AP Calculus BC Summer Assignment 2023

There are 3 parts to the AP Calculus BC Summer Assignment:

1. 4 Free Response 10-Point Problems to be submitted through Canvas on the first day of class (10 points each) - These are to be completed independently and without a calculator. You must show all work to receive full credit.

2. Multiple Choice Questions to review topics in Honors Calculus - These are to be completed and checked with the answers at the end of the packet. Show all work to receive full credit.

3. Antidifferentiation Circuit - The circuit is to be completed, showing all work in order to receive full credit.

Parts 2 and 3 will be graded on completion and will count 30 points. Please be sure to give yourself adequate time to complete this work. This will help refresh you going into the 2nd part of BC Calculus.

AP Calculus BC
JBHA - Strick
Summer Assignment - 4 10-Point Problems

Name:____________________________

Date:____________________________

1. (No calculator)

Given the function $g(x) = 3x^4 + x^3 - 21x^2$

a) Find an equation of the tangent line at the point (2, -28).

b) Find the absolute minimum value of the function. Show the analysis that leads to your conclusion.

c) Find the x-coordinate of each point of inflection. Show the analysis.

2. (No calculator) The velocity of a particle moving on the x-axis is given by $v(t) = 12t^2 - 36t + 15$ for $t \geq 0$. At t=1, the particle is at the origin.

a) Find the position s(t) of the particle at any time $t \geq 0$.

b) Find all values of t for which the particle is at rest.

c) Find the maximum velocity of the particle for $0 \leq t \leq 2$.

d) Find the total distance traveled by the particle from t=0 to t=2.

3. (No Calculator) Given f, a function defined for all $x \neq 0$ with $f(4) = -3$ and $f'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

a) Find all values of x for which the graph of f has a horizontal tangent. Then determine whether f has a relative maximum, a relative minimum or neither at each of these values. Justify your answer.

b) On what intervals, if any, is the graph of f concave up? Justify.

c) Write an equation for the tangent line of f at x=4.

d) Does the line tangent to the graph of f at x=4 lie above or below the graph of f for x>4? Why?

4. No Calculator

Given the curve $2 + xy = y^2$

a) Show that $\frac{dy}{dx} = \frac{y}{2y - x}$

b) Find all points on the curve where the line tangent to the curve has slope ½.

c) Show that there are no points (x,y) on the curve where the line tangent to the curve is horizontal.

d) Let x and y be functions of time t that are related by the equation $2 + xy = y^2$.

At time t=5, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time t=5.

MULTIPLE CHOICE (NO CALCULATORS)
Answers at the end of the problems.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. Let $f(x) = 4x^3 - 3x - 1$. An equation of the line tangent to $y = f(x)$ at $x = 2$ is

(A) $y = 25x - 5$

(B) $y = 45x + 65$

(C) $y = 45x - 65$

(D) $y = 65 - 45x$

(E) $y = 65x - 45$

2. If $g(x) = x + \cos x$, then $\lim_{h \to 0} \frac{g(x+h) - g(x)}{h} =$

(A) $\sin x + \cos x$

(B) $\sin x - \cos x$

(C) $1 - \sin x$

(D) $1 - \cos x$

(E) $x^2 - \sin x$

3. $\lim_{h \to 0}\left(\frac{\cos(x+h)-\cos x}{h}\right)=$

(A) $\sin x$

(B) $-\sin x$

(C) $\cos x$

(D) $-\cos x$

(E) does not exist

4. On which of the following intervals, is the graph of the curve $y = x^5 - 5x^4 + 10x + 15$ concave up ?

I. $x < 0$ II. $0 < x < 3$ III. $x > 3$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) II and III only

5. If $g(x) = \sqrt[3]{x-1}$ and f is the inverse function of g, then $f'(x) =$

(A) $3x^2$

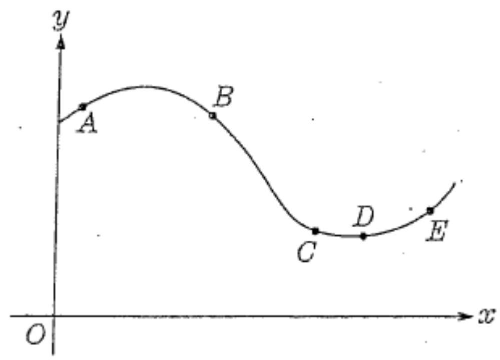
(B) $3(x-1)^2$

(C) $-\frac{1}{3}(x-1)^{-4/3}$

(D) $\frac{1}{3}(x-1)^{2/3}$

(E) does not exist

6. At which of the five points on the graph in the figure at the right are $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both negative?



(A) A
(B) B
(C) C
(D) D
(E) E

7. If the line $3x - y + 2 = 0$ is tangent in the first quadrant to the curve $y = x^3 + k$, then $k =$

(A) 5

(B) –5

(C) 4

(D) 1

(E) –1

8. $\frac{d}{dx}[\text{Arctan } 3x] =$

(A) $\frac{1}{1+9x^2}$

(B) $\frac{3}{1+9x^2}$

(C) $\frac{3}{\sqrt{4x^2-1}}$

(D) $\frac{3}{1+3x}$

(E) none of the above

9. What is $\lim_{x\to\infty} \frac{x^2-6}{2+x-3x^2}$?

(A) -3 (B) $-\frac{1}{3}$ (C) $\frac{1}{3}$ (D) 2 (E) The limit does not exist.

10. Let $y = 2e^{\cos x}$. Both x and y vary with time in such a way that y increases at the constant rate of 5 units per second. The rate at which x is changing when $x = \frac{\pi}{2}$ is

(A) 10 units/sec

(B) -10 units/sec

(C) -2.5 units/sec

(D) 2.5 units/sec

(E) -0.4 units/sec

11. If $g(x) = \dfrac{x-2}{x+2}$, then $g'(2) =$

(A) 1

(B) –1

(C) $\frac{1}{4}$

(D) $-\frac{1}{4}$

(E) 0

12. The slope of the tangent to the curve $y^3x + y^2x^2 = 6$ at (2, 1) is

(A) $-\frac{3}{2}$

(B) -1

(C) $-\frac{5}{14}$

(D) $-\frac{3}{14}$

(E) 0

13. The fourth derivative of $f(x) = (2x-3)^4$ is

(A) $24\left(2^4\right)$

(B) $24\left(2^3\right)$

(C) $24(2x-3)$

(D). $24\left(2^5\right)$

(E) 0

14. If $x^2 + 2xy - 3y = 3$, then the value of $\frac{dy}{dx}$ at $x = 2$ is

(A) 1

(B) 2

(C) -2

(D) $\frac{10}{3}$

(E) $-\frac{1}{2}$

15. If $\tan(x+y) = x$, then $\frac{dy}{dx} =$

(A) $\tan^2(x+y)$

(B) $\sec^2(x+y)$

(C) $\ln|\sec(x+y)|$

(D) $\sin^2(x+y) - 1$

(E) $\cos^2(x+y) - 1$

16. If $f(x) = e^{2x}$ and $g(x) = \ln x$, then the derivative of $y = f(g(x))$ at $x = e$ is

(A) e^2

(B) $2e^2$

(C) $2e$

(D) 2

(E) undefined

17. Let f be the function defined by $f(x) = x^{2/3}(5-2x)$. f is increasing on the interval

(A) $x < -\frac{5}{2}$ (B) $x > 0$ (C) $x < 1$ (D) $0 < x < \frac{5}{8}$ (E) $0 < x < 1$

18. If $h(x) = (x^2 - 4)^{3/4} + 1$, then the value of $h'(2)$ is

(A) 3

(B) 2

(C) 1

(D) 0

(E) does not exist

19. The derivative of $\sqrt{x} - \dfrac{1}{x\sqrt[3]{x}}$ is

(A) $\frac{1}{2}x^{-1/2} - x^{-4/3}$

(B) $\frac{1}{2}x^{-1/2} + \frac{4}{3}x^{-7/3}$

(C) $\frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-1/3}$

(D) $-\frac{1}{2}x^{-1/2} + \frac{4}{3}x^{-7/3}$

(E) $-\frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-1/3}$

20. The function f is continuous at $x = 1$.

If $f(x) = \begin{cases} \dfrac{\sqrt{x+3} - \sqrt{3x+1}}{x-1} & \text{for } x \neq 1 \\ k & \text{for } x = 1 \end{cases}$ then $k =$

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$ (E) none of the above

21. An equation of the normal to the graph of $f(x) = \dfrac{x}{2x-3}$ at $(1, f(1))$ is

(A) $3x + y = 4$

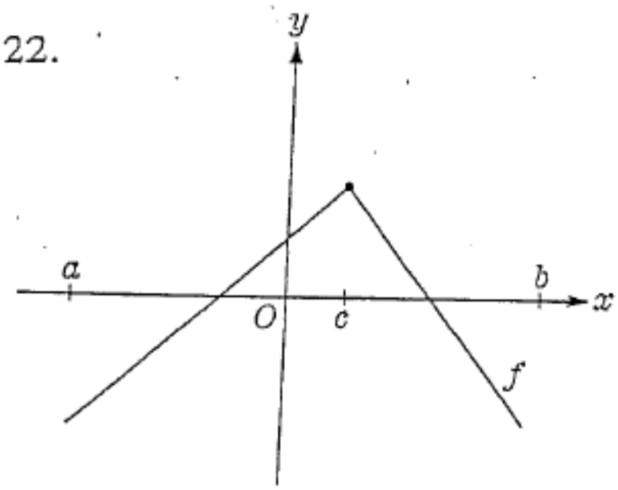
(B) $3x + y = 2$

(C) $x - 3y = -2$

(D) $x - 3y = 4$

(E) $x + 3y = 2$

22.



The function f, whose graph consists of two line segments, is shown above. Which of the following are true for f on the open interval (a, b)?

I. The domain of the derivative of f is the open interval (a, b).
II. f is continuous on the open interval (a, b).
III. The derivative of f is positive on the open interval (a, c).

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III

23. What are all values of k for which the graph of $y = 2x^3 + 3x^2 + k$ will have three distinct x-intercepts?

(A) all $k < 0$

(B) all $k > -1$

(C) all k

(D) $-1 < k < 0$

(E) $0 < k < 1$

24. The function f, for which $f'(x) = (x-2)^2(x-7)^3$, has an inflection point where $x =$

(A) 4 only
(B) 7 only
(C) 2 and 4 only
(D) 2 and 7 only
(E) 2 and 4 and 7

25. Suppose that g is a function that is defined for all real numbers. Which of the following conditions assures that g has an inverse function?

(A) $g'(x) < 1$, for all x

(B) $g'(x) > 1$, for all x

(C) $g''(x) > 0$, for all x

(D) $g''(x) < 0$, for all x

(E) g is continuous.

26. If $f(x) = \sin^2(3 - x)$, then $f'(0) =$

(A) $-2 \cos 3$
(B) $-2 \sin 3 \cos 3$
(C) $6 \cos 3$
(D) $2 \sin 3 \cos 3$
(E) $6 \sin 3 \cos 3$

27.

Which of the following statements about the function given by $f(x) = x^4 - 2x^3$ is true?

(A) The function has no relative extremum.
(B) The graph of the function has one point of inflection and the function has two relative extrema.
(C) The graph of the function has two points of inflection and the function has one relative extremum.
(D) The graph of the function has two points of inflection and the function has two relative extrema.
(E) The graph of the function has two points of inflection and the function has three relative extrema.

MULTIPLE CHOICE PART 2: GRAPHING CALCULATOR ACTIVE

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS IN THIS PART OF THE EXAMINATION.

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. Which of the following functions have a derivative at $x = 0$?

I. $y = \left|x^3 - 3x^2\right|$

II. $y = \sqrt{x^2 + .01} - |x - 1|$

III. $y = \dfrac{e^x}{\cos x}$

(A) None (B) II only (C) III only (D) II and III only (E) I, II, III

2. The derivative of the function g is $g'(x) = \cos(\sin x)$. At the point where $x = 0$ the graph of g

I. is increasing, II. is concave down, III. attains a relative maximum point.

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, III

3. Consider the function $f(x) = \dfrac{6x}{a + x^3}$ for which $f'(0) = 3$. The value of a is

(A) 5
(B) 4
(C) 3
(D) 2
(E) 1

4. Which of the following is true about the function f if $f(x) = \dfrac{(x-1)^2}{2x^2-5x+3}$?

I. f is continuous at $x = 1$.

II. The graph of f has a vertical asymptote at $x = 1$.

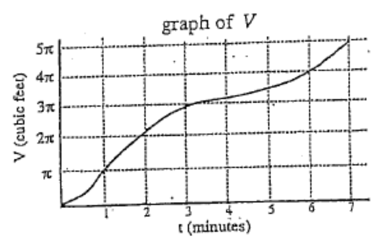
III. The graph of f has a horizontal asymptote at $y = \dfrac{1}{2}$.

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, III

5. If $y = u + 2e^u$ and $u = 1 + \ln x$, find $\dfrac{dy}{dx}$ when $x = \dfrac{1}{e}$

(A) e (B) $2e$ (C) $3e$ (D) $\dfrac{2}{e}$ (E) $\dfrac{3}{e}$

6. Sand is being dumped on a pile in such a way that it always forms a cone whose base radius is always 3 times its height. The function V whose graph is sketched in the figure gives the volume of the conical sand pile, $V(t)$, measured in cubic feet, after t minutes. $\left(V(t) = \frac{1}{3}\pi r^2 h\right)$ At what approximate rate is the radius of the base changing after 6 minutes?



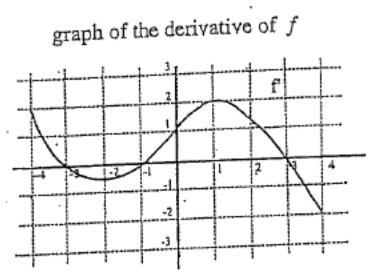
(A) 0.22 ft/min (B) 0.28 ft/min (C) 0.34 ft/min (D) 0.40 ft/min (E) 0.46 ft/min

7. The graph of the **derivative** of a function f is shown to the right. Which of the following is true about the function f?

I. f is increasing on the interval (–2, 1).

II. f is continuous at $x = 0$.

III. f has an inflection point at $x = -2$.



(A) I only (B) II only (C) III only (D) II and III only (E) I, II, III

8. The graph of $y = \frac{\sin x}{x}$ has

I. a vertical asymptote at $x = 0$

II. a horizontal asymptote at $y = 0$

III. an infinite number of zeros

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

9. If f is differentiable at $x = a$, which of the following could be false?

(A) f is continuous at $x = a$.
(B) $\lim_{x \to a} f(x)$ exists.
(C) $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ exists.
(D) $f'(a)$ is defined.
(E) $f''(a)$ is defined.

10. If $\frac{d}{dx} f(x) = g(x)$ and if $h(x) = x^2$, then $\frac{d}{dx} f(h(x)) =$

(A) $g(x^2)$
(B) $2xg(x)$
(C) $g'(x)$
(D) $2xg(x^2)$
(E) $x^2g(x^2)$

11. If $y = \sin u$, $u = v - \frac{1}{v}$, and $v = \ln x$, then value of $\frac{dy}{dx}$ at $x = e$ is

(A) 0

(B) 1

(C) $\frac{1}{e}$

(D) $\frac{2}{e}$

(E) $\cos e$

12. If $\frac{d}{dx}[f(x)] = g(x)$ and $\frac{d}{dx}[g(x)] = f(3x)$, then $\frac{d^2}{dx^2}\left[f(x^2)\right]$ is

(A) $4x^2 f(3x^2) + 2g(x^2)$

(B) $f(3x^2)$

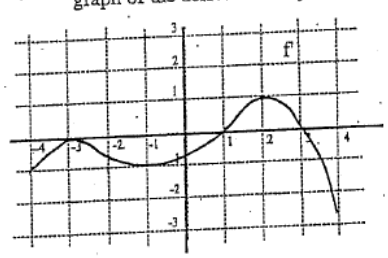
C) $f(x^4)$

(D) $2xf(3x^2) + 2g(x^2)$

(E) $2xf(3x^2)$

13. The figure shows the graph of f', the *derivative* of a function f. The domain of f is the interval $-4 \le x \le 4$. Which of the following are true about the graph of f?

graph of the derivative of f



I. At the points where $x = -3$ and $x = 2$ there are horizontal tangents.

II. At the point where $x = 1$ there is a relative minimum point.

III. At the point where $x = -3$ there is an inflection point.

(A) None (B) II only (C) III only (D) II and III only (E) I, II, III

14. Consider the function $f(x) = (x^2 - 5)^3$ for all real numbers x. The number of inflection points for the graph of f is

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

15. Let f be defined as follows, where $a \neq 0$.

$$f(x) = \begin{cases} \dfrac{x^2 - a^2}{x - a}, & \text{for } x \neq a, \\ 0, & \text{for } x = a. \end{cases}$$

Which of the following are true about f?

I. $\lim_{x \to a} f(x)$ exists.
II. $f(a)$ exists.
III. $f(x)$ is continuous at $x = a$.

(A) None
(B) I only
(C) II only
(D) I and II only
(E) I, II, and III

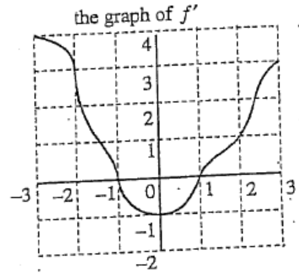
16. At how many points on the inteval $-2\pi \leq x \leq 2\pi$ does the tangent to the graph of the curve $y = x \cos x$ have slope $\frac{\pi}{2}$?

(A) 5
(B) 4
(C) 3
(D) 2
(E) 1

17. The graph of f', the derivative of a function f, is shown at the right.

Which of the following statements are true about the function f?

I. f is increasing on the interval $(-2, -1)$.
II. f has an inflection point at $x = 0$.
III. f is concave up on the interval $(-1, 0)$.



(A) I only (B) II only (C) III only (D) I and II only (E) II and III only

MULTIPLE CHOICE ANSWERS:

SECTION 1: NO CALCULATORS

1. C
2. C
3. B
4. C
5. A
6. B
7. C
8. B
9. B
10. C
11. C
12. C
13. A
14. C
15. E
16. C
17. E
18. E
19. B
20. D
21. D
22. D
23. D
24. C
25. B
26. B
27. C

SECTION 2: CALCULATOR ACTIVE

1. E
2. A
3. D
4. C
5. C
6. B
7. D
8. E
9. E
10.D
11. D
12. A
13. D
14. D
15. D
16. B
17. D

Circuit Training – Anti differentiation **Name** ________________________

Directions: Begin in cell #1. In each cell you will find the first derivative and a point on the original graph. Use that information to write the equation of the original function (or relation in the last few cases). You should not need a calculator to work any of these problems! Attach additional sheets of paper where you worked some of these if the box is too small. If you do everything correctly, you will have the supreme satisfaction of closing the circuit by coming back to the beginning.

Answer: $3 + e^3 + \ln 5$ # __1__ $f'(x) = 3x^2 - 2x + 7$, $f(0) = 5$ To advance in the circuit, locate $f(1)$.	Answer: $\frac{7\pi+18}{6}$ # ______ $f'(x) = 2x\cos(x^2)$, $f\left(\sqrt{\frac{7\pi}{6}}\right) = 2$ To advance in the circuit, locate $f\left(\frac{\sqrt{\pi}}{2}\right)$.
Answer: $\frac{5}{2}$ # ______ $h'(t) = -\frac{1}{t^2} + \frac{1}{t}$ $h(1) = 1$ To advance in the circuit, locate $h(\sqrt{e})$.	Answer: $4 + e$ # ______ $f'(x) = \frac{1}{\sqrt{1-x^2}}$ $f(0) = 2$ To advance in the circuit, locate $f(1)$.
Answer: $\frac{3}{2}$ # ______ $y' = \sec^2 x + 3x$, $y(0) = 3$ To advance in the circuit, locate $y\left(\frac{\pi}{4}\right)$.	Answer: $\frac{1}{3}$ # ______ $f''(x) = -10$, $f'(0) = 10$, $f(0) = 30$ To advance in the circuit, locate $f(-1)$.
Answer: $x^2 - y^2 = 25$ # ______ If $\frac{dy}{dx} = -\frac{x}{y}$, and $(-3, 4)$ solves the differential equation, then the original equation could have been…	Answer: $\frac{2+\sqrt{e}}{2\sqrt{e}}$ # ______ $f'(x) = 2x + \sin x$ $f(2\pi) = 4\pi^2 + 3$ To advance in the circuit, locate $f(0)$.
Answer: $2\sqrt{2} - 3$ # ______ $x'(t) = 2\cos t - 3\sin t$ $x(0) = \frac{7}{2}$ To advance in the circuit, locate $x\left(\frac{\pi}{2}\right)$.	Answer: $\frac{\sqrt{2}+5}{2}$ # ______ $f'(x) = \frac{1}{x}$ $f(e) = -3$ To advance in the circuit, locate $f(e^4)$.

Answer: 15 # ______ $f'(x) = \frac{72}{\pi^2}x + \cos x, \quad f\left(\frac{\pi}{2}\right) = 10$ To advance in the circuit, locate $f\left(\frac{\pi}{6}\right)$.	Answer: 12 # ______ $f'(x) = -8x^3 + 3, \quad f(1) = 3$ To advance in the circuit, locate $f(0)$.
Answer: $x^2 + y^2 = 25$ # ______ $f'(x) = \frac{3}{3x+2} + 3e^{3x} \quad f(0) = 4 + \ln 2$. To advance in the circuit, locate $f(1)$.	Answer: 3 # ______ Which of the following remaining answers has $\frac{dy}{dx} = \frac{x}{y}$ as its first derivative?
Answer: $\frac{128 + 3\pi^2}{32}$ # ______ $y' = \csc x \cot x, \quad y\left(\frac{\pi}{2}\right) = 6$ To advance in the circuit, locate $y\left(\frac{11\pi}{6}\right)$.	Answer: 2 # ______ $g'(x) = 5x^4 + 12x^2 - 9, \quad g(0) = 1$ To advance in the circuit, locate $g(-1)$.
Answer: 0 # ______ $f'(x) = \frac{1}{x} + 1 \quad f(1) = 4$ To advance in the circuit, locate $f(e)$.	Answer: $\frac{4+\pi}{2}$ # ______ $g'(x) = \frac{1}{2\sqrt{x}} \quad g(25) = 2$ To advance in the circuit, locate $g(8)$.
Answer: 5 # ______ $h'(t) = t^2 - t + 1, \quad h(0) = -\frac{1}{2}$ To advance in the circuit, locate $h(1)$.	Answer: 9 # ______ $f'(x) = \frac{2}{1+x^2} \quad f(-1) = 3$. To advance in the circuit, locate $f(\sqrt{3})$.