Course Title – Calculus

Implement start year – 2015-2016

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Unit # 3, topic – Applications of Derivatives

Transfer Goal –

Students will be able to independently use their learning to apply the rules of derivatives to solve application problems.

Stage 1 – Desired Results

Established Goals

2009 NJCCC Standard(s), Strand(s)/CPI #
(http://www.nj.gov/education/cccs/2009/final.htm)

Common Core Curriculum Standards for Math and English
(http://www.corestandards.org/)

Since the Calculus curriculum goes beyond the Common Core Curriculum Standards for Math, the NCTM standards have been adopted:

- Analyze change in various contexts.
- Represent and analyze mathematical situations using algebraic symbols.
- Use mathematical models to represent and understand quantitative relationships.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Apply appropriate techniques, tools, and formulas to determine measurements.
- Use the language of mathematics to express mathematical ideas precisely.
- Understand how mathematical ideas interconnect and build on one another.

21st Century Themes
(www.21stcenturyskills.org)

_x__ Global Awareness
_x__Financial, Economic, Business and Entrepreneurial Literacy
___Civic Literacy
_x__Health Literacy
_x__Environmental Literacy

21st Century Skills

Learning and Innovation Skills:
_x__Creativity and Innovation
_x__Critical Thinking and Problem Solving
_x__Communication and Collaboration

Information, Media and Technology Skills:
_x__Information Literacy
_x__Media Literacy
_x__ICT (Information, Communications and Technology) Literacy

Life and Career Skills:
_x_Flexibility and Adaptability
_x__Initiative and Self-Direction
_x__Social and Cross-Cultural Skills
_x__Productivity and Accountability
_x__Leadership and Responsibility

Enduring Understandings: *Students will understand that . . .* *EU 1* properties of calculus can be used to analyze functions. *EU 2* derivatives can be used to determine a functions behavior. *EU 3* the graph of a derivative can be used to draw conclusions about the original function. *EU 4* concepts of the derivative, along with intercepts, asymptotes, and symmetry, can be used to sketch the graph of a function without the use of a graphing calculator. *EU 5* applications of derivatives can be used to solve real-life problems.	**Essential Questions:** *EU 1* • What applications do Rolle's Theorem and Mean Value Theorem have in the real world? *EU 2* • How can a derivative, f '(x), be represented in a graph and how are the graphs of f(x), f '(x), and f "(x) related? *EU 3* • How can differentiation be used to determine the shape of a function? *EU 4* • How can Calculus be used to describe the behavior of a function? *EU 5* • How can Calculus be applied in the real world?
Knowledge: *Students will know . . .* *EU 1* • the purpose and criteria of Rolle's Theorem and Mean Value Theorem. *EU 2* • the test for increasing and decreasing functions (The First Derivative Test) • the test for relative extrema • the test for concavity • the test for a point of inflection and concavity • the Second Derivative Test	**Skills:** *Students will be able to . . .* *EU 1* • determine the validity of Rolle's Theorem or Mean Value Theorem. • use and apply Rolle's Theorem or Mean Value Theorem *EU 2* • determine intervals where the original function is increasing, decreasing, concave up, and concave down. • locate relative extrema and points of inflection

EU 3 • the relationships between the original function and its first and second derivatives *EU 4* • key concepts in graphing including intercepts, symmetry, and asymptotes *EU 5* • derivatives can be used to find minimum and maximum values in the real world (optimization)	*EU 3* • match the graphs of the original functions, first derivative function, and second derivative function. *EU 4* • sketch the graph of a function using the key concepts of graphing and the first and second derivatives *EU 5* • solve applied minimum and maximum word problems.

Stage 2 – Assessment Evidence

Recommended Performance Tasks:

Other Recommended Evidence: *Tests, Quizzes, Prompts, Self-assessment, Observations, Dialogues, etc.*

- Quiz on Rolle's Theorem, Mean Value Theorem, increasing, decreasing, relative extrema, concavity, and points of inflection
- Quiz on summary of curve sketching using domain, intercepts, symmetry, asymptotes, first derivative, and the second derivative
- Quiz on analyzing f from the graph of f' and f", table, and using the TI-Nspire
- Quiz on Optimization
- Assessed elements from recommended performance task.

Stage 3 – Learning Plan

Suggested Learning Activities to Include Differentiated Instruction and Interdisciplinary Connections:

- Activity #1: Discovering the Relationship between f' and f (M, T)

<u>Given:</u> $y = x^3 + 4x^2 + 3$
Look at the graph of the original function, f, and its first derivative, f'. Write out in a sentence or two how to find the various properties of the original function from the first derivative.

1. Where is the original function increasing? What is the relationship between that interval on both functions?

2. Where is the original function decreasing? What is the relationship between that interval on both functions?

3. Design a process for locating relative maximum points using the first derivative?

4. Design a process for locating relative minimum points using the first derivative?

5. From the first derivative, is it possible to identify where the original function is concave up?

6. From the first derivative, is it possible to identify where the original function is concave down?

7. Design a process to identify points of inflection of the original function from the first derivative?

- Activity #2: The Match Game (A)
  Each student is given a note card with the graph of f, f', or f". Students work with others to find the other two students holding cards that are related to their own. Each member of the group explains their graph and the relationship to the other two group members.

- Activity #3: Ticket-To-Leave (M, T)
  At the end of the unit, students will answer the following questions on a slip of paper and hand in to be discussed at the beginning of class next day:
  1. How is the derivative important in Rolle's Theorem and Mean Value Theorem?
  2. What information does the derivative give us? How does the derivative relate to relative extrema?
  3. How can Opimization be used in the real-world?

- Activity #4: TI-Nspire The First Derivative Test (M)
  http://education.ti.com/en/timathnspired/us/detail?id=50652E095CD94C31B85D5CAD599AA899&t=E990A0CFBE4A4B2CAF83A1416EA537B8

The following is a suggested sequence of learning activities for the Accelerated Calculus class. Approximate days for completion: 25.

- YWBAT explain the definition of extrema of a function on an interval. (A)
- YWBAT explain the definition of relative extrema of a function on an open interval. (A)
  - Activity #4
- YWBAT find extrema on a closed interval. (A)
- YWBAT explain the use Rolle's Theoerem and the Mean Value Theorem. (A, M)
  - Activity #3
- YWBAT determine intervals on which a function is increasing or decreasing. (A)
  - Activity #1
  - Activity #2
  - Activity #3
- YWBAT apply the First Derivative Test to find relative extrema of a function. (A, M)
  - Activity #1
  - Activity #2
  - Activity #3
- YWBAT determine intervals on which a function is concave upward or concave downward. (A)
  - Activity #1
  - Activity #2
  - Activity #3
- YWBAT find any points of inflection of the graph of a function. (A)
  - Activity #1
  - Activity #2
  - Activity #3
- YWBAT apply the Second Derivative Test to find the relative extrema of a function. (A, M)
- YWBAT determine finite and infinite limits at infinity. (A)
- YWBAT determine the horizontal asymptotes, if any, of the graph of a function. (A)
- YWBAT analyze and sketch the graph of a function. (A)
  - Activity #1
  - Activity #2
- YWBAT solve applied minimum and maximum problems. (A, M)
  - Activity #3
- Performance Task (M, T)

Critical Vocabulary:

- Extrema
- Relative extrema
- Critical numbers
- Increasing
- Decreasing
- Instantaneous rate of change
- Concavity
- Point of inflection
- Horizontal asymptote
- Limits at infinity
- Optimization