Math League SCASD
Study Packets

Meet #1

Geometry

Self-study Packet

2022-2023

Problem Categories for this Meet:

1. Mystery: Problem solving
2. Geometry: Angle measures in plane figures including supplements and complements
3. Number Theory: Divisibility rules, factors, primes, composites
4. **Arithmetic: Order of operations; mean, median, mode; rounding; statistics**
5. Algebra: Simplifying and evaluating expressions, solving equations with 1 unknown including identities

Geometry Information you need to know...

➤ **Supplements**: Two angles that add up to 180°

➤ **Complements**: Two angles that add up to 90°

➤ A **_regular_** **polygon** has all sides and all angles of equal measure.

➤ A straight angle measures 180°

➤ Angles are named with their vertex in the middle, or just by their vertex. For example,
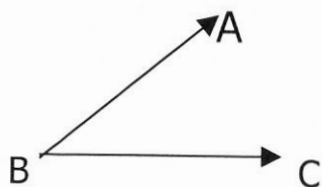
is named ∠ABC or just ∠B.

➤ **_Vertical angles_** are congruent (equal). In the diagram below, ∠ABC and ∠DBE are vertical angles. ∠ABD and ∠CBE are also vertical angles.
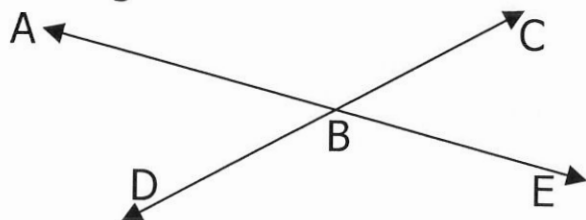
➤ If two lines are parallel, their **_corresponding angles_** are congruent. In the diagram below, corresponding angles are labeled with the same number.
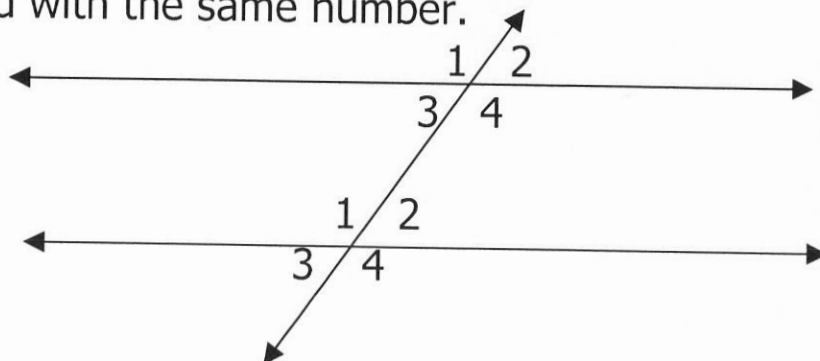
➢ Any polygon has a sum of its interior angles equal to 180(n – 2) with n being the number of sides of the polygon.

For example, A triangle has 180(3 – 2) or 180°; a quadrilateral 180(4 – 2) or 360°; a pentagon 180(5 – 2) or 540°, etc.

➢ To find the measure of an interior angle of a <u>regular</u> polygon, use the formula $\frac{180(n-2)}{n}$ with n being the number of sides of the polygon. ***Interior angles*** are the angles indicated by arcs in the following diagram.
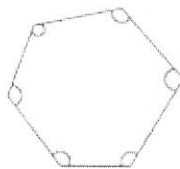
➢ To find the measure of an exterior angle of a <u>regular</u> polygon, simply divide 360 by the number of sides of the polygon. ***Exterior angles*** are the angles indicated by arcs in the following diagram. Formula: 360 ÷ n
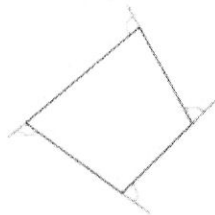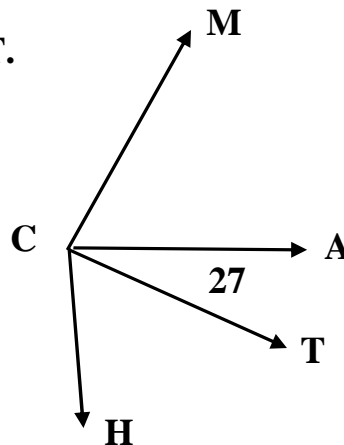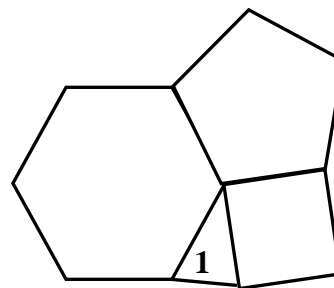
Category 2
Geometry
Meet #1 - November, 2020

1) Lines L1 and L2 intersect. Angle 1 measures 117 degrees. The measure of angle 3 is half the measure of angle 2. How many degrees are in the measure of angle 2 ?
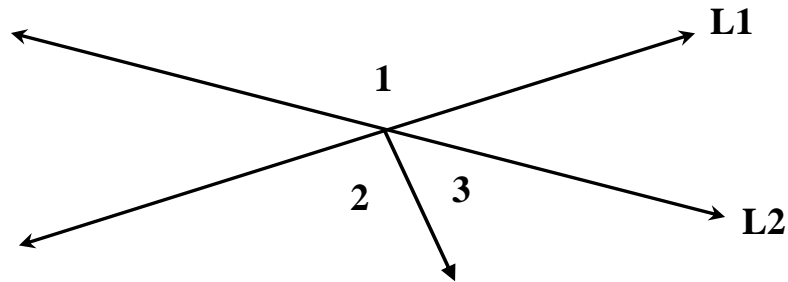
2) Angle MCA is complementary to angle ACT.
Angle ACT is complementary to angle TCH.
Angle ACT measures 27 degrees.
How many degrees are in angle MCH if
its measure is greater than 180 degrees?

3) A regular hexagon and a regular pentagon share a side. The pentagon shares a different side with a square. How many degrees are in the measure of angle 1 ?

Answers

1) _____

2) _____

3) _____

Solutions to Category 2
Geometry
Meet #1 - November, 2020

1) **When two lines intersect in a plane, the opposite, or "vertical" angles, are congruent and have the same measure. Since angle 1 measures 117 degrees, then the combined sum of angles 2 and 3 is also 116 degrees. Angle 3 is half of angle 2, so   X + 2X = 117**
$$3X = 117$$
$$X = 39 \text{ and } 2X = 78.$$
Therefore, the measure of angle 2 is 78 degrees.

2) **Angle MCH has two measure - one less than 180 degrees and one greater than 180 degrees, such that their sum is 360 degrees. The smaller of the two angles is the sum of right angles MCT and ACH, minus 27 degrees. 90 + 90 - 27 = 153 degrees. So, the larger angle MCH is 360 - 153, or 207 degrees.**

3) **Since the triangle is isosceles, as every side length in the diagram is the same, angle 1 is equal to the other base angle. We just need to find the measure of the remaining angle.**
Each measure of the regular hexagon is (4)(180) / 6, or 120 degrees.
Each measure of the regular pentagon is (3)(180) / 5. or 108 degrees.
Each measure of the square is 90 degrees.
At the center point where the polygons meet, let X = the measure of the apex of the isosceles triangle. Then
$$120 + 108 + 90 + X = 360$$
$$318 + X = 360$$
$$X = 42$$
So, the apex angle of the triangle measures 42 degrees and the sum of the base angles is 180 - 42, or 138 degrees. The two base angles are congruent, so angle 1 measures half of 138, or 69 degrees.

Category 2
Geometry
Meet #1 - October, 2018
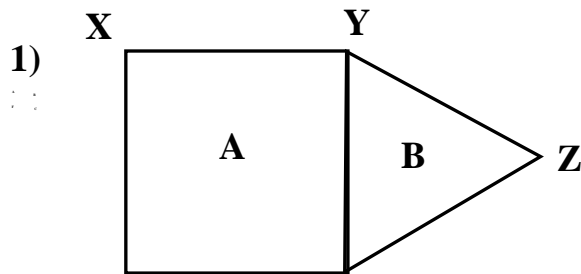
1)

X   Y

A   B   Z

Figure A is a square and figure B is an equilateral triangle. How many degrees are in exterior angle XYZ if its measure is greater than 180 degrees? The square and triangle share a common side.

2) The two horizontal lines are parallel. Segment AD bisects angle EAF. Angle CAE measures 72 degrees. How many degrees are in angle ADG if its measure is less than 180 degrees?

C
A
E
F   D   G
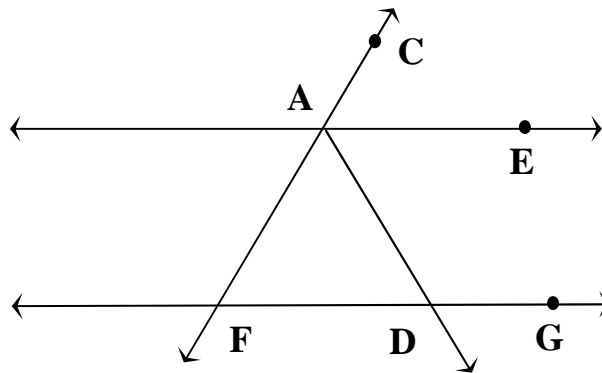
3) The horizontal lines are parallel. How many degrees are in the complement of acute angle A? The measures of two obtuse angles are given in the diagram.

137

106

A

Answers

1) _____

2) _____

3) _____
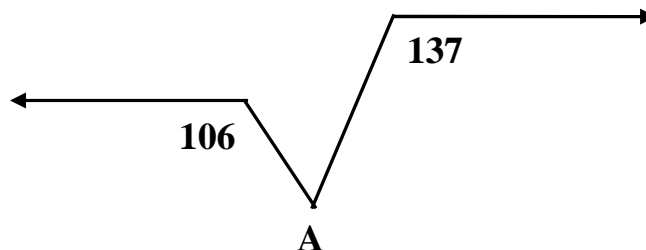
Solutions to Category 2
Geometry
Meet #1 - October, 2018

1) **One interior angle of the square measures 90 degrees. One interior angle of the triangle measures 60 degrees. The combined interior angle XYZ measures 90 + 60, or 150 degrees. So, the exterior angle XYZ measures 360 - 150, 210 degrees.**

<table>
<tr><td colspan="2" align="center"><u>Answers</u></td></tr>
<tr><td>1)</td><td>210</td></tr>
<tr><td>2)</td><td>126</td></tr>
<tr><td>3)</td><td>27</td></tr>
</table>

2) **Since angle CAE measures 72 degrees, then so does angle AFD, as alternate interior angles are congruent. Angle FAE is the supplement of each of those angles and measures 180 - 72, or 108 degrees. Since AD bisects angle FAE, then each of the half-angles measures half of 108 degrees, or 54 degrees each. Angle FDA is the remaining as-yet unmeasured angle of triangle FDA. Since the sum of the measures of a triangle is 180 degrees, then angle FDA measures 180 - (72 + 54), or 54 degrees. Its supplement, angle ADG - the angle requested in the question - measures 180 - 54, or 126 degrees.**

3) **Drawing an auxiliary line through the lowest point in the diagram, parallel to the two horizontal lines, can be helpful. The two interior angles on the left are supplementary, as are the two on the right, hence the meas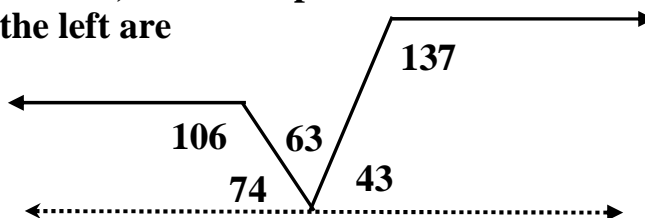ures of 74 and 43 degrees in the diagram. Angle A, combined with the 74 and 43 degree angles, form a straight line. Angle A = 180 - (74 + 43), or 180 - 117, or 63 degrees. The question asks for the complement of angle A, so 90 - 63 = 27 degrees.**

137

106 63

74 43

Category 2
Geometry
Meet #1 - October, 2016

For all problems below, angles are
marked in degrees as shown. *Figures
are not necessarily drawn to scale.*

1) Two horizontal lines are parallel.
   Ray BD bisects angle ABC. How
   many degrees are in the measure
   of acute angle CBD?

42

B   C

A
   D
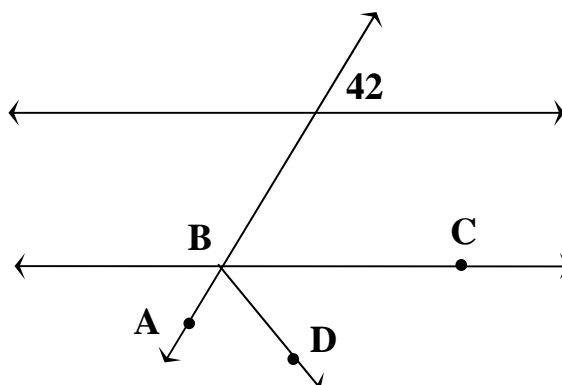
2) Angle 1 is the complement of
   angle 2. MATH is a rectangle.
   Angle 3 measures 57 degrees.
   How many degrees are in the
   measure of angle NLT ?

M   A
     1   2
   3
              N

H
            L   T
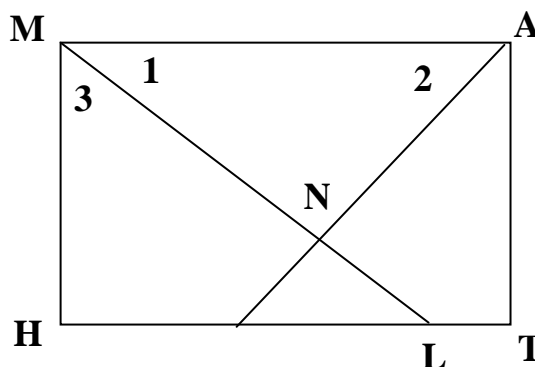
3) Ray FE is parallel to ray JK and to ray HM. Angle 2 is twice the
   measure of angle 1. How many degrees are in the measure of acute
   angle L if angle LMN = 41 degrees, angle 4 = 38 degrees, and
   angle 5 = 116 degrees?

K

J
                    1
E   F   2
                         L
   5

   H
      4   M   N

   G

Answers

1) _____

2) _____

3) _____
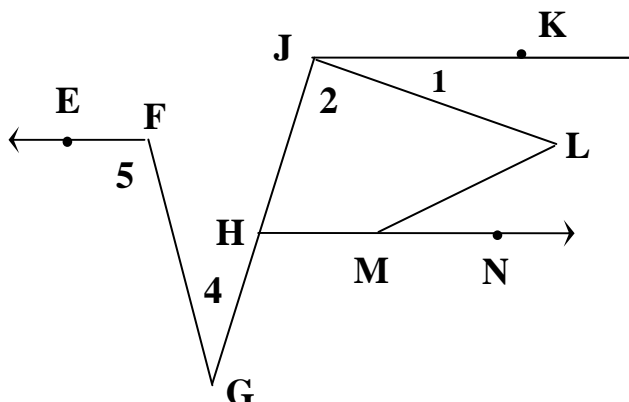
Solutions to Category 2
Geometry
Meet #1 - October, 2016

1) **Angle ABC is the supplement of the 42-degree angle**
   and equals 138 degrees. Bisecting angle ABC yields
   a pair of 69-degree angles.

2) **By definition, each vertex of the rectangle measures**
   90 degrees. Angles 3 and 1 are complementary, as are
   angles 1 and 2 (given), so angles 3 and 2 are equal.
   Angle 3 = 57 degrees, so does angle 2, therefore angle
   **NAT = 33 degrees. Now, referring to quadrilateral**
   NATL: Angle ANL + angle NAT + angle T + angle NLT = 360
   90 + 33 + 90 + angle NLT = 360
   213 + angle NLT = 360
   angle NLT = 147

+---------------------+
| **Answers** |
| |
| **1)   69** |
| |
| **2) 147** |
| |
| **3)   75** |
+---------------------+

3) **The key to solving this puzzle lies in the drawing of two lines, each**
   parallel to the given parallel lines, passing through the points G
   and L. Using the following properties of parallel lines being cut by
   transversals produces these properties and allow for filling in the
   diagram with the given angle measures: 1) collinear angles have a
   sum of 180 degrees, and 2) alternate interior angles are congruent,
   and 3) same side interior angles are supplementary.
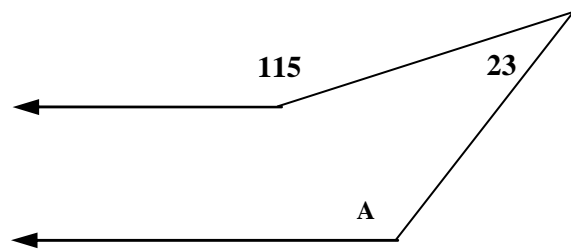   **Angle L = 34 + 41, or 75 degrees.**
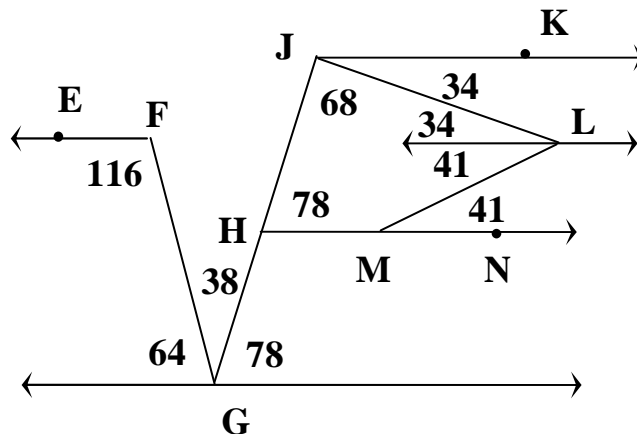
For all problems below, angles
are marked in degrees as shown.
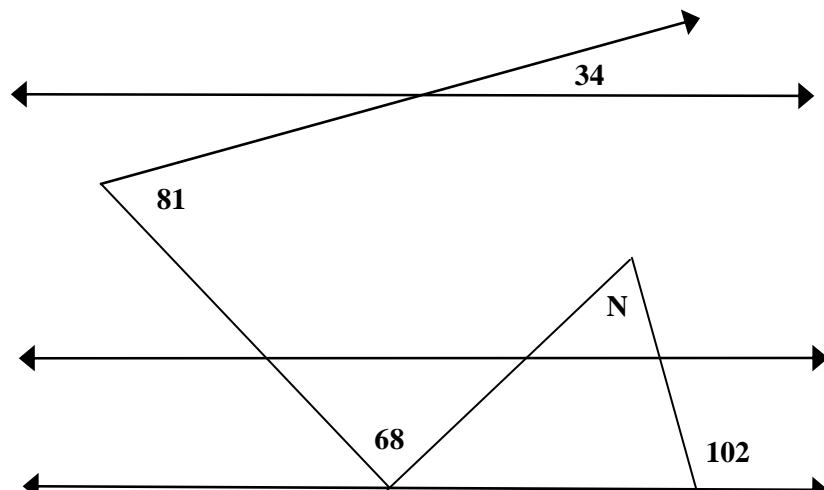*Figures are not necessarily drawn
to scale.*

1) **In the figure at the right, how
   many degrees are in the
   complement of angle W ?**

72

W
127
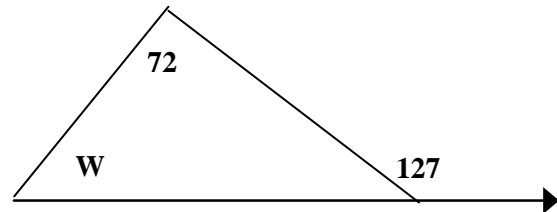
2) **The two horizontal lines are
   parallel. How many degrees
   are in angle A if it is less
   than 180 degrees ?**

115
23
A

3) **All three horizontal lines are parallel. How many degrees are in the
   supplement of angle N ?**

34

81

N

68
102

Answers

1) _____

2) _____

3) _____

Solutions to Category 2
Geometry
Meet #1 - October, 2014

1) The supplement of the 127-degree angle is (180 - 127), or 53 degrees and lies in the interior of the triangle. The remaining angle, W is (180 - (72 + 53)), or 55 degrees, as the sum of the angles of a triangle is 180 degrees. The question, however, asks for the *complement* of angle W. So, 90 - 55 = 35 degrees.

Answers
1)   35
2)   92
3)   143

2) **Strategy: Extend the parallel lines to the right, creating angles with the far-right slanted line segment that serves as a transversal.** The triangle in the upper-right of the figure has three angles: B, C, and 23. Angle B is the supplement of 115 and measures 65 degrees. Angle C = 180 - (65 + 23), or 92 degrees. Angle A is equal to angle C (corresponding angles). Therefore, angle A = 92 degrees.
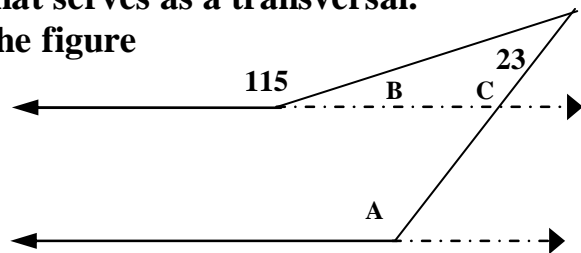
3) **Strategy: draw a line through the vertex of the 81 degree angle that is parallel to the three horizontal parallel lines.** The 81 degree angle is now subdivided into two angles: A = 34 (corresponding angles are equal) and B = 47 (81 - 34 = 47).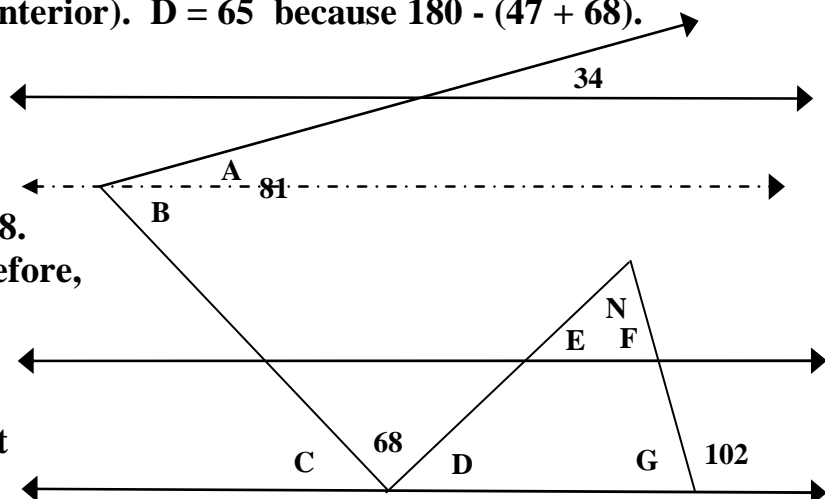 B = C = 47 (alternate interior). D = 65 because 180 - (47 + 68). E corresponds to D = 65. G i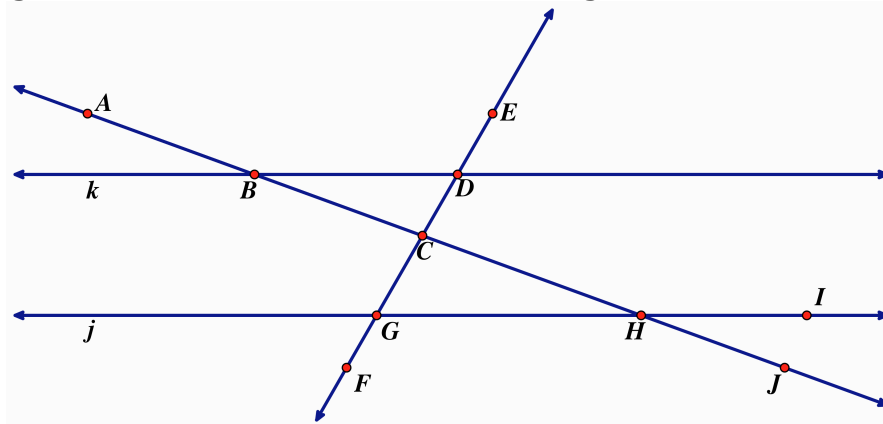s the supplement of 102, so G = 78. F corresponds to G = 78. N + E + F = 180. Therefore, N = 180 - (65 + 78) = 180 - 143 = 37. Finally, the supplement of N is (180 - 37) = 143.
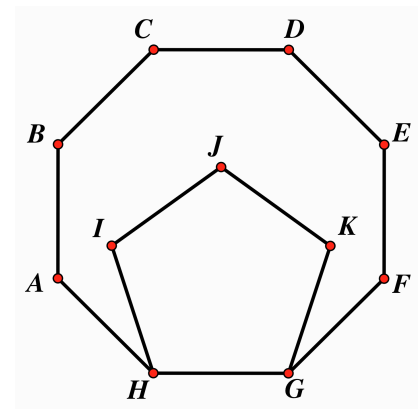
Category 2
Geometry
Meet #1, October 2012

1. In the diagram below, lines k and j are parallel. The measure of angle IHJ is 20 degrees and the measure of angle DCH is 80 degrees. How many degrees are there in the measure of angle CGH?

2. The supplement of a certain angle is 5 more than 6 times the angle itself. How many degrees are there in the measure of the complement of this angle?

3. In the figure at right, regular pentagon GHIJK is constructed inside regular octagon ABCDEFGH so that the two polygons share side GH. If line segments BC and IJ are extended so that they meet at a point X, how many degrees are there in the measure of angle BXI?

Answers	
1. _____degrees	
2. _____degrees	
3. _____degrees	

Solutions to Category 2
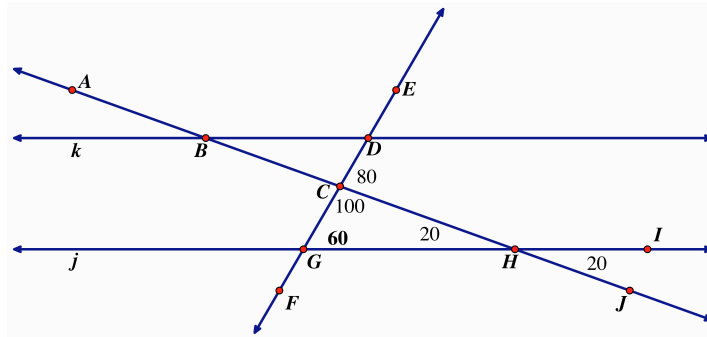# Geometry
Meet #1, October 2012

1. Angle CHG = 20 degrees (vertical angles). Angle GCH = 100 degrees (supplementary angles). Thus angle CGH = 180 – 100 – 20 = **60 degrees**.

2. Translating the words to algebra, we can write the equation 180 – x = 6x + 5. This simplifies to 7x = 175, so x = 25. The complement of x is therefore 90 – 25 = **65 degrees**.

3. Imagine that you are taking a walk along the path CBAHIJ. You will take two left turns of 45 degrees, one left turn of 45 + 108 = 153 degrees and one right turn of 72 degrees. The end result is that you have turned 45 + 45 + 153 – 72 = 171 degrees, which is 9 degrees less than 180 degrees. There must be **9 degrees** in the measure of angle BXI.
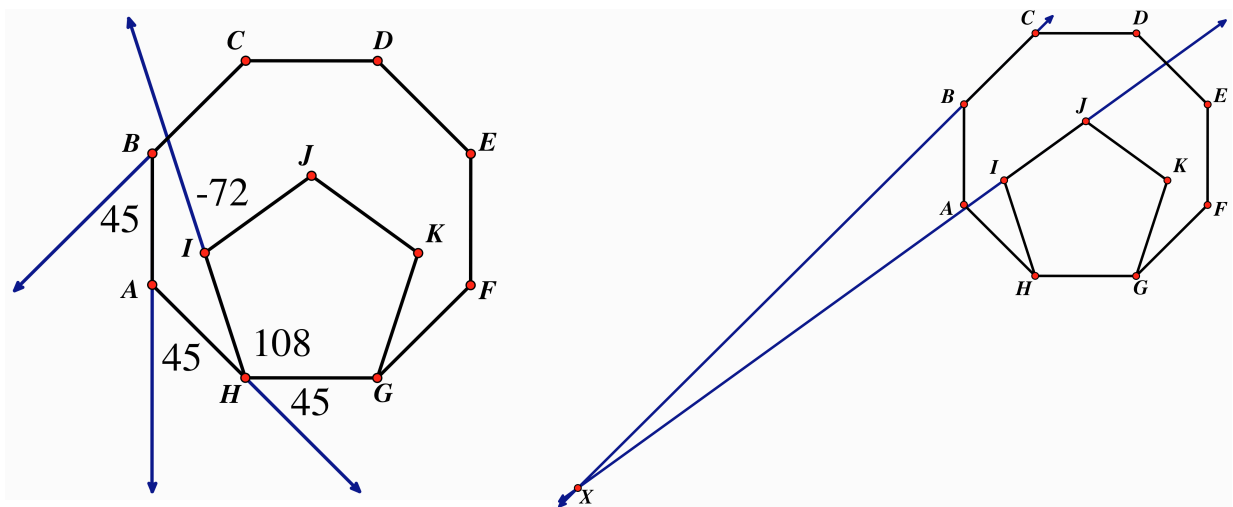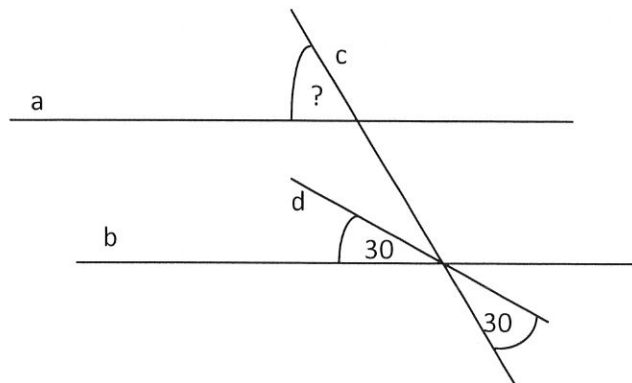
Category 2 – Geometry

1. How many degrees are in the measure of angle ∠*CED*?
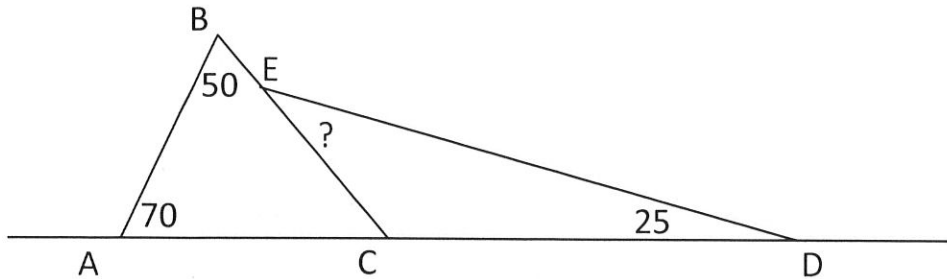
2. In the drawing below, parallel lines **a** and **b** are both intersected by line **c**. Line **d** passes through the intersection of lines **b** and **c**. Two of the angles are given. How many degrees are in the angle marked by a question mark?

3. How many degrees are in the angle between the hour-hand and the minute-hand of an analog clock when the time is 2:20?
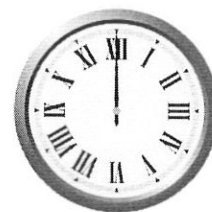
Answers
1. _____
2. _____
3. _____

Solutions to Category 2 – Geometery

	Answers
1.	35
2.	60
3.	50

1. To complete the angles in triangle ABC to 180 degrees, the angle $\angle ACB$ must measure 60 degrees. $\angle ECD$ is its supplement, and so measures 120 degrees. Lastly, $\angle CED$ must complete the angles in triangle ECD to 180 degrees, and so will measure $180 - 120 - 25 = 35$ degrees.

2. The angle in question must equal $(x + 30)$ degrees, as **a** and **b** are parallel (Correspoinding angles). x must equal 30 since it is opposite the 'bottom' 30 degree angle in the diagram (so the line **d** bisects the angle between **b** and **c**). So $x + 30 = 60$.
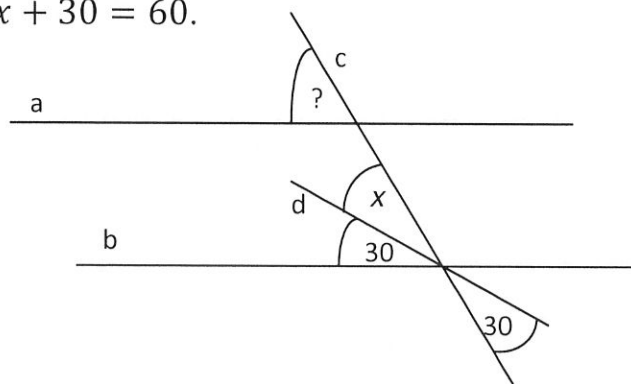
3. Since the dial of a clock is divided to 12 hours, there are $\frac{360}{12} = 30$ degrees between neigboring hours. At 2:20, the minute-hand points to 4, and the hour-hand should be a third of the way between 2 and 3. So the angle between them is equal to $1\frac{2}{3}$ hours or $1\frac{2}{3} * 30 \ degrees = 50 \ degrees$.