2024 AP Calculus SUMMER PACKET

DUE SEPTEMBER 4, 2024

NORTH PLAINFIELD HIGH SCHOOL

YOUR NAME: _____ **GRADE:** $\frac{(\%\ COMPLETE\ +\ \%\ CORRECT)}{2} =$ _____

DEAR STUDENT,

The purpose of the summer packet is to make sure you are prepared with the prerequisite skills necessary to be successful in your AP Calculus course.

Page 2 of this packet lists the skills and shows the rubric your teacher will use to grade your packet. Make sure you read both carefully so that you start off the year on the right foot.

Follow these guidelines while completing the assignment:

- You will not receive credit unless you SHOW YOUR WORK!
- Do NOT use a calculator! The purpose of the packet is to sharpen your skills, not how to use a calculator.
- If you do not remember how to solve a problem, go to Khan Academy (https://www.khanacademy.org/) and watch a lesson on the topic.
- A great website to practice math skills https://www.ixl.com/signin/nplainfield.

This packet is due to your math teacher on **September 4, 2024.** If it is turned in late, you will lose points as noted in the grading rubric. If it is not turned in by Friday, September 7th, you will receive a 0. **There will be a review of this material along with a quiz upon your return to school in September.**

If you need a paper copy of this packet, they are available in the NPHS central office during summer hours (Monday-Thursday 8am - 12pm and 1pm - 2pm).

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MATH SUPERVISOR, K-12
NORTH PLAINFIELD HIGH SCHOOL

Grading rubric: 100 point quiz grade (your first grade of the year in math!!)

Your score will be graded using this formula: $\frac{(\%\ complete\ +\ \%\ correct)}{2}$

The completion percent only includes problems with all work shown. An additional 10 points will be deducted for each day your packet is late.

THIS IS AN IMPORTANT ASSIGNMENT. BE SMART and start off your year with your best effort!!!

Special Cases:

- If you are absent in the first 3 days of school, you are allowed 1 day per day absent to turn in your packet.
- If you start NPHS in September, you have 2 weeks after your first day in school to turn this packet into your math teacher.
- If you transfer from one course to another in September, your summer packet score from your original class will transfer with you.
- If you start NPHS after September, you are exempt from completing this packet.

NORTH PLAINFIELD
AP Calculus Summer Assignment

Name: _____ Class: _____

Please show your work to all problems on separate paper. Check that your solution is the same as given on the assignment paper. Put a question mark on the question(s) that your answer is not the same as given on the paper. Be ready to explain your solutions to all the questions that you got right. No work equals no points.

1. The horsepower H required to overcome wind drag on a certain automobile is approximated by

$$H(x) = 0.005x^2 + 0.003x - 0.025, 10 \leq x \leq 100,$$

where x is the speed of the car in miles per hour. Find $H\left(\frac{x}{1.8}\right)$. Round the numerical values in your answer to five decimal places.

2. Students in a lab measured the breaking strength S (in pounds) of wood 2 inches thick, x inches high, and 12 inches long. The results are shown in the table below. Use a graphing utility to plot the data and graph the quadratic model.

x	4	6	8	10	12
S	2370	4460	9310	13,250	24,860

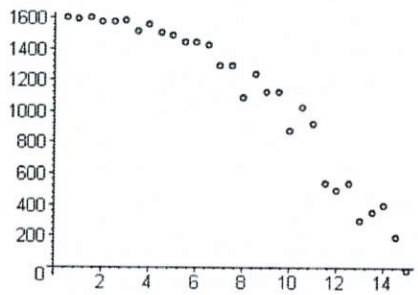
3. A company reimburses its sales representatives \$185 per day for lodging and meals plus 45¢ per mile driven. Write a linear equation giving the daily cost C to the company in terms of x, the number of miles driven.

4. Sketch the graph of the equation $x = 4 - y^2$.

5. Let $g(x) = \frac{1}{\sqrt{x+6}}$. Evaluate the expression $\frac{g(x) - g(10)}{x - 10}$ and then simplify the result.

6. Find an equation of the line through the points of intersection of $y = x^2$ and $y = 8x - x^2$.

7. Which function below would be the most appropriate model for the given data?

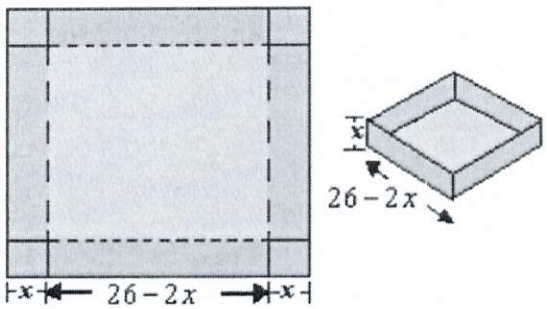


A) trigonometric
B) linear
C) no apparent relationship between x and y
D) quadratic

8. Write the equation of a line whose slope is $m = -7$ and passes through the point (2, 6). Then find another point which lies on the line.

9. A real estate office handles an apartment complex with 50 units. When the rent is \$840 per month, all 50 units are occupied. However, when the rent is \$885, the average number of occupied units drops to 47. Assume that the relationship between the monthly rent p and demand x is linear. Predict the number of units occupied if the rent is raised to \$975.

10. An open box of maximum volume is to be made from a square piece of material 26 centimeters on a side by cutting equal squares from the corners and turning up the sides (see figure). Write the volume V as a function of x, the length of the corner squares.



11. Test for symmetry with respect to each axis and to the origin.

$$y = \frac{x^2 + 9}{x}$$

12. Find the sales necessary to break even ($R = C$) if the cost C of producing x units is $C = 5.2\sqrt{x} + 50,000$ and the revenue R for selling x units is $R = 3.29x$. Round your answer to the nearest integer.

13. Find all intercepts:
$y = x^2 - 5x - 24$

14. Write an equation of the line that passes through the given point and is parallel to the given point.

Point	Line
(-1,7)	$2x + y = 18$

15. Specify a sequence of transformations for the function $h(x) - \sin\left(x + \frac{\pi}{3}\right) + 2$ that will yield the graph of h from the graph of the function $f(x) = \sin x$.

A) The function $h(x) = \sin\left(x + \frac{\pi}{3}\right) + 2$ is a vertical shift $\frac{\pi}{3}$ units downwards, followed by a horizontal shift 2 units to the right.

B) The function $h(x) = \sin\left(x + \frac{\pi}{3}\right) + 2$ is a horizontal shift $\frac{\pi}{3}$ units to the left, followed by a vertical shift 2 units upwards.

C) The function $h(x) = \sin\left(x + \frac{\pi}{3}\right) + 2$ is a horizontal shift $\frac{\pi}{3}$ units to the right, followed by a vertical shift 2 units downwards.

D) The function $h(x) = \sin\left(x + \frac{\pi}{3}\right) + 2$ is a vertical shift $\frac{\pi}{3}$ units upwards, followed by a horizontal shift 2 units to the left.

E) The function $h(x) = \sin\left(x + \frac{\pi}{3}\right) + 2$ is a horizontal shift $\frac{\pi}{3}$ units to the left, followed by a horizontal shift 2 units to the right.

16. Determine whether the function is even, odd or neither.
$f(x) = x^2(3 - x)^2$

17. Find the domain and range of the function $h(x) = \dfrac{2}{x+1}$.

18. Test for symmetry with respect to each axis and to the origin.
$x^2y^2 = 6$

19. Each ordered pair gives the exposure index x of a carcinogenic substance and the cancer mortality y per 100,000 people in the population. Use the model $y = 9.2x + 108.4$ to approximate y if $x = 6$. Round your answer to one complete decimal place.

(3.50, 150.1), (3.58, 133.1), (4.42, 132.9), (2.26, 116.7), (2.63, 140.7), (4.85, 165.5),

(12.65, 210.7), (7.42, 181.0), (9.35, 213.4)

20. Write an equation of the line that passes through the given point and is perpendicular to the given line.

Point	Line
(-8,-9)	$x = -1$

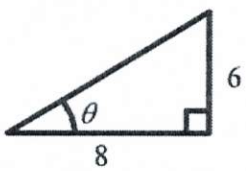
21. Find the slope of the line passing through the pair of points.
(6, 9), (9, -11)

22. Evaluate (if possible) the function $f(x) = \sqrt{x-4}$ at $x = 3$. Simplify the result.

23. Use the result, "the line with intercepts (a, 0) and (0, b) has the equation $\dfrac{x}{a} + \dfrac{y}{b} = 1,\ a \neq 0, b \neq 0$", to write the equation of the line with x-intercept: (2, 0) and y-intercept: (0, 5).

24. Use the figure below to find the exact value of the given trigonometric expression.

$\sin \frac{\theta}{2}$



(figure not necessarily to scale)

25. Write the equation of the line that passes through the point $\left(\frac{5}{4}, \frac{7}{8}\right)$ and is parallel to the line $5x - 7y = 0$.

26. Determine the domain and range of the function

$$f(x) = \begin{cases} 4x + 2, & x < 0, \\ 4x + 4, & x \geq 0. \end{cases}$$

27. Let $f(x) = -4x + 7$. Then simplify the expression $\dfrac{f(x) - f(-2)}{x+2}$.

28. Solve the following equation.
$2\cos x - 1 = 0$

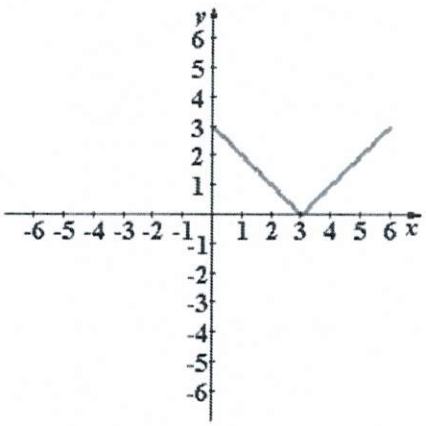
29. Given $f(x) = \cos x$ and $g(x) = \frac{\pi}{x}x$, evaluate $f(g(1))$.

30. Find the coordinates of a second point on the graph of a function f if the given point $\left(-\frac{9}{8}, 3\right)$ is on the graph and the function is even.

31. Evaluate (if possible) the function $g(x) = x^2(x-3)$ at $x = t - 6$. Simplify the result.

32. Determine whether y is a function of x.
$xy - x^2 = 4y + x$

33. Use the graph of $y = f(x)$, given below to find the graph of function $y = f(x+5) - 1$.



34. Find the distance between the point (-14, 3) and line $x - y - 2 = 0$, using the formula,
Distance $= \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$ for the distance between point (x_1, y_1) and the line $Ax + By + C = 0$.

A) $\frac{17\sqrt{2}}{2}$

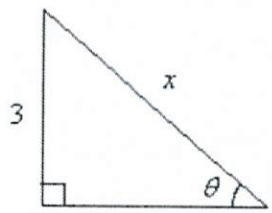
B) $\frac{16\sqrt{3}}{3}$

C) $\frac{19\sqrt{2}}{2}$

D) $\frac{14\sqrt{3}}{3}$

E) $\frac{5\sqrt{2}}{2}$

35. Use an inverse function to write θ as a function of x.



36. Given $A = 96°, b = 9$, and $c = 5$, use the Law of Cosines to solve the right triangle for the value of a. Round answers to two decimal places.

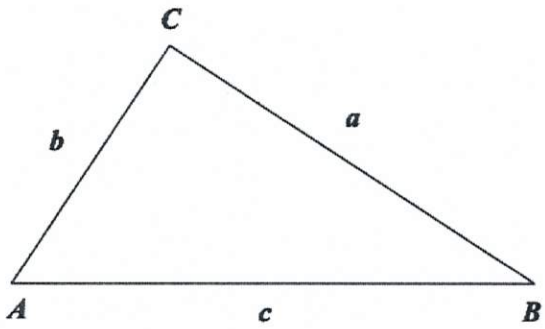
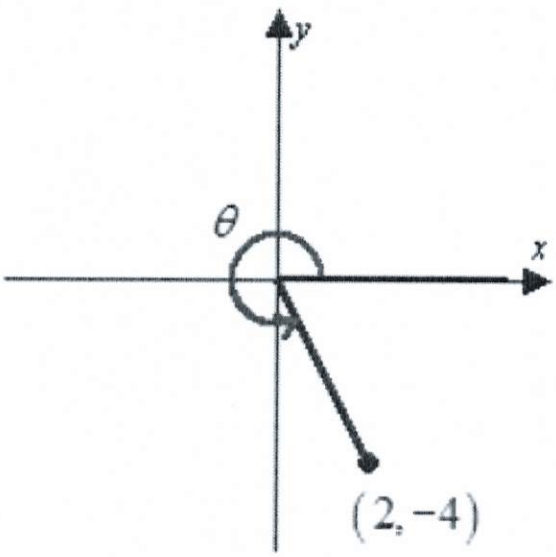


Figure not drawn to scale

37. A moving conveyor is built to rise 8 meters for every 9 meters of horizontal change. Find the slope of the conveyor.

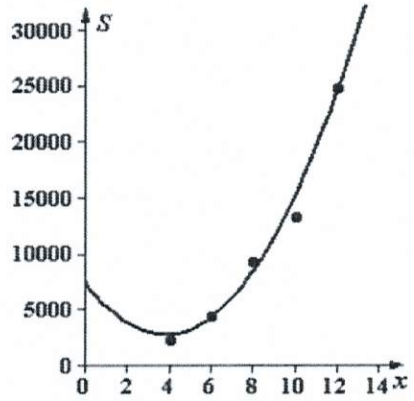
38. Using the figure below, determine the exact value of the given trigonometric function.

$\sec\theta$

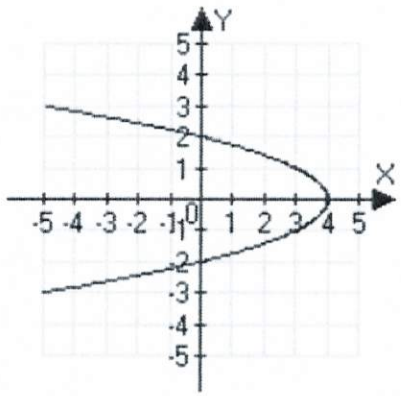


Answer Key

1. $H\left(\frac{x}{1.8}\right) = 0.00154x^2 + 0.00167x - 0.02500$

2. 

3. $C = 0.45x + 185$

4. 

5. $\dfrac{4\sqrt{x+6} - x - 6}{4(x-10)(x+6)}$

6. $y = 4x$

7. Quadratic

8. $y = -7x + 20$; Point answer varies.

9. 41 units

10. $V = x(26 - 2x)^2$

11. Symmetric with respect to the origin

12. $x \approx 15,394$ units

13. x-intercepts: (-3, 0), (8, 0); y-intercept: (0, -24)

14. $2x + y = 5$

15. B

16. neither

17. domain: $(-\infty, -1) \cup (-1, \infty)$ range: $(-\infty, 0) \cup (0, \infty)$

18. Symmetric to x-axis, y-axis and origin

19. 163.6

20. $y = -9$

21. $-\frac{20}{3}$

22. undefined

23. $5x + 2y - 10 = 0$

24. $\frac{\sqrt{10}}{10}$

25. $40x - 56y - 1 = 0$

26. domain: $(-\infty, \infty)$ range: $(-\infty, 2) \cup [4, \infty)$

27. -4

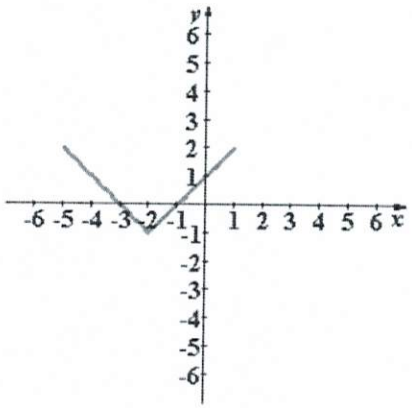
28. $x = \frac{\pi}{3} + 2n\pi$ and $x = \frac{5\pi}{3} + 2n\pi$, where n is an i

29. -1

30. $\left(\frac{9}{8}, 3\right)$

31. $t^3 - 21t^2 + 144t - 324$

32. yes

33. 

34. C

35. $\theta = \arcsin \frac{3}{x}$

36. 10.74

37. $\frac{8}{9}$

38. $\sqrt{5}$