Course Title – Calculus

Implement start year – 2015-2016

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Unit # 2, topic – Differentiation

Transfer Goal –

Students will be able to independently use their learning to evaluate and understand rate of change.

Stage 1 – Desired Results

Established Goals

2009 NJCCC Standard(s), Strand(s)/CPI #
(http://www.nj.gov/education/cccs/2009/final.htm)

Common Core Curriculum Standards for Math and English
(http://www.corestandards.org/)

Since the Calculus curriculum goes beyond the Common Core Curriculum Standards for Math, the NCTM standards have been adopted:

- Analyze change in various contexts.
- Represent and analyze mathematical situations using algebraic symbols.
- Use mathematical models to represent and understand quantitative relationships.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Apply appropriate techniques, tools, and formulas to determine measurements.
- Use the language of mathematics to express mathematical ideas precisely.
- Understand how mathematical ideas interconnect and build on one another.

21st Century Themes
(www.21stcenturyskills.org)

_x__ Global Awareness
_x__Financial, Economic, Business and Entrepreneurial Literacy
___Civic Literacy
_x__Health Literacy
_x__Environmental Literacy

21st Century Skills

Learning and Innovation Skills:
_x__Creativity and Innovation
_x__Critical Thinking and Problem Solving
_x__Communication and Collaboration

Information, Media and Technology Skills:
_x__Information Literacy
_x__Media Literacy
_x__ICT (Information, Communications and Technology) Literacy

Life and Career Skills:
_x_Flexibility and Adaptability
_x__Initiative and Self-Direction
_x__Social and Cross-Cultural Skills
_x__Productivity and Accountability
_x__Leadership and Responsibility

Enduring Understandings:

Students will understand that . . .

EU 1
slope can be used to describe the rate at which a function changes.

EU 2
a limiting process can be applied to real life situations.

EU 3
derivatives can be used to describe the rate of change of a function at a given point.

EU 4
besides the limit process, there are many "shortcut" rules for coming up with the derivative of a variety of functions.

EU 5
a static relationship between two or more quantities can be used to derive a dynamic relationship between the rates at which those quantities are changing.

Essential Questions:

EU 1
- How can the concept of slope be transferred from the slope of a line to the slope of a curve?

EU 2
- How can the slope of a curve be estimated at only a single point?

EU 3
- How can a formula for the slope at an arbitrary x-value be determined?

EU 4
- Is there a pattern when finding the derivatives of polynomial functions?

EU 5
- When a quantity is changing with respect to time, how are associated quantities changing?

Knowledge:

Students will know . . .

EU 1
- the meaning of slope.

EU 2
- when and why to evaluate limits.

EU 3
- the definition of a derivative.

Skills:

Students will be able to . . .

EU 1
- describe the slope of tangent lines at a point.

EU 2
- evaluate the slope at a point by the limit definition and The Alternative Formula.

EU 3
- evaluate the derivative of a function by the limit definition.

EU 4 • methods to calculate derivatives for a variety of functions. *EU 5* • quantities that are related algebraically have related rates of change.	*EU 4* • use the power, product, quotient, chain, trigonometric, exponential and logarithmic rules when each is appropriate. • use implicit and logarithmic differentiation when each is appropriate. *EU 5* • model, differentiate and evaluate application problems.

Stage 2 – Assessment Evidence

Performance Task #1:

Other Recommended Evidence: *Tests, Quizzes, Prompts, Self-assessment, Observations, Dialogues, etc.*

- Quiz on The Derivative
- Quiz on Basic Differentiation Rules
- Quiz on Product and Quotient Rules, Trig Rules and Higher-Order Derivatives
- Quiz on Chain Rule
- Quiz on Implicit Differentiation
- Quiz on Related Rates
- Quiz on The Natural Log Function: Differentiation
- Quiz on Inverse Functions
- Quiz on Exponential Functions: Differentiation
- Quiz on Bases Other than e and Applications
- Quiz on Inverse Trig Functions: Differentiation
- Unit Assessment
- Assessed elements from recommended performance task

Stage 3 – Learning Plan

Suggested Learning Activities to Include Differentiated Instruction and Interdisciplinary Connections:

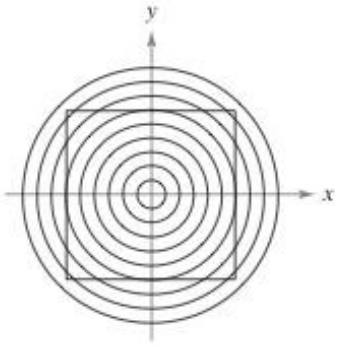
- Activity #1: The Tangent Line Problem. Given a function and only one point, have the students design an innovative way to calculate the slope at that point using only prior knowledge from previous courses. (T)
- Activity #2: The Power Rule. Break the students up into small groups to use the limit definition of the derivative to calculate derivatives for various power (and radical) functions. Then have them analyze the results and generalize a rule for power functions. (M)
- Activity #3: Flashcards. Have students create flashcards for the various differentiation rules. (A)
- Activity #4: Students solve related rates problems and then design their own to be posed to the rest of the class (M, T)
- Activity #5: Optical Illusions (M, T)

Optical Illusions

In each graph below, an optical illusion is created by having lines intersect a family of curves. In each case, the lines appear to be curved. Find the value of dy/dx for the given values of x and y.

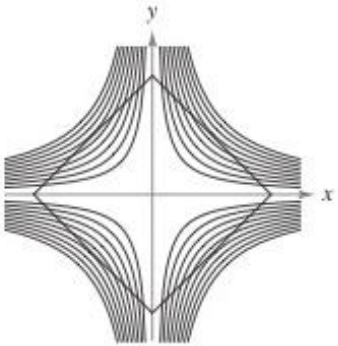
(a) Circles: $x^2 + y^2 = C^2$

$x = 3, y = 4, C = 5$



(b) Hyperbolas: $xy = C$

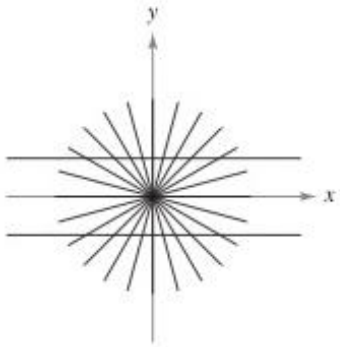
$x = 1, y = 4, C = 4$



(c) Lines: $ax = by$

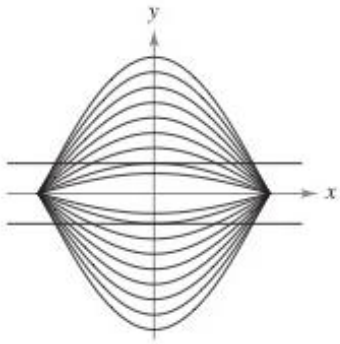
$x = \sqrt{3}, y = 3,$

$a = \sqrt{3}, b = 1$



(d) Cosine curves: $y = C \cos x$

$x = \frac{\pi}{3}, y = \frac{1}{3}, C = \frac{2}{3}$



■ **FOR FURTHER INFORMATION** For more information on the mathematics of optical illusions, see the article "Descriptive Models for Perception of Optical Illusions" by David A. Smith in *The UMAP Journal*.

- Activity #6: TI-Nspire Continuity and Differentiability (M)

http://education.ti.com/en/timathnspired/us/detail?id=50518F52B6FE49E6A44FA667A6D90D8E&t=E990A0CFBE4A4B2CAF83A1416EA537B8

- Activity #7: Tootsie Roll Pop - How Many Licks does it take? (M, T)

Objective:
In this activity you will determine the rate of change of volume of a Tootsie Roll Pop (TRP) as you consume it.

Equipment:
• Tootsie Roll Pop
• Dental Floss & Ruler
• Graphing Calculator

Data Collection:
• Step 1: Determine the initial circumference of the TRP and calculate the radius. (Assume it is a perfect sphere.)

• Step 2: Place it in your mouth and carefully suck for 30 seconds.

• Step 3: Measure the circumference and record.

• Step 4: Repeat step two and three a total of 10 times after the initial reading.

• Step 5: Record your measured data in the table below.

Time (minutes)	Measured Circum (cm)	Calculated Radius (cm)
0		
.5		
1		
1.5		
2		
2.5		
3		
3.5		
4		
4.5		
5		

Data Analysis:
• Step 1: Graph a scatter plot of the radius as a function of time to see how the radius changes with time. Determine the best fitting curve for this scatter plot. Choose from linear, quadratic, cubic, or exponential. Use this graph to determine the average rate of change of the radius (dr/dt) of the TRP from your mouth power during the first five minutes.

Best fitting curve is (state the equation) _____

because _____.

Average $\frac{dr}{dt}$ using graph= _____ Equation for $\frac{dr}{dt}$= _____

Average $\frac{dr}{dt}$ using table= _____

• Step 2: Graph the volume as a function of time so you see how the volume can be modeled as a function of time. Determine the best fitting curve for this scatter plot. Choose from linear, quadratic, cubic, or exponential.

Best fitting curve is (state the equation) _____

because _____.

Average $\frac{dv}{dt}$ using graph= _____ Equation for $\frac{dv}{dt}$= _____

Average $\frac{dv}{dt}$ using table= _____

• Step 3: Using what you found above, predict $\frac{dV}{dt}$ at t = 5 minutes. Compare this to using just the table to find $\frac{dV}{dt}$.

• Step 4: Using the equations found above, estimate how fast the volume is changing when the radius is three-fourths of its original radius.

• Step 5: Using only the table with the recorded data, estimate how fast the volume is changing when the radius is three-fourths of its original radius.

The following is a suggested sequence of learning activities for the Accelerated Calculus class. Approximate days for completion: 48.

- YWBAT find the slope of a tangent line to a curve at a point and use the limit definition to find the derivative of a function.
  - Activity #1
- YWBAT understand the relationship between differentiability and continuity.
  - Activity #6
- YWBAT differentiate simple functions using the Constant and Power rules as well as Sine and Cosine functions.
  - Activity #2
- YWBAT find the derivative of a function using the Product and Quotient rules, find the derivative of all trig functions and find higher order derivatives.
- YWBAT use the chain rule to differentiate composite functions.
  - Activity #3
- YWBAT distinguish between functions written in implicit and explicit form and use implicit differentiation to find the derivative of functions.
  - Activity #5
- YWBAT use derivatives to find related rates and use it to solve real-life problems.
  - Activity #4
  - Activity #7
- YWBAT develop and use properties of the natural logarithmic function.
- YWBAT find derivatives of functions involving the natural logarithmic function.
- YWBAT find the derivative of an inverse function.
- YWBAT differentiate natural exponential functions.
- YWBAT differentiate and integrate exponential functions that have bases other than e.
- YWBAT differentiate an inverse trigonometric function.
- Performance Task

Critical Vocabulary

- Tangent Line
- Difference Quotient
- Slope
- Derivative
- Limit Process
- Differentiability
- Cusp
- Rate of Change
- Derivative
- Differentiation
- Implicit
- Explicit
- Related Rates
- Logarithm
- Natural Logarithmic Function
- Base
- Inverse
- Horizontal Line Test
- One-to-One
- Natural Exponential Function
- Common Logarithmic Function
- Transcendental