Course Title – Calculus

Implement start year – 2015-2016

Revision Committee Members, email, extension –
Paula Marques, pmarques@lrhsd.org, ext. 8981, Deborah Jenson, djenson@lrhsd.org, ext. 8560,
Dana Palumbo, dpalumbo@lrhsd.org, ext. 8422, Brian Moore, bmoore@lrhsd.org, ext. 8129

Unit # 1, topic – Limits

Transfer Goal – Students will be able to independently use their learning to analyze and evaluate various functions.

Stage 1 – Desired Results

Established Goals

2009 NJCCC Standard(s), Strand(s)/CPI #
(http://www.nj.gov/education/cccs/2009/final.htm)

Common Core Curriculum Standards for Math and English
(http://www.corestandards.org/)

Since the Calculus curriculum goes beyond the Common Core Curriculum Standards for Math, the NCTM standards have been adopted:

- Analyze change in various contexts.
- Represent and analyze mathematical situations using algebraic symbols.
- Use mathematical models to represent and understand quantitative relationships.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Apply appropriate techniques, tools, and formulas to determine measurements.
- Use the language of mathematics to express mathematical ideas precisely.
- Understand how mathematical ideas interconnect and build on one another.

21st Century Themes
(www.21stcenturyskills.org)

_x__ Global Awareness
_x__Financial, Economic, Business and Entrepreneurial Literacy
___Civic Literacy
_x__Health Literacy
_x__Environmental Literacy

21st Century Skills

Learning and Innovation Skills:
_x__Creativity and Innovation
_x__Critical Thinking and Problem Solving
_x__Communication and Collaboration

Information, Media and Technology Skills:
_x__Information Literacy
_x__Media Literacy
_x__ICT (Information, Communications and Technology) Literacy

Life and Career Skills:
_x_Flexibility and Adaptability
_x__Initiative and Self-Direction
_x__Social and Cross-Cultural Skills
_x__Productivity and Accountability
_x__Leadership and Responsibility

Enduring Understandings: *Students will understand that . . .*	**Essential Questions:**
EU 1 limits of functions can be evaluated numerically, graphically, and algebraically.	*EU 1* • What is a limit? • What methods can be used to find the limit of a function?
EU 2 properties of calculus can be used to describe the behavior of real world functions.	*EU 2* • How can calculus be used to describe the behavior of a function?
EU 3 some graphs demonstrate asymptotic and unbounded behavior.	*EU 3* • How do limits approaching infinity help describe the asymptotic/unbounded behavior of a function? • How do limits that don't exist help describe the behavior of a function?
EU 4 the relationship between limit and continuity at a given point.	*EU 4* • How do limits help determine the continuity of a function?
Knowledge: *Students will know . . .*	**Skills:** *Students will be able to . . .*
EU 1 • the most appropriate technique for determining the limit of a function.	*EU 1* • calculate limits using algebra. • estimate limits from graphs or tables of data.
EU 2 • the various behaviors of a function using calculus techniques.	EU 2 • discuss the properties of a graph using calculus techniques
EU 3 • the methods used to determine the asymptotes of a function	*EU 3* • describe asymptotic behavior in terms of limits involving infinity
EU 4 • the methods used to determine the continuity of a function at a point	EU 4 • determine continuity in terms of limits. • apply the Intermediate Value Theorem

Stage 2 – Assessment Evidence

Performance Task for Limits

Other Recommended Evidence: *Tests, Quizzes, Prompts, Self-assessment, Observations, Dialogues, etc.*

- Quiz on limits numerically and graphically
- Quiz on limits algebraically
- Quiz on the definition of continuity and Intermediate Value Theorem
- Assessed elements from recommended performance task

Stage 3 – Learning Plan

Suggested Learning Activities to Include Differentiated Instruction and Interdisciplinary Connections:.

- Activity #1: Give each student an index card with a limit problem which must be specifically solved either numerically, graphically, or algebraically. Each student must then find two other students with the same limit answer. (A)

- Activity #2: Investigating Limits on the TI calculators. (M)

  The expression $\lim_{x \to a} f(x) = L$ means that you can find a value L that the function f(x) approaches as x approaches a. To examine this limit idea on the TI-Nspire graphing calculator, you will use two features of the calculator: The graph and table features.

  Step 1: To investigate the $\lim_{x \to 0} \frac{1 - \cos x}{x}$ enter the expression in $f_1(x)$.

  - Graph the function $f_1(x)$ in the standard window and look at the behavior of the function as x approaches 0.
  - Trace along this curve. What happens at x = 0?

  Step 2: To observe more closely what this function is doing in a neighborhood of zero, use the table feature by hitting Ctrl T. Once the table appears, hit menu, Table, Edit Table and change Table Start to -0.1 and Table Step to .01.

  - Observe what happens when the function approaches 0 at increments of .01.
  - Re-observe the function when changing Table Start to -0.01 and Table Step to .001 and then again to Table Start of -0.001 and Table Step to .0001. What is your conclusion about $\lim_{x \to 0} \frac{1 - \cos x}{x}$?

- Activity #3: What Happens as X Approaches Zero. (M)

Let $f(x)=\frac{\sin x}{x}$

Describe the $\lim_{x\to 0} f(x)$

First, create a graph of *f* in an appropriate window.
Find the value of *f* at zero. Explain your answer.
View a table associated with $f(x)$ near zero. What does the table tell about the value of $f(x)$ when x is near zero?

- Activity #4: The Hot Cup of Tea Problem (T)

x	0	3	4	6	8	9
T(x)	180°	174°	172°	168°	164°	162°

A hot cup of tea is placed on a counter and left to cool. The temperature of the tea, in degrees Fahrenheit (correct to the nearest degree), x minutes after the cup is placed on the counter is modeled by a continuous function T(x) for $0 \le x < 10$. Values of T(x) at various times x are shown in the table above.

a) Evaluate: $\lim_{x\to 4} T(x)$. Justify the answer.

b) Using the data in the table, find the average rate of change in the temperature of the tea for $3 \le x \le 8$. Include units in the final answer.

c) Identify, using the times listed in the table, the shortest interval during which there must exist a time x for which the temperature of the tea is 166.5°. Justify the answer.

d) Use the data in the table to find the best estimate of the slop of the line tangent to the graph of T at x = 8.

- Activity #5: The Motion Problem (T)

  The position function $s(t) = -4.9t^2 + 396.9$ gives the height (in meters) of an object that has fallen from a height of 396.9 meters after t seconds.

  a) Explain why there must exist a time t, $1 < t < 2$, at which the height of the object must be 382 meters above the ground.

  b) Find the time at which the object hits the ground.

  c) Find the average rate of change in s over the interval [8, 9]. Include units of measure. Explain why this is a good estimate of the velocity at which the object hits the ground. How can this estimate be improved?

  d) Evaluate $\lim_{t \to 3} \frac{s(t) - s(3)}{t - 3}$. Show the work that leads to your answer. Include units.

- Activity #6: TI-NSpire Basic Limits (M)
  http://education.ti.com/en/timathnspired/us/detail?id=CC9F89F49B89439993768B40FC9CFEBF&t=5E2A88F117944527ACAEC97F6BF4FEB3

The following is suggested sequence of learning activities for the Accelerated Calculus class. Approximate days for completion : 15.

- YWBAT estimate a limit using a numerical or graphical approach
  - Activity #6
  - Activity #2
  - Activity #3
- YWBAT learn different ways that a limit can fail to exist
- YWBAT evaluate a limit using properties of limits
  - Activity #1
- YWBAT develop and use a strategy for finding limits
- YWBAT determine continuity at a point and continuity on an open interval
- YWBAT determine one-sided limits and continuity on a closed interval
  - Activity #6
- YWBAT use properties of continuity
- YWBAT understand and use the Intermediate Value Theorem
  - Activity #4
  - Activity #5
- YWBAT determine infinite limits from the left and from the right
- YWBAT find and sketch vertical asymptotes of the graph of a function
- Performance Tasks

Critical Vocabulary

- Limit
- Oscillating Behavior
- Unbounded Behavior
- Rationalizing
- Continuity
- One-sided Limit
- Open Interval
- Closed Interval
- Removable Discontinuity
- Non-removable Discontinuity
- Everywhere Continuous
- Continuous
- Infinite Limit
- Vertical Asymptote