

Pre-Calculus

Implement start year – 2014-2015

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Unit #4, Basic Trig and Trig Functions

Transfer Goal: Students will be able to independently use their learning to identify periodic phenomena and patterns used in real life.

Stage 1 – Desired Results

Established Goals

2009 NJCCC Standard(s), Strand(s)/CPI #

(<http://www.nj.gov/education/cccs/2009/final.htm>)

Common Core Curriculum Standards for Math and English

(<http://www.corestandards.org/>)

HSF-TF.A.3

- Extend the domain of trigonometric functions using the unit circle

HSF-TF.B.6

- Model periodic phenomena with trigonometric functions.

HSG-SRT.C

- Define trigonometric ratios and solve problems involving right triangles

21st Century Themes

(www.21stcenturyskills.org)

- Global Awareness
- Financial, Economic, Business and Entrepreneurial Literacy
- Civic Literacy
- Health Literacy
- Environmental Literacy

21st Century Skills

Learning and Innovation Skills:

- Creativity and Innovation
- Critical Thinking and Problem Solving
- Communication and Collaboration

Information, Media and Technology Skills:

- Information Literacy
- Media Literacy
- ICT (Information, Communications and Technology) Literacy

Life and Career Skills:

- Flexibility and Adaptability
- Initiative and Self-Direction
- Social and Cross-Cultural Skills
- Productivity and Accountability
- Leadership and Responsibility

<p><u>Enduring Understandings:</u> <i>Students will understand that . . .</i></p> <p><i>EU 1</i></p> <ul style="list-style-type: none"> • There are multiple ways to measure the various aspects of the Unit Circle <p><i>EU 2</i></p> <p>Periodic phenomena can be represented by trigonometric functions</p> <p><i>EU 3</i></p> <ul style="list-style-type: none"> • Different trigonometric function have different characteristics <p><i>EU 4</i></p> <ul style="list-style-type: none"> • Vocabulary plays a critical role in understanding trigonometric functions 	<p><u>Essential Questions:</u></p> <p><i>EU 1</i></p> <p>How is an appropriate unit of measure determined?</p> <p><i>EU 2</i></p> <p>How can periodic phenomena be represented using trigonometric functions?</p> <p><i>EU 3</i></p> <p>What are the characteristics of trigonometric functions?</p> <p><i>EU 4</i></p> <p>What role does critical vocabulary play in understanding trigonometric functions?</p>
<p><u>Knowledge:</u> <i>Students will know . . .</i></p> <p><i>EU 1</i></p> <p>Recognize the relationship between arc length and the measure of the angle in radians trigonometric values are ratios, that are constant regardless of the size of the triangle</p> <p><i>EU 2</i></p> <ul style="list-style-type: none"> • The period of a trigonometric function can represent different real life situations <p><i>EU 3</i></p> <p>The general shape of all six trigonometric functions</p> <p><i>EU 4</i></p> <ul style="list-style-type: none"> • The correct terms and usage for all six trigonometric functions 	<p><u>Skills:</u> <i>Students will be able to . . .</i></p> <p><i>EU 1</i></p> <p>Convert given angles from radians to degrees and vice versa Use right triangles to find trigonometric ratios</p> <p><i>EU 2</i></p> <ul style="list-style-type: none"> • Find and analyze examples in the real world that occurs in cycles <p><i>EU 3</i></p> <p>Identify and graph period, amplitude, vertical and horizontal shift for all six trigonometric functions</p> <ul style="list-style-type: none"> • Identify the domain and range, in interval notation, of the six trigonometric functions <p><i>EU 4</i></p> <ul style="list-style-type: none"> • Distinguish between each six trigonometric functions • Define the six trigonometric functions as they relate to the sides of a right triangle

Stage 2 – Assessment Evidence

Recommended Performance Tasks:

Performance Task Helpful Directions:

1. Your link on the Web is:
<http://tbone.biol.sc.edu/tide/sitesel.html>
2. Select a region from this page and then choose a site from the next page. Do NOT choose a site that ends in “current”. Do NOT choose a basin, bay or river.
3. Scroll down and select the prompt
4. Set the following two options: Change “Select Presentation Options” to 3 days
Change “Starting Time and Time Display Options” to start sometime between April 1 and August 15, 2009 at 0:00.
5. Click on
6. Print out the data (just the first page) that shows the dates, times and tidal heights. You will include this print-out in your project. *Be sure that you have 2 high tides and 2 low tides for ALL THREE days.* Pick a location where the tides are at least 3 feet or meters in difference.
7. Using the data from your print-out, convert all times (hours and minutes) into decimal hours by dividing the minutes by 60 and round to 2 decimal places. Ex. 2:15 = 2.25
8. Find the equation of the sine curve that best fits this data by following the directions on the subsequent data page.
9. Put your information into a T-chart, using time for the independent variable (x-axis) and tide height for the dependent variable (y-axis). Your project begins at “time 0.00 on Day 1 and ends with time 72.00 on Day 3”. You need to add 24 hours to all your times on Day 2 and 48 hours to all your times on Day 3 before you graph, so that your x-values run from 1 -72.
10. Using graph paper or the computer, plot the data from your T-chart, connecting the points with a smooth curve (not segments) and scaling the axes according to your needs. The two axes may be scaled differently. You will have two graphs on one set of axes – one from the raw data and the second from your cosine equation. Be sure to state the cosine equation with your graph.

Presentation: Your final project is due on _____. You may turn your project in early and late projects will lose 10% for each

day late. Your project must be typed, double-spaced and must include, in this order

*A cover page that includes:

* Page 1: An introduction to the project, including the location you chose, the dates selected and why you chose this particular place and time of year. Give a little background of the place you selected. Is it a tourist place? Are the tides consistent all year round or are they higher at certain times of the year? In other words, do some research on your location, Include a picture, if possible.

* Page 2: The print-out of the tides from the Internet

* Page 3: The “data page”, including the T-chart with your original data. Make sure your mathematical calculations are correct!

* Page 4: The graph of the raw data and the cosine equation on the same set of axes. State the cosine equation.

* Page 5: A summary of the project answering these questions –

1) State any tendencies that you saw in the high and low tides. For example, were there any consistencies between the time or height of the high/low tides from day to day

2) Is there a predictable pattern?

3) Might the moon have had an influence on this pattern? (your print-out might have information about this)

4) State at least one other natural phenomena that is also predictable by means of a periodic sine or cosine wave or curve.

5) Predict the height of the tidal wave at 11am on the 6th day. Explain how you got it.

6) Predict the height of the tidal wave at 6 am on March 1, 2011. Explain how you got it.

FINDING THE EQUATION OF YOUR SINUSOID

Analysis: Follow the directions below for finding a cosine equation that best fits your data. Your equation will be in the general form $y = a \cos (bx-h) + d$. Throughout this page, round your decimals to three places.

a) Find “d”, the vertical shift. The vertical shift is the **average** of the average of your high tides and the average of your low tides (the average of the averages)

b) Find “a”, the amplitude. The amplitude is the distance between your average height and “d” (or your average low and “d”). (Hint-you may need to make “a “ negative. Look at your data and decide.)

c) Enter “b”. Use a half a lunar day as the period for the tides, 12 hours and 24 minutes (12.4).

Since $P = 12.4$, $b = 2\pi/12.4$ or $5\pi/31$. Your value of “b” is : 5π/31

d) Find “h”, the horizontal shift. Let h equal the time of your first tide.

e) Put it all together and write the equation of the cosine curve in the form from $y = a \cos (b(x-h)) + d$.

Create a T-chart like the one in the sample below and include it with your project. The y-values for your sinusoid can be found by calculating the max and min of the cosine equation using you graphing calculator. Enter your equation into $y_1=$. Remember that all high tides will be 12.4 hours apart and all low tides will be 12.4 hours apart.

Other Recommended Evidence: *Tests, Quizzes, Prompts, Self-assessment, Observations, Dialogues, etc.*

- Miniquiz on all angles in Unit Circle in degrees and radians
- Quiz on degrees, radians, and ordered pairs in the Unit Circle
- Quiz on Right Triangle Trig and Unit Circle
- Quiz on graphing all six trig functions

- Quiz on evaluating inverse trig functions
- Quiz on right triangle trig word problems
- Assessed elements from recommended performance task.

Stage 3 – Learning Plan

Suggested Learning Activities to Include Differentiated Instruction and Interdisciplinary Connections: *Consider the WHERETO elements. Each learning activity listed must be accompanied by a learning goal of A= Acquiring basic knowledge and skills, M= Making meaning and/or a T= Transfer.*

- Activity #1: Unit Circle Contest – Students form a circle. Teacher stands in the center and holds up giant flash cards containing Unit Circle angles (example: $\sin(5\pi/6)$). A student must answer correctly within a certain time limit, or the student is eliminated. Continue the game until there is one student left. They win the PRIZE (example: a Unit Circle Cookie Cake)! (A)
- Activity #2: Students construct a circle and cut out a piece of string that is the length of the radius. Then, they count how many times the piece of string makes one revolution around the circle. This represents the circumference for any circle. (M)
 - Activity Guidelines are attached.
- Activity #3: Find 5 real life examples that have a periodic phenomena, explain why they are periodic and how they relate to sine and cosine functions. (T)
- More activities (including instructions and rubrics) are available at <http://precalculus.lrhdsd.org>

The following is a suggested sequence of learning activities for the Accelerated Pre-Calculus class. Approximate days for completion: 35. Adjustments should be made accordingly for other levels.

- YWBAT covert radians to degrees and degrees to radians (A, M)
- YWBAT label special angles in Unit Circle in radians and degrees (A)
- YWBAT find coterminal angles (A)
- YWBAT find reference angles (A)
- YWBAT derive the ordered pairs in the Unit Circle using right triangles (A, M)
 - Activity #1
 - Activity #2
- YWBAT find all six trig values of any angle (special or non-special) (A)
- YWBAT find the angle given the trig value (special angles) (A)

- YWBAT graph sine and cosine functions (A)
 - Activity #3
- YWBAT graph cosecant and secant (A)
- YWBAT graph tangent and cotangent (A)

- YWBAT evaluate inverse trig functions (A)

- YWBAT solve right triangle trig word problems involving angle elevation, angle of depression, and bearing angle (M)

- Performance Task (M, T)

Critical Vocabulary

Radians	Degrees	Arc Length	Terminal Side	Initial Side	Standard Position
Coterminal Angle	Reference Angle	Sine	Cosine	Tangent	Cosecant
Secant	Cotangent	Pythagorean Theorem	Period	Amplitude	Vertical/Horizontal Shifts
Inverse Trig	Angle Elevation	Angle Depression	Bearing Angle		

Activity #2 Worksheet – Discovering the Sine and Cosine Curves

Objective: Today we will discover the graphs of sine and cosine by transferring the Unit Circle to an x-y coordinate plane.

Warm Up Question:

1.) When we used the Unit Circle for Trigonometry, everything we talked about was angles (θ) and values (coordinates). Given that a trig function is generally written as $y = \sin \theta$ or $y = \cos \theta$, when we transfer to an x-y coordinate plane, what would represent the x axis? What would represent the y axis?

2.) When we were first learning trig functions and the Unit Circle, we said that sine represented the _____ side of the reference triangle or the _____ coordinate, and cosine represented the _____ side of the reference triangle or the _____ coordinate.

Set Up:

Each group of 4 people will need:

Unit Circle paper

1 Sine Grid and 1 Cosine Grid

1 piece of tin foil

1 bottle of glue

Dry Spaghetti (I will hand out)

Twizzler Pull and Peels (I will hand out)

DO NOT EAT THE SPAGHETTI!!!!

You can eat the left over Twizzlers AFTER the activity is over.

NOTE – THERE SHOULD NOT BE ANY PIECES OF SPAGHETTI ON THE FLOOR WHEN YOU LEAVE THIS ROOM!!!!!!

Activity:

Step 1 – Set Up

The grid paper that you have is missing some of our Unit Circle Angles. Label the angles formed by the 45° reference angles on both grids; write the angles right above the x-axis.

Step 2

On the unit circle, use the spaghetti to measure the length of the side associated with your trig function (see warm up question #2), for each degree measurement marked out on the grid. Break the spaghetti to match this length.

Step 3

Glue each piece of spaghetti to its corresponding mark on the x-axis, perpendicular to the x-axis.

REPEAT STEPS 2 AND 3 FOR EACH ANGLE MARKED ON THE GRAPH.

***** Don't forget about 90, 180, 270, and 360!!!**

Are there any patterns you've noticed that can help you save time?

If the distance you measured was negative, where on the graph should the spaghetti be glued???

Step 4

Write the correct y-values on the y-axis, marking the end of each piece of spaghetti.

Step 5

After you've finished with all of the angles, the ends of the spaghetti will form a curve that resembles a wave. Pull the Twizzlers apart and glue them to the graph to follow the wave. The Twizzlers show the actual sine or cosine wave.

Step 6

Cover the pieces of spaghetti with tape to prevent them from falling off (we're going to hang them in the room), and tape all three pieces of paper together in this order: sine curve, unit circle, cosine curve.

NOTE – THERE SHOULD NOT BE ANY PIECES OF SPAGHETTI ON THE FLOOR WHEN YOU LEAVE THIS ROOM!!!!

After you have finished making the curve, answer the following questions.

- 1.) What is the radius of the unit circle?

- 2.) What is the highest point on your graph?

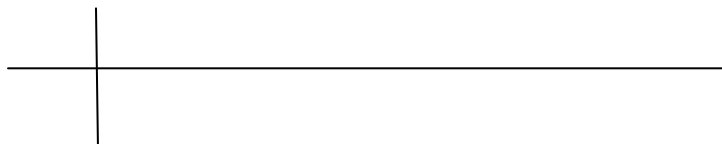
- 3.) What is the range of your graph?

- 4.) How many degrees are in a circle?

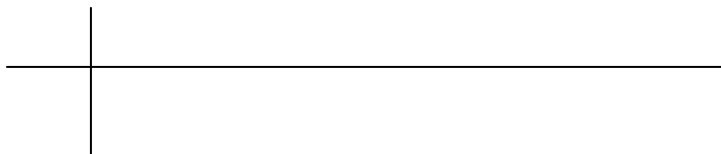
- 5.) What would happen if we went around the circle more than once? How long does it take your graph to repeat? (This is called the period of the curve.)
How many times does it repeat?

6.) What is the domain of your graph?

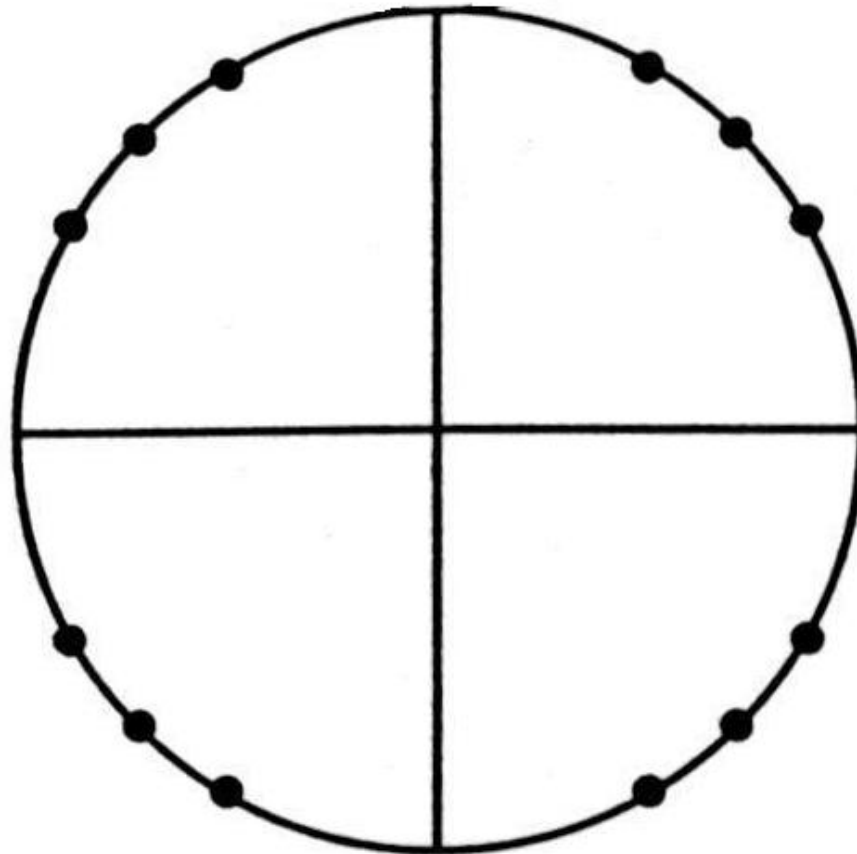
7.) Draw two periods of the sine curve below.



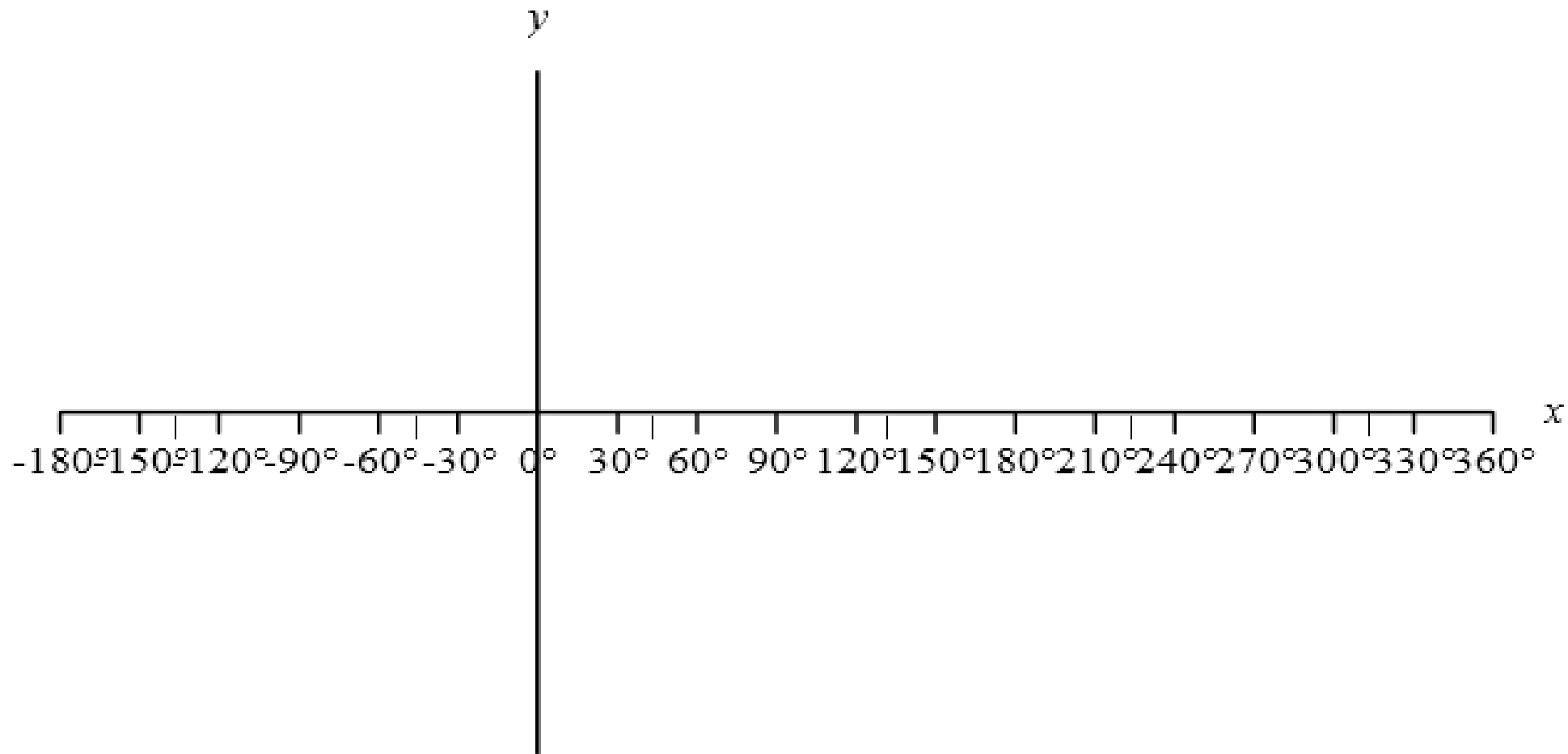
8.) Draw two periods of the cosine curve below.



The Unit Circle



The Sine Curve

$$y = \sin x$$


The Cosine Curve

$y = \cos x$

