Discrete Mathematics

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Unit 4: Graph Theory and Geometry

Transfer goal: Students will be able to independently use their learning to investigate multiple representations to efficiently make decisions.

Stage 1 – Desired Results

Established Goals

NJ Student Learning Standards for Mathematics
http://www.state.nj.us/education/cccs/2016/math/hs

Number and Quantity N-QA: Reason quantitatively and use units to solve problems.
Geometry G-COA: Experiment with transformations in the plane.
Geometry G-COA: Make geometric constructions.
Geometry G-GMD: Visualize the relationship between two- and three-dimensional objects.
Geometry G-MG: Apply geometric concepts in modeling situations.

NCTM Principles and Standards of School Mathematics
www.cssu.org/cms/lib5/VT01000775/Centricity/Domain/32/CSSUMathCurric

A3: Use mathematical models to represent and understand quantitative relationships.
G1: Analyze characteristics and properties of 2- and 3-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
G2: Specify locations and describe spatial relationships using coordinate geometry and other representational systems
G3: Apply transformations and use symmetry to analyze mathematical situations.
G4: Use visualization, spatial reasoning, and geometric modeling to solve problems.
M2: Apply appropriate techniques, tools, and formulas to determine measurements.

21st Century Themes

www.21stcentruyskills.org

_x__ Global Awareness
_x__Financial, Economic, Business and Entrepreneurial Literacy
_x__Civic Literacy
_x__Health Literacy
_x__Environmental Literacy

21st Century Skills

Learning and Innovation Skills:
_x__Creativity and Innovation
_x__Critical Thinking and Problem Solving
_x__Communication and Collaboration

Information, Media and Technology Skills:
_x__Information Literacy
___Media Literacy
_x__ICT (Information, Communications and Technology) Literacy

Life and Career Skills:
_x__Flexibility and Adaptability
_x__Initiative and Self-Direction
_x__Social and Cross-Cultural Skills
_x__Productivity and Accountability
_x__Leadership and Responsibility

Enduring Understandings:	**Essential Questions:**
Students will understand that . . . *EU 1* graphs can be used to help solve problems from a wide variety of applications that relate to everyday life. *EU 2* there are various types of geometry (Euclidean and Non-Euclidean) that can be used to analyze shape and structure. *EU 3* topology is used to solve many real-world problems including those involving maps, computer science, robotics and science.	*EU1* ● What was Ben Franklin thinking when he planned the design for Philadelphia streets? *EU 2* ● What do our bodies, bacteria, animation and cell phone data storage all have in common? *EU 3* ● How can colors be used to analyze two-dimensional maps?
Knowledge: *Students will know . . .* *EU 1* ● the types of graphs that can be used to model real world situations. ● graphs can be used to represent problems. ● paths, circuits and bridges are essential components of graphs. *EU 2* ● the existence of parallel lines determines the various branches of geometry. ● the basic principles and uses of Euclidean, Non-Euclidean and Fractal Geometries. *EU 3* ● Mobius strips, Klein bottles, maps, and Jordan curves can be used to solve problems. ● the importance of topological equivalence.	**Skills:** *Students will be able to . . .* *EU 1* ● solve problems involving Euler and Hamiltonian paths and circuits. ● use graph theory terminology to analyze graphs. ● determine the chromatic number of a graph. ● analyze graphs to find trees, and minimum spanning trees. ● use the Nearest Neighbor Algorithm. *EU 2* ● describe Euclid's Parallel Postulate. ● apply the Fifth Axiom in Euclidean and Non-Euclidean Geometries. ● compare and contrast elliptical, hyperbolic and fractal geometries. *EU 3* ● explain characteristics and real-world applications of Mobius bands, Klein bottles and Jordan curves. ● use the four-color theorem to analyze maps. ● determine if figures are topologically equivalent. ● determine the genus of an object.

Stage 2 – Assessment Evidence

Recommended Performance Task: EU 1

A friend living in South Jersey wants to visit 4 destinations for summer vacation: Chicago, San Francisco, Dallas, and Orlando. They want to spend at least 2 days visiting each city before returning home to the South Jersey region. They want to start and finish their trip at either Philadelphia International Airport or Newark Liberty International Airport. The friend also doesn't want to use more than two different airlines. As their travel agent, you are to use graph theory to determine the best travel plan for them to minimize cost. The travel agent must present the friend with a travel portfolio including:

- An accurate graph with all necessary vertices and edges shown, and the minimum spanning tree highlighted.
- A summary paragraph explaining the best option, how the best option was found using graph theory vocabulary, how the travel agent gathered the data, the method(s) used to determine the minimal cost, what the departing city should be, the order the cities should be visited, and the actual flights to be used (including airlines, times, and costs).

Other Evidence: *Tests, Quizzes, Prompts, Self-assessment, Observations, Dialogues, etc.*

Tests, quizzes and activities to include

- identifying paths and circuits involving transportation maps and floor planes
- explaining the differences among graphs, paths, circuits, Euler Paths and Hamiltonian Paths
- using Euler's Theorem and Fleury's Algorithm
- finding minimum spanning trees by and using weighted graphs
- applying Kruskal's Algorithm
- using graph vocabulary
- traveling salesperson problems involving various algorithms (brute force, nearest neighbor)
- explanations of the Fifth Axiom and Euclid's Parallel Postulate
- syntheses of Two Dimensional, Euclidean, Spherical and Hyperbolic Geometries
- descriptions of Mobius bands, Klein Bottles and Jordan Curves
- identifying topologically equivalent objects
- calculating the genus of an object
- identifying the location of points inside and outside of curves
- using the four-color theorem
- applications of fractals

Stage 3 – Learning Plan

Suggested Learning Activities to Include Differentiated Instruction and Interdisciplinary Connections: *Consider the WHERETO elements*

Activity 1: The Konigsberg Bridge Problem (T)

- Students will use graph theory concepts to analyze a real-world situation involving graphs
- Students will use vocabulary associated with graphs to summarize the graph
- Students will apply concepts from graphs to other real-world situations.
- http://mathworld.wolfram.com/KoenigsbergBridgeProblem.html

Activity 2: Social Media Friend Graphing (T)
Students will select at least 3 of their friends on a social media platform and see what mutual friends they have in order to construct a complete graph.

Activity 3: The Four Color Theorem Game (T)

- Students will be able to discover the Four-Color Theorem
- Students can compete against each other to use the Four-Color Theorem in a variety of situations
- http://www.novelgames.com/en/fourcolour/

Activity 4: Flatland Novel by Edwin Abbott (M)

- Students will read and compare and contrast 2 and 3 dimensional geometries
- Students will apply Euclidean Geometric concepts involving 2-dimensions
- Students will extend concepts of geometry in 2-dimensions to those in other dimensions and geometries
- Online Text: http://www.geom.uiuc.edu/~banchoff/Flatland/

Key Vocabulary Includes:

graph	Euler circuit	tree	parallel
vertex	algorithm	spanning tree	plane
edge	traveling salesman problems	minimum-cost spanning tree	sphere
loop	weighted graph	Euclidean Parallel Postulate	pseudosphere
path	Hamilton path	non-Euclidean geometry	topology
circuit	Hamilton circuit	elliptical geometry	Mobius strip
connected	complete graph	hyperbolic geometry	Mobius band

disconnected	factorial	geodesic	Klein bottle
bridge	optimal solution	fractal geometry	Jordan curve
traversable	complete, weighted graph	recursion	topologically
Euler path	approximate solution	Koch snowflake	genus

Approximate Timeline: 16 Days

- Analyze real world situations by using graph theory concepts and appropriate vocabulary (A)
- Activity 1: Konigsberg Bridge Problem (T)
- Explain whether graphs contain Euler Circuits and Paths (M)
- Use Hamiltonian Paths and Circuits to solve real world problems (T)
- Activity 2: Social Media Friends (T)
- Summer Vacation Performance Task (T)
- Explain minimum spanning trees (A)
- Use Kruskal's Algorithm to determine minimum spanning trees (T)
- Use chromatic numbers to resolve conflicts (T)
- Apply the four-color theorem to graphs (M)
- Activity 3: Four-color theorem game (T)
- Explain topological vocabulary (A)
- Apply topological vocabulary to objects (M)
- Calculate the genus of objects (T)
- Explain Euclid's Parallel Postulate (A)
- Apply variations of the Fifth Axiom to other non-Euclidean Geometries (M)
- Compare and contrast concepts from Euclidean and Spherical Geometries (M)
- Activity 4: Flatland Reading/Video (M)