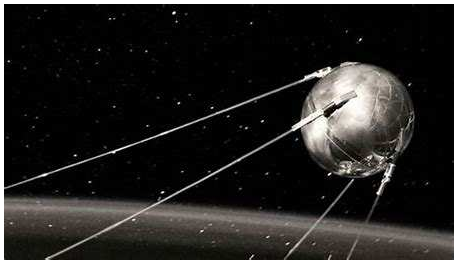
**Answers**

**1)** _____

**2)** _____

**3)** _____

**The Space Age began on October 4, 1957 when Russia launched the first satellite, Sputnik, into orbit, thus propelling the USA to promise to be the first to land a man on the moon.**



**Solutions to Category 3**
**Number Theory**
**Meet #1 - October, 2018**

**1) The factors of 18 are: 1, 2, 3, 6, 9, and 18. The sum of the missing factors is 3 + 9 + 18 = 30.**

**2) The sum of the digits must be a multiple of nine in order for the 4-digit number to be divisible by nine. If that sum were 9, then 2N = 5 and N = 2.5 (not a whole number). If that sum were 18, then 2N = 14 and N = 7 (a whole number). If that sum were 27, then 2N = 23 and N = 11.5 (not a whole number and not a single digit). So, the answer is 7.**

**3) The composite numbers between 140 and 160 are 141, 142, 143, 144, 145, 146, 147, 148, 150, 152, 153, 154, 155, 156, 158, and 159 and their sum is 2393. The prime numbers between 140 and 160 are 149, 151, and 157 and their sum is 457.**
**The difference between these two sums is 2393 - 457, or 1936.**

**Answers**

**1) 30**

**2) 7**

**3) 1936**

**Category 3**
**Number Theory**
**Meet #1 - October, 2016**

**1) The factors of a positive whole number, N, include the numbers 1, N, and all positive numbers, W, such that N divided by W is a positive whole number. How many factors of 36 are also multiples of 4 ?**

**2) There are three whole numbers, A, B, and C, that are between 120 and 130 that are each the product of exactly two different prime numbers. What is the sum A + B + C ?**

**3)**  **is a positive integer greater than 1.**

 **is the square of an integer.**

 **is the cube of an integer.**

**What is the smallest possible number of different positive factors (divisors) that**  **can have ?**

**Answers**

**1)** _____

**2)** _____

**3)** _____

**Solutions to Category 3**
**Number Theory**
**Meet #1 - October, 2016**

**1) The factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18, 36. Those factors that are multiples of four are: 4, 12, and 36. So, there are three.**

**2)**

| | |
|---|---|
| **121 = 11 x 11** | **No - the factors must be different.** |
| **122 = 2 x 61** | **Yes - both 2 and 61 are prime.** |
| **123 = 3 x 41** | **Yes - both 3 and 41 are prime.** |
| **124 = 2 x 2 x 31** | **No - too many factors.** |
| **125 = 5 x 5 x 5** | **No - too many factors.** |
| **126 = 2 x 3 x 3 x 7** | **No - too many factors.** |
| **127 = prime** | **No - prime** |
| **128 = 2 x 2 x 2 x 2 x 2 x 2 x 2** | **No - too many factors.** |
| **129 = 3 x 43** | **Yes - both 3 and 43 are prime.** |

**Therefore, A + B + C = 122 + 123 + 129 = 374.**

**3) Sixty-four is the smallest positive integer that meets all the criteria. The factors of 64 are: 1, 2, 4, 8, 16, 32, and 64, so there are seven in all.**

**Answers**

**1) 3**

**2) 374**

**3) 7**

**Category 3**
**Number Theory**
**Meet #1 - October, 2014**

**1) What is the sum of all the prime numbers between 40 and 60 ?**

**2)**  **has an odd number of factors.**

 **is divisible by 3.**

 **> 20.**

 **< 70.**

**What is the value of**  **?**

**3) How many of the three-digit numbers that can be made using the digits 1, 3, and 5 are not multiples of three ? Each digit can be used more than once.**

**Answers**

**1)** _____

**2)** _____

**3)** _____

**Solutions to Category 3**
**Number Theory**
**Meet #1 - October, 2014**

**1)** **41 + 43 + 47 + 53 + 59 = 243**

**2)**  **is a perfect square if it has an odd number of factors. If that square is between 20 and 70, then it could be any of these: 25, 36, 49, or 64. The only one divisible by 3 is 36.**

**3)** **The full list of three-digit numbers is: 111, 113, 115, 131, 133, 135, 151, 153, 155, 311, 313, 315, 331, 333, 335, 351, 353, 355, 511, 513, 515, 531, 533, 535, 551, 553, and 555. Checking that there should be twenty-seven numbers: 3x3x3 = 27.**

**The numbers that are divisible by 3 are those whose digit-sum is a multiple of 3: 111, 135, 153, 315, 333, 351, 513, 531, and 555.**

**That is nine numbers. Therefore, there are 27 - 9, or 18 numbers, that are not multiples of three.**

**Answers**

**1) 243**

**2) 36**

**3) 18**

Category 3
Number Theory
Meet #1, October 2012

**1.** Two primes $p$ and $q$ have a sum of 38. Given that $p > q$, find the value of $p - q$.

**2.** What single-digit value of $N$ will make the 7-digit number 1,295,$N$84 divisible by 18?

**3.** What number is the least three-digit multiple of 5 that has exactly six factors?

Answers

1. _____
2. _____
3. _____

# Solutions to Category 3
# Number Theory
# Meet #1, October 2012

| Answers |
|---|
| 1. **24** |
| 2. **7** |
| 3. **175** |

**1.** The two primes must be 31 and 7. Their positive difference is 31 – 7 = **24**.

**2.** The 7-digit number 1,295,*N*84 is clearly even, so we only need to make sure it is divisible by 9. The sum of the known digits is 1 + 2 + 9 + 5 + 8 + 4 = 29. *N* will have to be **7** to get us to the next multiple of 9.

**3.** Numbers with exactly six factors must have a prime factorization of the form $p^5$ or $p^2 \times q$, where $p$ and $q$ are primes. The smallest multiple of 5 of the form $p^5$ is $5^5 = 3125$, which is not three digits. The other options are the forms $p^2 \times 5$ or $5^2 \times q$. The least three-digit number of the $p^2 \times 5$ form is $7^2 \times 5 = 49 \times 5 = 245$. The least three-digit number of the $5^2 \times q$ form is $5^2 \times 7 = 25 \times 7 =$ **175**, which is our desired answer.

Category 3 – Number Theory

1. Thinking of the natural numbers from 2 to (and including) 20, what is the positive difference between the sum of all composites and the sum of all primes?

2. What value of the digit $A$ will make the number $567{,}88A$ be divisible by 12?

3. Find the sum of all factors of the number 96 (including 1 and 96).

| Answers |
| --- |
| 1. _____ |
| 2. _____ |
| 3. _____ |

Solutions to Category 3 – Number Theory

| Answers | |
|---|---|
| 1. | 55 |
| 2. | 8 |
| 3. | 252 |

1. Sum of primes: $2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 = 77$

   Sum of composites:

   $4 + 6 + 8 + 9 + 10 + 12 + 14 + 15 + 16 + 18 + 20 = 132$

   The difference is $132 - 77 = 55$

2. In order to be divisible by 12, a number should be divisible both by 3 and by 4. The sum of digits of $567{,}88A$ is $(7 + A)$, so $A$ can be 2, 5, or 8 in order to make it divisible by 3. However, only a value of 8 will make it divisible by 4.

3. The factors of 96 are:

   $96 = 1 \times 96 = 2 \times 48 = 3 \times 32 = 4 \times 24 = 6 \times 16 = 8 \times 12$

   The sum therefore is:

   $1 + 2 + 3 + 4 + 6 + 8 + 12 + 16 + 24 + 32 + 48 + 96 = 252$

   Of all numbers under 100, the number 96 has the largest sum of factors.

   As preparation for meet #2 try to figure out why.