Discrete Mathematics

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Unit 1: Problem Solving and Number Theory

Transfer goal: Students will be able to independently use their learning to construct valid conclusions, effectively communicate those conclusions and critique the reasoning of others.

Stage 1 – Desired Results

Established Goals

NJ Student Learning Standards for Mathematics
http://www.state.nj.us/education/cccs/2016/math/hs

Number and Quantity N-QA: Reason quantitatively and use units to solve problems.
Number and Quantity N-VMC: Perform operations on matrices and use matrices in applications.
Algebra A-SSE: Interpret the structure of expressions.

NCTM Principles and Standards of School Mathematics
www.cssu.org/cms/lib5/VT01000775/Centricity/Domain/32/CSSUMathCurric

N1: Understand numbers, ways of representing numbers, relationships among numbers, and number systems
N2: Understand meanings of operations and how they relate to one another.
N3: Compute fluently and make reasonable estimates.
A2: Represent and analyze mathematical situations and structures.
A3: Use mathematical models to represent and understand quantitative relationships
M1: Understand measurable attributes of objects and the units, systems, and processes of measurement.
M2: Apply appropriate techniques, tools, and formulas to determine measurements.

21st Century Themes

www.21stcentruyskills.org

_x__ Global Awareness
_x__Financial, Economic, Business and Entrepreneurial Literacy
_x__Civic Literacy
___Health Literacy
___Environmental Literacy

	21st Century Skills *Learning and Innovation Skills:* _x__Creativity and Innovation _x__Critical Thinking and Problem Solving _x__Communication and Collaboration *Information, Media and Technology Skills:* _x__Information Literacy ___Media Literacy ___ICT (Information, Communications and Technology) Literacy *Life and Career Skills:* _x__Flexibility and Adaptability _x__Initiative and Self-Direction _x__Social and Cross-Cultural Skills _x__Productivity and Accountability _x__Leadership and Responsibility
Enduring Understandings: *Students will understand that . . .* *EU 1* being a successful problem solver is vital to succeeding in every discipline and aspect of life. *EU 2* math is a universal language that spans various times and civilizations. *EU 3* mathematics is the foundation to cryptography, a coding system necessary for all modern technology as well as the security and safety of our world.	**Essential Questions:** *EU1* ● What strategies and techniques can be used to become a better problem solver? *EU 2* ● How did ancient Babylonians help in the development of modern technologies? *EU 3* ● How does Amazon ensure your cyber security?
Knowledge: *Students will know . . .* *EU 1* ● inductive and deductive reasoning is used to solve problems. ● general procedures can be used to streamline the problem solving process.	**Skills:** *Students will be able to . . .* *EU 1* ● use inductive reasoning to create conjectures and state counterexamples. ● use Polya's problem solving process to solve non-routine problems. ● use permutations, combinations and Pascal's triangles to solve problems.

EU 2 ● the language of math consists of symbols, bases, and processes which are the foundation of numeration systems. ● a variety of numeration systems have been used throughout history and are used throughout the world today. ● modern methods of calculations have been derived from methods used thousands of years ago. *EU 3* ● encryption systems involve many areas of mathematics. ● it's important to not only look at numbers as a whole but what they become when broken down into its parts. ● series and sequences can be used to make predictions.	*EU 2* ● explain connections between various civilizations and their role in the development of mathematics. ● discuss the similarities and differences of numbers among Hindu-Arabic, Babylonian, Egyptian, Roman, Chinese and Greek numeration systems. ● solve math problems in a variety of bases other than base ten ● explain, use and apply ancient calculation methods and devices. *EU 3* ● identify special sets of numbers and describe properties of the real number system. ● discuss and apply divisibility rules. ● use arithmetic, geometric and Fibonacci sequences. ● discuss the requirements that make a set of numbers a group. ● use modular arithmetic to perform operations with numbers. ● decode and encode messages using encryption. ● apply real number and group concepts to matrices.

Stage 2 – Assessment Evidence

Recommended Performance Task: EU 2

A new numerations system is being invented for secret communications within the U.S. military. A cryptographer will be hired to create the numeration system. As a cryptographer contending for this prestigious position, create your proposal that contains:

a cover page stating your name, the name of the numeration system, and the base that the system is written in (It can't be any base already discussed in class);
a second page listing all of the numerals/symbols for your numeration system, stating what the symbols represent in the new system as well as in base 10;
a third page that lists at least 5 numerals from the numeration system. At least one numeral from each of the following ranges must be represented (1-50, 51-100, 250-1000, 1001-50,000, and a fraction);
a fourth page demonstrating a combination of at least three addition and three subtraction problems in your numeration system, at least one of which requires carrying and at least one requiring regrouping;
a fifth page explaining advantages and disadvantages of the numeration systems and an argument as to why the U.S. should use this new numeration system compared to others that already exist (comparing to those studied in class);
explanations and work that are well organized, neat, and mathematically correct.

Other Evidence: *Tests, Quizzes, Prompts, Self-assessment, Observations, Dialogues, etc.*

Tests, quizzes and activities to include

non-routine problems assessed with rubrics
number patterns
uses of inductive and deductive reasoning
applications of combinatorics and Pascal's Triangle
solving math problems in other bases and numeration systems
Venn diagrams comparing different historical numeration systems
presentations about historical mathematicians and events regarding the history of math
demonstrating the use of ancient calculational devices (abacus, Napier's Bones, lattice multiplication)
calculating terms in various series and sequences
uses of cryptography to locate "hidden treasures" in the classroom or school
using divisibility rules
recreational number theory problems
summarizers involving groups and fields

Stage 3 – Learning Plan

Understanding the Problem

3	2	1	0
Problem is rewritten **clearly, accurately in your own words** and demonstrates **understanding**. Explanation includes what information in the problem is given, what answer is being looked for, and what the answer will/will not look like.	Problem is **rewritten in your own words** and demonstrates **understanding**. Explanation is missing either the given information or what the answer will/will not look like.	Problem is **restated** or rewritten in your own words demonstrating **mistunderstanding**. A minimal explanation is given regarding the given information or what the answer will/will not look like.	Problem is **not restated** or rewritten in your own words. A **complete misunderstanding** is represented in the explanation of the problem. No given information is written and/or no description of the final answer is

Devise a Plan

3	2	1	0
At least two reasonable problem solving stragegies are listed. These strategies are **justified completely** in terms of the context of the problem.	**At least one reasonable** problem solving strategy is listed. **Justifications** for the strategy(ies) is **adequate**.	**At least one** problem solving strategy is listed although it may **not be reasonable** based on the problem context. **Justifications** for the strategy(ies) is weak.	**No** problem solving stragety is listed or the problem solving strategy is **not justified**.

Carry out the Plan

3	2	1	0
One or more plans is carried out and shows **complete understanding** of the problem's mathematical concepts. The response contains **few minor , if any, mathematical errors**. **All** work is **clear and explained**. (*correct answer*)	One or more plans is carried out and shows a **nearly complete understading** of the problem's mathematical concepts. The response **may have minor errors**. **Most**, if not **all** work is shown and explained. (*correct or incorrect answer*)	One or more plans is carried out and demonstrates a **limited understanding** of the problem's mathematical concepts. The response contains **major errors**. **Little** work is shown and explained. (*correct or incorrect answer*)	Either **no plan is carried out**, or the plan shows **insufficient understanding** of the problem's mathematical concepts. There may be **no** solution, no work shown, and/or no explanation. (*correct or incorrect answer*)

Look Back

3	2	1	0
A **complete, clear and insightful explanation is provided** explaining **why** the plan worked, and **how** you know it worked or, if the plan didn't work, why the plan failed, and how you know it failed.	An **explanation is provided** explaning why the plan did or did not work. The explanation may be **brief**, or **lacking** insight/clarity.	An **explanation is provided** explaning why the plan did or did not work. The explanation is **weak** and contains **little, if any**, insight or clarity to the work done.	An explanation is either **missing**, or contains **no** justification or insight into why the plan worked or did not work.

Suggested Learning Activities to Include Differentiated Instruction and Interdisciplinary Connections: *Consider the WHERETO elements*

Activity 1: Polya's Process (T)

- Students will solve non-routine problems which will be assessed using Poly's Problem Solving Rubric and/or the LRHSD Math Department Progress Assessment Rubric.

Activity 2: Egyptian Fractions (M)

- Students will apply concepts regarding fractions from the Hindu-Arabic Numeration System and the Egyptian Numeration System.
- Students will use the Egyptian Numeration System to compare fractions.
- Students will use the Egyptian Numeration System to explain fundamental concepts in the Hindu-Arabic Numeration System.

http://www.nctm.org/Publications/Student-Explorations-in-Mathematics/2009/Egyptian-Fractions(March-2009)/

Activity 3: 20 Questions (T)
Students will use the game of 20 questions to explain the use of the binary numeration system.

Activity 4: Ancient Egyptians and Russian Peasants (M)

- Students will be able to apply ancient Egyptian and Russian multiplication techniques to solve problems.
- Students will be able to explain connections between Egyptian and Russian multiplication methods to the use of Napier's Bones.
- Students will be able to explain how multiplication by hand works.

http://www.nctm.org/Publications/mathematics-teacher/2007/Vol100/Issue9/Ancient-Egyptians-and-Russian-Peasants-Foretell-the-Digital-Age/

Activity 5: Cryptography (T)
Students will cipher and decipher messages using various cryptography techniques.
Students will apply modular arithmetic concepts to encoding and decoding.
https://blossoms.mit.edu/sites/default/files/video/download/aurangzeb-activities.pdf

Students will practice decoding messages
Students will make connections between other branches of discrete math and other disciplines while practicing cryptograph2
https://www.braingle.com/Cryptography.html

Key Vocabulary Includes:

natural numbers	expanded form	unit	counterexample
counting numbers	bases	sieve of Eratosthenes	inverse
ellipsis	binary	prime factorization	group
inductive reasoning	octal	theorem	commutative group

scientific method	hexadecimal	Mersenne primes	modulo m system
hypothesis	bit	Fermat number	modulo classes
conjecture	byte	Goldbach's conjecture	modulus
deductive reasoning	number theory	twin prime conjecture	remainder
number	counting numbers	twin prime	congruent
numeral	natural numbers	whole numbers	cipher
Hindu-Arabic system	factors	integers	dimensions
place-value system	divisor	inverse	matrix
positional-value system	divides	base	square
base	divisible	mathematical system	scalar
decimal number system	prime number	binary operation	
digits	composite number	closure	

Approximate Timeline: 22 days

Explain inductive and deductive reasoning (A)
Apply inductive and deductive reasoning to state conclusions and counterexamples (T)
Explain the steps of Polya's Problem Solving Process (A)
Activity 1: Use Polya's process to solve non-routine problems (T)
Use combinatorics and Pascal's Triangle to solve non-routine problems (M)
Explain concepts associated with numeration systems (A)
Solve math problems involving additive and ciphered numeration systems (T)
Use place value numeration systems (M)
Activity 2: Egyptian Fractions (M)
Perform mathematical operations in bases other than base ten (M)
Activity 3: 20 Questions (T)
Explain and use historic calculational methods and devices (M)
Activity 3: Ancient Egyptians and Russian Peasants (M)
Explain divisibility rules (A)
Use divisibility rules without using a calculator (M)
Solve problems involving number theory (T)
Calculate terms of arithmetic and geometric sequences (A)
Use golden ratios (M)
Describe mathematical systems and properties of those systems (A)
Determine whether mathematical systems are groups (M)
Solve problems involving modulo systems (M)
Use modulo systems in cryptography (T)

Activity 5: Cryptography (T)
Apply group theory to matrices (T)