Name _____ Summer 2024
Anderle/Bode/Kahan/Yoo

Calculus Summer Assignment 2024
No Calculator Allowed
To Be Completed by: Friday, August 30th, 2024

The problems in this assignment are to be completed without the use of a calculator of any kind. Show all of your work and clearly indicate final answers; **any form of a correct answer will receive full credit**. You may use your notes from Precalculus, textbooks, online sources or a peer as needed. Your work must be completed by the date above, as your first exam will include questions on the topics included in this assignment.

These topics are:

1. Limits
   a) by substitution
   b) using algebraic simplification then substitution
   c) one-sided limits and piecewise functions
   d) around a vertical asymptote
   e) trigonometric limits
   f) as x approaches $\pm\infty$
   g) graphically

2. Derivatives
   a) Definition of the Derivative
   b) Power Rule
   c) Equation of a tangent line
   d) Horizontal tangent line
   e) Product rule
   f) Quotient rule
   g) Trigonometric Functions
   h) Chain Rule

3. Continuity of a Function
   a) Definition of Continuity
   b) Types of discontinuities – P.O.D., Vertical Asymptote, Jump

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I. Calculate each limit:

1. $\lim_{x\to 0} \dfrac{x-1}{x^2-1}$

2. $\lim_{x\to 3} \dfrac{x-3}{\frac{1}{x}-\frac{1}{3}}$

I. Calculate each limit (cont.)

3. $\lim_{x\to 9} \dfrac{\sqrt{x}-3}{x-9}$

4. $\lim_{x\to 0} \dfrac{(x-1)^2-1}{x}$

Remember: $\lim_{x\to a} f(x)$ **exists if and only if** $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$**.**

5. $\lim_{x\to 3} f(x)$ if $f(x) = \dfrac{x}{x-3}$

6. $\lim_{x\to 3} g(x)$ if $g(x) = \dfrac{x}{(x-3)^2}$

7. Let $f(x) = \begin{cases} x^2+3, & x \le -2 \\ 5-x, & x > -2 \end{cases}$

Find: a) $\lim_{x\to -2^-} f(x)$ b) $\lim_{x\to -2^+} f(x)$ c) $\lim_{x\to -2} f(x)$

I. Calculate each limit (cont.)

8. Let $f(x)=\begin{cases} x+2, & x<0 \\ \sqrt{x}+2, & 0\le x<1 \\ \ln x, & x\ge 1 \end{cases}$

Find: a) $f(0)$ b) $\lim_{x\to 0} f(x)$

c) $f(1)$ d) $\lim_{x\to 1} f(x)$

e) Is $f(x)$ continuous at $x=0$? Justify your answer.

f) Is $f(x)$ continuous at $x=1$? Justify your answer.

Some special trigonometric limits

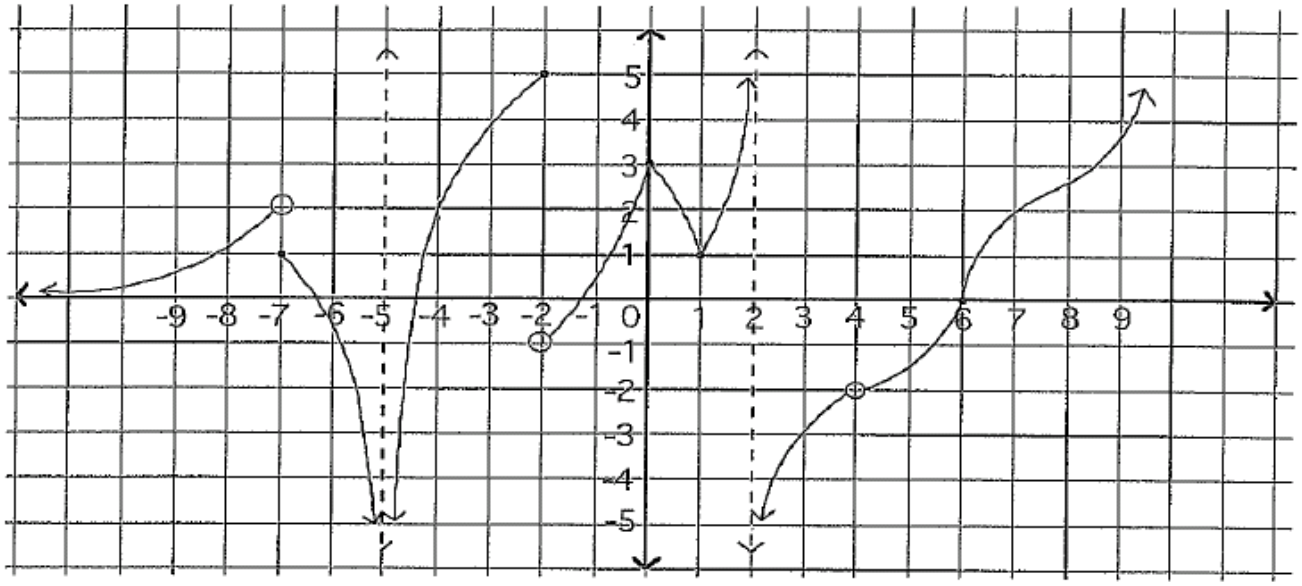
9. $\lim_{x\to 0} \dfrac{\sin x}{x}$

10. $\lim_{x\to 0} \dfrac{5\sin x}{4x}$

11. $\lim_{x\to 0} \dfrac{\sin x - x}{3x}$

12. $\lim_{x\to 0} \dfrac{\cos x \tan x}{x}$

Calculate each limit based on the graph of $f(x)$ is shown below:



13 a) $\lim_{x\to -2^-} f(x)$ b) $\lim_{x\to -2^+} f(x)$ c) $\lim_{x\to -2} f(x)$

d) $\lim_{x\to 4^-} f(x)$ e) $\lim_{x\to 4^+} f(x)$ f) $\lim_{x\to 4} f(x)$

g) $\lim_{x\to 2^-} f(x)$ h) $\lim_{x\to 2^+} f(x)$ i) $\lim_{x\to 2} f(x)$

j) $\lim_{x\to -7} f(x)$ k) $\lim_{x\to -5} f(x)$ l) $\lim_{x\to 1} f(x)$

m) $\lim_{x\to -\infty} f(x)$ n) $\lim_{x\to \infty} f(x)$

p) State each value of x at which $f(x)$ is not continuous and state the type of discontinuity.

Limits as $x \to \infty$

14. $\lim_{x\to\infty} \dfrac{1-x^2-x^4}{3x^4+1}$

15. $\lim_{x\to\infty} \dfrac{1-x^2-x^3}{3x^4+1}$

16. $\lim_{x\to\infty} \dfrac{1-x^2-x^4}{3x^3+1}$

17. $\lim_{x\to-\infty} \dfrac{1-x^2-x^4}{3x^3+1}$

II. Derivatives

Limits involving the definition of the derivative…each limit is not calculated as a limit!

18. $\lim_{h\to0} \dfrac{\tan(x+h)-\tan(x)}{h}$

19. $\lim_{x\to9} \dfrac{\sqrt{x}-3}{x-9}$

Derivatives Power Rule

Find y' for each:

20. y = 3

21. $y = -3x^2 + 5x - 7$

22. $y = \sqrt{x} - \frac{5}{\sqrt{x}} + \frac{3}{x^3}$

Derivatives Product Rule

23. Find $\frac{dy}{dx}$ using the product rule. Verify by distributing first then finding $\frac{dy}{dx}$ if $y = (x^2 + 3x)(x + 5)$.

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24. Find $\frac{dy}{dx}$ if $y = x^2 \sin x$

Derivatives Quotient Rule

Find y' for each:

25. $y = \dfrac{3x-2}{2x-3}$

26. $y = \dfrac{x+1}{\sqrt{x}}$

Derivatives Trigonometric Functions

Find y' for each:

27. y = sin(x) cot(x)

28. y = sec(x) tan(x)

29. $y = \dfrac{\cos(x)}{\sec(x)}$

Derivatives Using the Chain Rule

Find y' for each:

30. $y = (4x^2 + 1)^3$

31. $y = \cos(x^2)$

32. $y = \sin^3(\cos(x^2))$

33. $y = \sqrt{x + \tan x}$

Tangent Line Equations

34. Find the slope of the normal line of $f(x) = \dfrac{x}{x-1}$ at $(2, 2)$.

35. Write an equation of the tangent line of $f(x) = -3x^2 + 5x - 7$ at $x = 1$.

36. State the coordinates of each point(s) at which the graph of the equation $y = \frac{x^3}{3} - \frac{3x^2}{2} + 2x$ has horizontal tangent line(s).

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37. Given $f(x) = \frac{x^2 + x - 2}{x^2 + 3x - 4}$, find:

a) the coordinates of all points of discontinuity,

b) an equation of all vertical asymptotes and

c) an equation of the horizontal asymptotes.