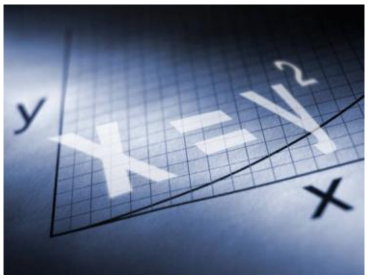
Amity Mathematics Department
Summer Math Packet
Incoming AP Calculus Students



~ (Σ er) (Force*Distance) and ($L_1, L_2, \ldots\ldots L_n$) of Topical Understandings ~

Directions

Attached is the Summer Review Packet for AP Calculus containing many of the prerequisite skills from Algebra and Pre-Calculus which we will use in calculus and with which you should be familiar.

Do all work neatly on the packet. The packet will be due the **first day back to school**. It will be collected and graded and count will as the first Problem Set for the course. Best of luck and if you have any questions, feel free to contact us.

We are looking forward to seeing you in September.

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The importance of a solid foundation in algebra cannot be over-emphasized. Algebra is used throughout Calculus as it is in all subsequent courses. The student cannot be expected to grasp the new concepts encountered if s/he is deficient in this important basic tool. The following are some of the skills with which a student should be very familiar.

1. Manipulate algebraic expressions involving exponents and radicals
2. Manipulate algebraic fractions
3. Factor algebraic expressions
4. Solve equations through quadratics; complete the square in a quadratic expression
5. Solve simultaneous equations
6. Solve "word problems", i.e., translate words into algebraic expressions
7. Functions and graphs (rectangular coordinates)
8. Solve inequalities
9. Find roots of polynomials using synthetic division
10. Use the binomial theorem
11. Manipulate complex numbers
12. Manipulate logarithmic expressions, graph logarithmic functions and solve logarithmic equations
13. Graph exponential functions and solve exponential equations
14. Find equations of straight lines and conic sections
15. Determine inverse functions

A reference sheet of these skills & concepts can be found here:
http://tutorial.math.lamar.edu/pdf/Algebra_Cheat_Sheet.pdf

Trigonometry also plays an important role in calculus and is used throughout this course. The student must know trigonometry in order to be successful in the course. In particular, the student should be familiar with the following:

1. Fundamental definitions
2. Basic identities
3. Application of basic identities to the solution of trigonometric equations and proving identities
4. Graphing of trigonometric functions
5. Radian measure
6. Graphing of the inverse trigonometric functions

A reference sheet of these skills & concepts can be found here:
http://tutorial.math.lamar.edu/pdf/Trig_Cheat_Sheet.pdf

1. Simplify:

a. $(4a^{5/3})^{3/2}$

b. $(5a^{2/3})(4a^{3/2})$

c. $e^{\ln 3}$

d. $\ln 1$

e. $e^{(1+\ln x)}$

f. $\ln e^7$

g. $\log_{\frac{1}{2}} 8$

h. $e^{3\ln x}$

i. $\dfrac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$

j. $\dfrac{9-x^{-2}}{3+x^{-1}}$

k. $2\log(x-3)+\log(x+2)-6\log x$

l. $\dfrac{\frac{2}{x}-3}{1-\frac{1}{x-1}}$

m. $x(1-2x)^{-\frac{3}{2}}+(1-2x)^{-\frac{1}{2}}$

n. $\dfrac{4}{\sqrt{3}+\sqrt{5}}$

o. $\dfrac{x-4}{x^2-3x-4}$

p. $\dfrac{5-x}{x^2-25}$

q. $\dfrac{\frac{2}{x^2}}{\frac{10}{x^5}}$

r. $\dfrac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}}$

s. $x^{\frac{3}{2}}(x+x^{\frac{5}{2}}-x^2)$

t. $\dfrac{1}{x+h}-\dfrac{1}{x}$

u. $\dfrac{2x}{x^2-6x+9} - \dfrac{1}{x+1} - \dfrac{8}{x^2-2x-3}$

2. Using the point-slope form $y - y_1 = m(x - x_1)$, write an equation for the line

a. with slope -2, containing the point (3, 4)

b. containing the points (1, -3) and (-5, 2)

c. with slope 0, containing the point (4, 2)

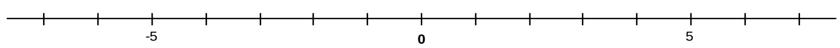
d. parallel to 2x – 3y = 7 and passes through (5, 1)

e. perpendicular to the line in (a.), containing the point (3, 4)

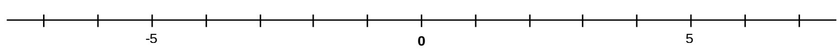
f. passes through (3, -4) and is perpendicular to the line $2x - 3y = 5$.

3. Solve the following inequalities and graph the solutions on a number line.

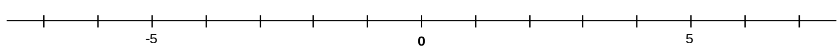
a. $5 < 2x + 7 < 12$



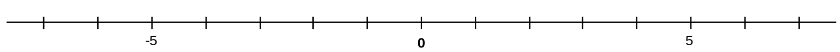
b. $1 - 3x \leq 10$



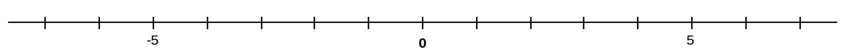
c. $|2x+5| < 7$



d. $|3x-4| > 8$



e. $x^3 - 6x^2 + 8x \leq 0$



4. Without a calculator, determine the exact value of each expression.

a. $\sin 0$

b. $\sin \frac{\pi}{2}$

c. $\sin \frac{3\pi}{4}$

d. $\cos \pi$

e. $\cos \frac{7\pi}{6}$

f. $\cos \frac{\pi}{3}$

g. $\tan \frac{7\pi}{4}$

h. $\tan \frac{\pi}{6}$

i. $\mathrm{Tan}^{-1}\, 1$

j. $\tan \frac{\pi}{2}$

k. $\cos(\mathrm{Sin}^{-1} \frac{1}{2})$

l. Sin^{-1}(sin $\frac{7\pi}{6}$)

5. Factor completely

a. $3x^3 + 192$

b. $2x^3 - 11x^2 + 12x + 9$

c. $4x^4 - 64$

d. $9x^2 - 3x - 2$

6. Solve the following.

a. $4t^3 - 12t^2 + 8t = 0$

b. $3\sqrt{x-2} - 8 = 8$

c. $\log x + \log(x-3) = 1$

d. $\frac{x-5}{3-x} \geq 0$

e. $\left|2 - \frac{x}{3}\right| < 5$

f. $4e^{2x} = 5$

g. $(x-4) - 5(x-4)^{1/2} = 6$

h. $2\sin^2 x = \sin x + 1; \quad 0 \leq x \leq 2\pi$

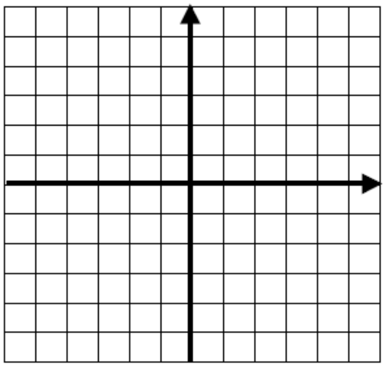
i. $27^{2x} = 9^{x-3}$

j. $(x+1)^2(x-2) + (x+1)(x-2)^2 = 0$

k. $4\sin(x-\frac{\pi}{2})+1=-3$

7. Sketch the graph of each function. Give the domain and range.

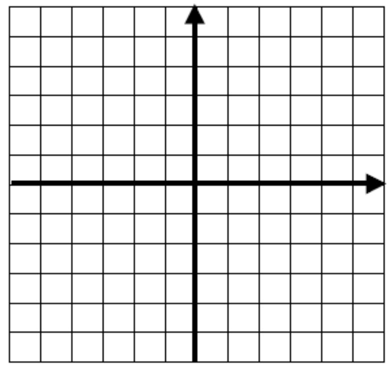
a. The square function, $y = x^2$



Domain _______________

Range _______________

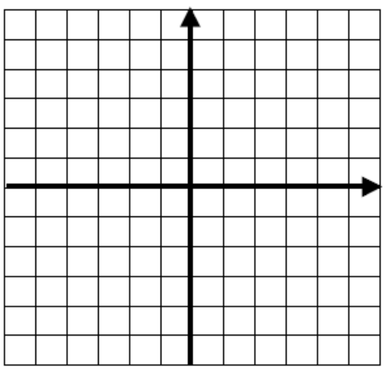
b. The cube function, $y = x^3$



Domain _______________

Range _______________

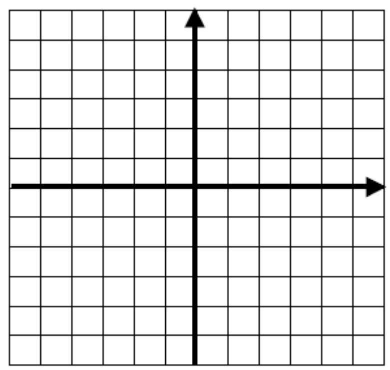
c. The square root function, $y=\sqrt{x}$



Domain _______________

Range _______________

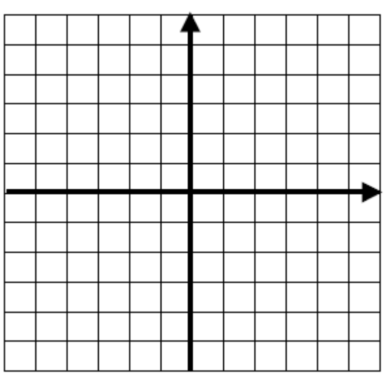
d. The absolute value function, $y=|x|$



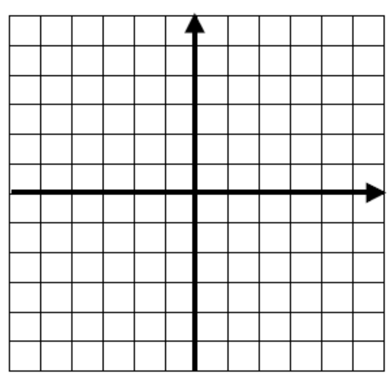
Domain _______________

Range _______________

e. The identity function, $y = x$



f. The cube root function, $y=\sqrt[3]{x}$



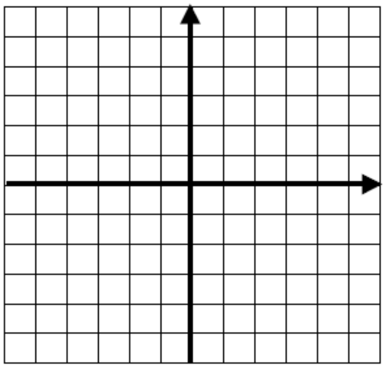
Domain _______________

Range _______________

Domain _______________

Range _______________

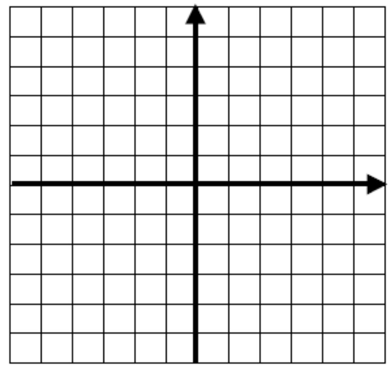
g. The natural log function, $y = \ln x$



Domain _______________

Range _______________

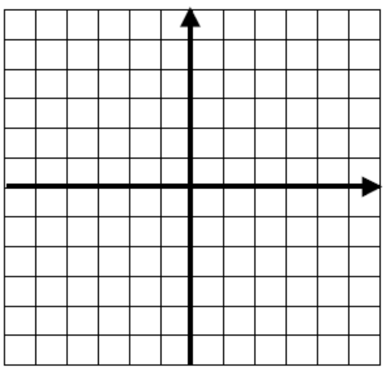
h. The exponential function, $y = e^x$



Domain _______________

Range _______________

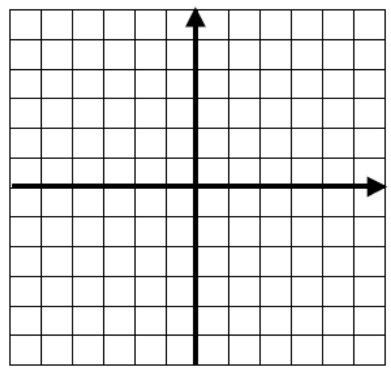
i. The sine function, y = sin x



Domain _______________

Range _______________

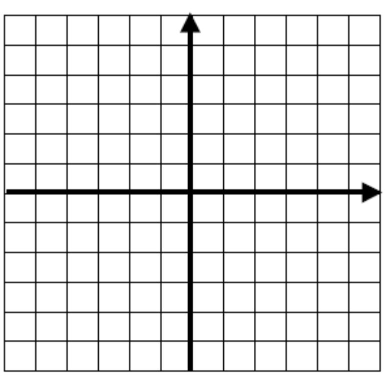
j. The cosine function, y = cos x



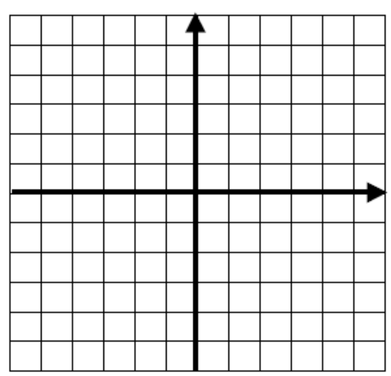
Domain _______________

Range _______________

k. The tangent function, y = tan x



l. The inverse function, $y = \frac{1}{x}$



Domain ________________ Domain ________________

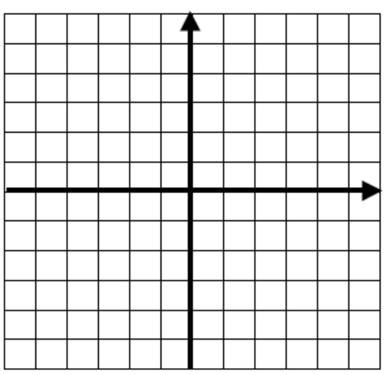
Range ________________ Range ________________

8. For each function, determine its domain and range.

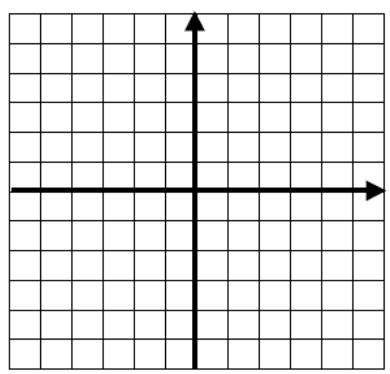
Function	Domain	Range
a. $y=\sqrt{x-4}$	____________________	____________________
b. $g(x)=\dfrac{x-2}{x^2-4}$	____________________	____________________
c. $y=\sqrt{4-x^2}$	____________________	____________________
d. $y=\sqrt{x^2+4}$	____________________	____________________
e. $h(x)=2^x+1$	____________________	____________________
f. $f\left(x\right)=\lvert x+3\rvert$	____________________	____________________
g. $f\left(x\right)=x^3+x$	____________________	____________________
h. $g(x)=4-3\sin(\dfrac{x}{3})$	____________________	____________________

9. Sketch the graph of each curve without a calculator.

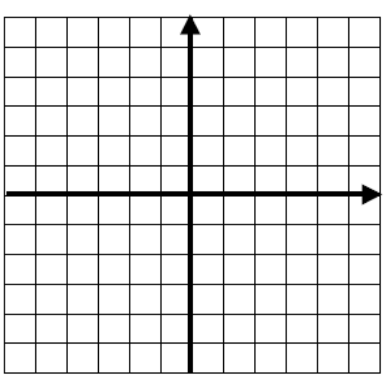
a. $y=(x-3)^2+2$



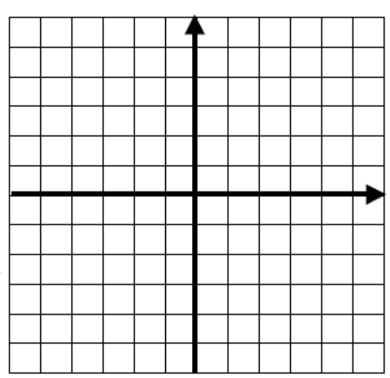
b. $y=(x+1)^3$



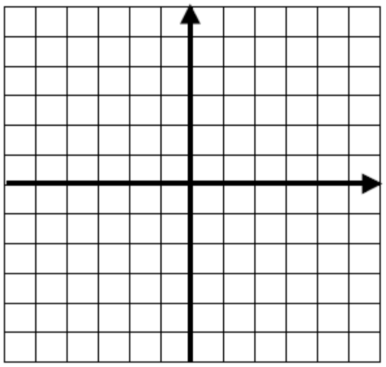
c. $y=2-\sqrt{x}$



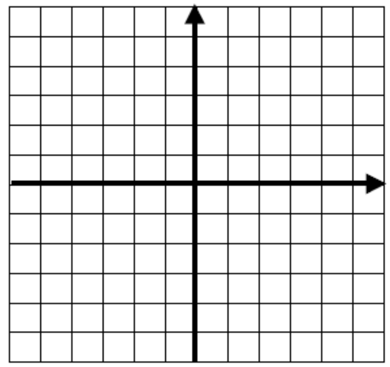
d. $y=2^{-x}$



e. $y = 2|x-3|$



f. $y = \dfrac{1}{x+1}$



10. If $f(x) = \sqrt{x-3}$ and $g(x) = x^2 + 5$, determine each of the following: [Note $(f \circ g)(x) = f(g(x))$]

a. $(f \circ g)(4)$ ______________

b. $(g \circ f)(4)$ ______________

c. $(f \circ g)(x)$ ______________

d. $f^{-1}(x)$ ______________

11. If $f(x) = \dfrac{x}{x-1}$ and $g(x) = \dfrac{1}{x-1}$, determine each of the following:

a. $(f+g)(2)$.

b. $(f \bullet g)(-1)$

c. $(f \circ g)(3)$

d. $(f \circ g)(x)$

e. $(g \circ f)(x)$

12. Given that $h(x) = \tan^{-1}(1-\pi x)$ is a composite function of the form $h(x) = f(g(x))$, find *f* and *g*.

13. Determine whether the function is odd, even, or neither:

a. $f(x) = x^3 + x$

b. $f(x) = x^2 + 1$

14. Find the inverse of the following function. Determine if the inverse is a function.

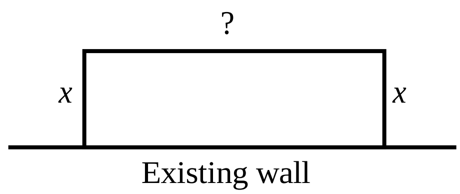
a. $y = 7x - 3$

b. $f(x) = 2x^2 + 5$

c. $g(x) = x^3 - 1$

15. Three sides of a fence and an existing wall form a rectangular enclosure. The total length of a fence used for the three sides is 240 ft. Let x be the length of two sides perpendicular to the wall as shown.

a. Write an equation of area A of the enclosure as a function of the length x of the rectangular area as shown in the above figure.

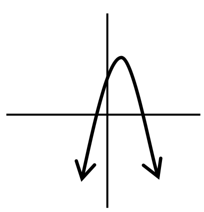


b. The find value(s) of x for which the area is 5500 ft^2

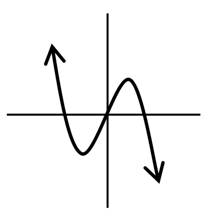
c. What is the maximum area?

16. Which of the following could represent a complete graph of $f(x) = ax - x^3$, where a is a real number?

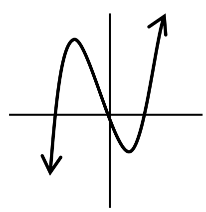
A.



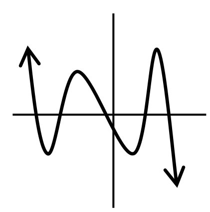
B.



C.



D.



17. Find a degree 3 polynomial with zeros -2, 1, and 5 and going through the point $(0, -3)$.

18. Expand $(x+y)^3$

19. Find the limit.

a. $\lim_{x\to 2}(4x^2-5x+3)$

b. $\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}$

c. $\lim_{x\to 0}\sqrt{x^2+4}$

d. $\lim_{x\to 0}\frac{(x-5)^2-25}{x}$

e. $\lim_{x\to -3}\frac{x^2+x-6}{x+3}$

f. $\lim_{x\to \infty}\frac{3x-5x^2}{4x^2+1}$

g. $\lim_{x\to -6}\frac{x+6}{x^2+3x-18}$

h. $\lim_{x\to -\infty}\frac{2+3x^3}{x^2+4}$

i. $\lim_{x\to -2}\frac{x^3+8}{x+2}$

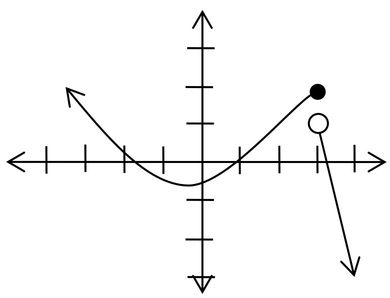
20. For the function $f(x)$ graphed, evaluate $\lim_{x\to 3} f(x)$ (multiple choice)

A. $\lim_{x\to 3} f(x)=2$

B. $\lim_{x\to 3} f(x)=3$

C. $\lim_{x\to 3} f(x)=1$

D. $\lim_{x\to 3} f(x)$ DNE



21. Find the limit (if it exists). $\lim_{x\to -2}\frac{x^2-4}{x^3+8}$

22. Find the one-sided limit. $\lim_{x\to 0^+}\frac{\sin 4x}{5x}$

23. The function f is defined as follows. $f(x)=\frac{\tan 2x}{x},\ x\neq 0$

(a) Find $\lim_{x\to 0}\frac{\tan 2x}{x}$ (if it exists).

(b) Can the function f be defined at x = 0 such that it is continuous at x = 0?

24. Determine the intervals on which the function is continuous.

$$f(x) = \begin{cases} 5-x, & x \le 2 \\ 2x-3, & x > 2 \end{cases}$$

25. Determine the value of c such that the function is continuous on the entire real line.

$$f(x) = \begin{cases} x+3, & x \le 2 \\ cx+6, & x > 2 \end{cases}$$

26. Average Rate of Change and Linear Functions

x	0	1	2	3	4	5	6
f(x)	-10	-8	-1	0	5	7	2

Using the table of values given above, find the average of change for *f* on the interval $(2, 5)$ and write an equation for the secant line passing through the corresponding points.

27. Consider the piecewise function $f(x) = \begin{cases} x-1, \text{ if } x < 2 \\ \sqrt{x-1}, \text{ if } x > 2 \end{cases}$. Answer the following:

a) Calculate $f(-3)$

b) Calculate $f(2)$

c) Calculate $f(10)$

d) Graph $f(x)$

The following problems are for students in AP Calculus BC only.

1. Determine the sum, if it exists, of the infinite geometric series… $4+\frac{4}{3}+\frac{4}{9}+\frac{4}{27}+...$

2. Determine the sum, if it exists, of the infinite geometric series… $3-6+12-24+...$

Determine whether the following sequences are arithmetic, geometric, or neither

3. $\sqrt[3]{5}, \sqrt[3]{10}, \sqrt[3]{20}, \sqrt[3]{40}, \cdots$

4. $15, 11, 7, 3, \cdots$

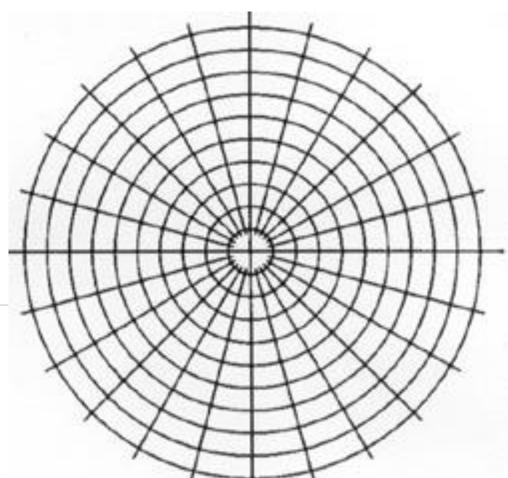
5. $f(n) = \frac{1}{2}(2)^n$

6. Simplify: $\frac{(2n+2)!}{(2n+4)!}$

7. Find the intersection of the two polar curves, $r=2$ and $r=4\cos 2\theta$, on $[0,2\pi]$.

8. State the smallest interval $(0 \le \theta \le k)$ that gives the complete graph of the polar equation $r=4\cos(5\theta)$

   b. Sketch the graph of #7 on the axes to the right.



9. The graph of the polar equation $r = 5\sin(3\theta)$ is a…

A. 3 petal rose, starting at the polar axes
B. 3 petal rose, starting in the 1st quadrant
C. 6 petal rose, starting at the polar axes
D. 6 petal rose, starting in the 1st quadrant

10. What is the sum of the geometric series $\sum_{i=0}^{\infty}\left(\frac{2}{3}\right)^i$

11. Expand and simplify:

a. $\sum_{n=0}^{4}\frac{n^2}{2}$

b. $\sum_{n=1}^{4}\frac{1}{n^2}$

c. $\sum_{n=0}^{5}(n+1)^2$

d. $\sum_{n=1}^{10}30*0.9^n$

12. Using partial fraction decomposition, decompose the fraction

a. $\frac{5x+3}{x^2+x-2}$

b. $\frac{5x+1}{x^2-1}$

13. Find the sum of the infinite geometric series... $21-3+\frac{3}{7}-\dots$